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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

 Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

HERBERT B. ENDERTON. ***A Mathematical Introduction to Logic.*** Harcourt/Academic Press, New York and London, 2001 (Second edition), xii + 317 pp.

This text gives a clearly written, rigorous and detailed presentation of, in the words of the author, "the basic concepts and results of logic: the topics are proofs, truth, and computability" (ix). After over thirty years of wide use in colleges and universities throughout the United States, the first edition has earned the reputation of being the definitive text for a junior-senior level undergraduate course in mathematical logic. The reviewer projects the second edition will continue to hold this status.

The material and its organization remain almost exactly the same as in the first edition. The book is composed of four weighty chapters. Propositional logic is covered in Chapter 1. Chapter 2 gives a thorough introduction to first-order logic, covering truth and models, formal deductions, and the Compactness, Soundness, and Completeness Theorems. Chapter 3 covers number theory and reducts and subtheories of number theory, Gödel's First and Second Incompleteness Theorems, and recursive functions and undecidability. An introduction to second-order logic is given in Chapter 4.

There is greater emphasis on computability throughout this edition, and the section on "Effectiveness and Computability" has been non-trivially expanded. The section on models of theories now includes material on finite models. Indicated are some sections which can be postponed or even omitted. For instance, the section on "Induction and Recursion," which can be difficult for many students, occurs two sections later, and can even be postponed

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further. In addition, there are sections on switching circuits, nonstandard analysis, and representing exponentiation which may be included or omitted from a course, as the instructor wishes.

As in the first edition, the proofs are rigorous and well-written. In addition, the readability has been improved. Many informative details and comments have been added which render the proofs easier to read and more self-contained, reducing the number of times the reader must check references to notation or theorems in other sections. The presentation and motivation also have often been augmented to provide more clarity of purpose and draw more connections between different sections.

The other major change in the second edition is the inclusion of additional exercises. In most sections there are 2–4 new exercises. The additional exercises certainly help students develop a more comprehensive understanding of the material, but still require a certain level of mathematical maturity. Very few of the new or old exercises are purely routine. The additional exercises also provide the instructor with more flexibility in choosing problems corresponding to the level of the students and in choosing which aspects of the material to emphasize.

The reviewer has had two first-hand experiences with this text. The first was as a first-year graduate student using the first edition. The class consisted of three graduate students in mathematics and four undergraduate students—three in computer science and one in mathematics. In one semester we covered up to Section 3.5, Gödel's Incompleteness Theorem. This course was enjoyed by most, and considered extremely difficult by all the undergraduates, and rigorous, but not unduly difficult by the graduate students.

Recently, the reviewer used the second edition to teach a one-semester undergraduate course. The majority of the students were undergraduates with little or no experience in rigorous proofs; only one student was majoring in mathematics. The intended pace of the text was beyond their level, so in order to keep the students learning without exasperating them, the reviewer decided to cover less material and assign most of the homework problems to build understanding. By the end of the semester, we had only covered up to Section 2.5, just touching on the Completeness Theorem. For such a class, the reviewer advises covering Section 1.6, "Switching Circuits," which provides a much needed respite for the students after Section 1.4, "Induction and Recursion." It might also be wise to postpone Section 1.4 until completing Section 1.6.

Given the completeness, rigor, and readability of the text and given the experiences of the reviewer and some colleagues, the reviewer highly recommends this book for a class consisting of students with a medium to high level of mathematical maturity. For instance, it is a great text for senior-level mathematics majors, juniors who have completed a rigorous upper-division course in which proofs play an integral part, and first-year graduate students who have not yet taken a course in mathematical logic. It may also work well for junior-level students with little experience with rigorous proofs, but who are highly motivated and planning to attend graduate school in mathematics. For students not fitting one of these descriptions, one can make this text work, but it requires much planning on the part of the instructor. In this case, a less complete, slower-paced text with more routine problems might serve the students and instructor better.

The reviewer would like to conclude by saying that it has been a pleasure reading the second edition and seeing the improvements that have been made to a text which was already excellent.

Natasha Dobrinen

Department of Mathematics, Pennsylvania State University, University Park, PA 16802, USA. dobrinen@math.psu.edu.

PETER B. ANDREWS. ***An introduction to mathematical logic and type theory: to truth through proof.*** Applied logic series, vol. 27. Kluwer Academic Publishers, Dordrecht, Boston, and London, 2002, xviii + 390 pp.

The first edition of this book was published in 1986, and there is a review of it in The Journal of Symbolic Logic, vol. 53, pp. 312–314. The difference between this new edition and the old is succinctly explained by the author as follows: "The basic structure of the book is unchanged, but improvements and enhancements may be found throughout the text. There is more introductory material and motivation. Historical notes and bibliographical references have been added." And thus, merits are retained and demerits are removed—adds the reviewer, gladly. For a description of the contents, refer to the previous review: it has not changed. Improvements will be mentioned below.

In this edition, the first chapter, Propositional Calculus, does not begin abruptly with a list of symbols, but with examples that explain propositional reasoning. Then follow discussions of connectives and truth tables. After six pages the old "The primitive symbols of P are the following:" appears. The axiomatic system is also introduced after good motivation. This chapter is reworked and is more oriented to computer science than the first edition was. The chapter on first-order logic also begins with introductory material now. It includes exercises of translating ordinary sentences (into first-order). [On local trains in New Jersey, we hear announcements like "All doors will not open at the next station." Passengers are advised to notice the difference between $\sim\forall x$ and $\forall x\sim$.] Also, in this edition, the Gödel sentence "I am not provable" does not appear all of a sudden in its formal form. It is preceeded by discussions of the liar paradox, etc., and followed by 'gossip' about consistency proofs, lengths of proofs and so forth. These additions make the great(est) theorem of logic more accessible and appetizing. The author provides Supplementary Exercises "which are not explicitly tied to any particular section", and so offer a good test of the reader's understanding.

All in all, this new edition is a good improvement over the old, and much more useable for those who don't have a tutor/guru nearby. In the intervening years since the first edition, type theory has received some attention, and now there are some research monographs and survey articles in numerous "Handbook of … ". But books that address students/learners and carry out the details neatly are rare, if not none except for the present one.

MITSURU YASUHARA
66 Pine St., Princeton, NJ 08542, USA. miyapinpri@earthlink.net.

DAVID MARKER. ***Model Theory: An Introduction.*** Graduate Texts in Mathematics, vol. 271. Springer Verlag, New York, Berlin and Heidelberg, 2002, 342 pp.

Model theory has needed a good introductory text for some time. Practicing model theorists know what I mean; a book that would convey to our mathematical colleagues exactly what the deal is with this subject that they see solving open problems in their fields but which remains mysterious. A book that will give strong graduate students a taste of the modern subject, which will contain the essential basics but will be written in context and will discriminate with an eye toward applications. This is that book. Marker has written an elegant introductory text which covers the basic material without falling into the many syntactical traps or adopting any convention which is at odds with the working community. He has included applications and problems which should be of interest to and will challenge the best graduate students, and at the same time will introduce them to modern model theory without unnecessary barriers. The book is not perfect and one can debate whether this subject or that should have been included or could have been done differently but this is immaterial; the book has an internal cohesion and logic which should make it the primary choice for graduate courses in model theory for many years.

The tone is already set in the first chapter where the absolute basics of language and structures must be introduced. The style is to emphasize semantics over syntax and already the problems, which appear at the end of each chapter, are challenging. The reader is forewarned that doing the exercises is not optional for many are referred to in the later text.

In the second chapter, the completeness theorem is discussed briefly but there is no formal discussion of proof systems. Instead, the compactness theorem takes center stage and is proved via a Henkin construction. Concrete non-trivial examples of complete theories are introduced here such as algebraically closed fields of characteristic p (ACF_p) and applications such as Ax's famous proof that an injective map from a variety to itself is surjective is given. The fundamental back-and-forth technique is highlighted and Scott's isomorphism theorem is proved. Once again, strong exercises appear at the end of the chapter including an example of a finitely axiomatizable uncountably categorical theory.

Quantifier elimination is introduced in the third chapter and again, the focus is on practical ways to prove it. The examples chosen are classical, ACF_p and real closed fields (RCF) but again, applications in each case rule the day. In the case of RCF, Marker gives an entirely elementary proof of cell decomposition in real closed fields which could stand in any o-minimal theory (a subject which is discussed in the notes at the end of the chapter).

No model theory book would be complete without a discussion of types and this appears in chapter 4. The omitting types theorem is proved and homogeneous and saturated models are introduced. This is a natural time to discuss two-cardinal models which are needed for later chapters but Marker sneaks in several other things. Differentially closed fields are introduced at this point ostensibly to provide a good example of quantifier elimination via saturated models; a look ahead to the final chapter indicates why this might not be the only reason.

Indiscernibles rounds out the list of basic techniques discussed in this book and again, Marker takes a straight-forward approach not spending too much time on partition theorems. He proves the basics—Ramsey's theorem and the Erdös-Rado theorem—and highlights the applications. The highlighted applications are both challenging: a many models argument for unstable theories and the Paris-Harrington theorem.

The final three chapters can be seen as making a bold attempt to bring the reader to the forefront of modern model theory while at the same time covering all the important material necessary to understand these results. The starting point is Morley's theorem that any countable theory categorical in one uncountable power is categorical in all uncountable powers. A proof is given building on material from early chapters and then the critical definitions of Morley rank and forking are given (the latter only in the context of ω-stable theories). It is when one reaches the seventh chapter that one must realize that the end goal here is to look at Hrushovski's applications of model theory to diophantine geometry. In order to follow the proof, one must understand something about groups interpretable in an ω-stable theory. Hence, the seventh chapter is devoted to such groups. It also contains a very important section on the group configuration, a method of identifying in abstract theories, the presence of a group. The final chapter is the payoff with a discussion of a version of Hrushovski's theorem and even material on Zariski geometries, an abstract characterization of the topology of an algebraic curve.

This is a serious book meant for strong graduate students and working mathematicians with interest in learning model theory. The exercises are an important part of reading this book and the appendices at the end of each chapter provide both a history and a guide to the literature. The book is not without typos but errata can be found on the author's website `www.math.uic.edu/~marker`.

BRADD HART

Department of Mathematics and Statistics, McMaster University, Hamilton ON L8S 4K1, Canada. hartb@mcmaster.ca.

Paraconsistency: the logical way to the inconsistent, edited by Walter A. Carnielli, Marcelo E. Coniglio, and Itala M. Loffredo D'Ottaviano, Marcel Dekker, New York, 2002, xiv + 552 pp.—therein:

Walter A. Carnielli and João Marcos. *A taxonomy of C-systems*. Pp. 1–94.

Bryson Brown. *Paraconsistent classical logic*. Pp. 95–107.

F. G. Asenjo. *The logic of opposition*. Pp. 109–140.

Fred Johnson and Peter W. Woodruff. *Categorical consequence for paraconsistent logic*. Pp. 141–150.

Guido Vanackere. *Ontological causes of inconsistency and a change-adaptive, logical solution*. Pp. 151–165.

Joke Meheus. *An adaptive logic for pragmatic truth*. Pp. 167–185.

Ana Teresa Martins, Marcelino Pequeno, and Tarcísio Pequeno. *A multiple worlds semantics for a paraconsistent nonmonotonic logic*. Pp. 187–211.

Newton C. A. da Costa and Décio Krause. *An inductive annotated logic*. Pp. 213–225.

José Eduardo de A. Moura and Itala M. Loffredo D'Ottaviano. *On NCG_ω: a paraconsistent sequent calculus*. Pp. 227–240.

Robert K. Meyer and John K. Slaney. *A, still adorable*. Pp. 241–260.

Graham Priest. *Fuzzy relevant logic*. Pp. 261–274.

Diderik Batens. *On some remarkable relations between paraconsistent logics, modal logics, and ambiguity logics*. Pp. 275–293.

Shahid Rahman and Jean Paul van Bendegem. *The dialogical dynamics of adaptive paraconsistency*. Pp. 295–322.

Timothy Vermeir. *An inconsistency-adaptive proof procedure for logic programming*. Pp. 323–339.

Grzegorz Malinowski. *Referential and inferential many-valuedness*. Pp. 341–352.

Marcelo Finger. *When is a substructural logic paraconsistent? Structural conditions for paraconsistency in ternary frames*. Pp. 353–367.

R. E. Jennings and D. Sarenac. *Beyond truth(-preservation)*. Pp. 369–380.

Renato A. Lewin and Marta S. Sagastume. *Paraconsistency in Chang's logic with positive and negative truth values*. Pp. 381–396.

Daniele Mundici. *Fault tolerance and Rota-Metropolis cubic logic*. Pp. 397–409.

Guillermo Ortiz Rico. *The annotated logics OP_{BL}*. Pp. 411–433.

Larisa Maksimova. *Definability and interpolation in extensions of Johansson's minimal logic*. Pp. 435–443.

Chris Mortensen. *Towards a mathematics of impossible pictures*. Pp. 445–453.

B. H. Slater. *Ambiguity is not enough*. Pp. 455–463.

Jean-Yves Béziau. *Are paraconsistent negations negations?* Pp. 465–486.

Max Urchs. *On the role of adjunction in para(in)consistent logic*. Pp. 487–499.

Don Faust. *Between consistency and paraconsistency: perspectives from evidence logic*. Pp. 501–510.

Gregory R. Wheeler. *Kinds of inconsistency*. Pp. 511–522.

Jacek Pásniczek. *Paraconsistent logic vs. Meinongian logic*. Pp. 523–534.

Otávio Bueno. *Can a paraconsistent theorist be a logical monist?* Pp. 535–552.

This volume brings together twenty-nine papers that were presented at the Second World Congress on Paraconsistency, held in São Paulo, Brazil in 2000. The research collected here is, in large part, a continuation of the research collected in two earlier volumes: G. Priest, R. Routley, and J. Norman (eds.), ***Paraconsistent Logic: Essays on the Inconsistent*** (Philosophia Verlag 1989) and D. Batens, C. Mortensen, G. Priest, and J. P. van Bendegem (eds.), ***Frontiers of Paraconsistent Logic*** (Research Studies Press 2000).

A *paraconsistent* logic is one which can be used to formalize inconsistent but non-trivial theories. In particular, the principle of *ex contradictione sequitur quodlibet* or *explosion*

$(A, {\sim}A \vdash B)$ cannot hold in a paraconsistent logic. First developed by S. Jaśkowski and N. C. A. da Costa, paraconsistent logics have been proposed for, among other things, drawing reasonable inferences from inconsistent bodies of data, modeling the process of belief revision, capturing the dynamic structure of dialogue, and taming the semantical and set-theoretical paradoxes.

The contents of the book are divided into three parts: "Paraconsistency and Systems of Paraconsistent Logic," "Paraconsistency as Related to Other Logics," and "Philosophical Aspects of Paraconsistency." I will discuss each part in turn, giving a very general overview of the papers therein and discussing one of them in a little depth.

The nine papers in Part I introduce, discuss, compare, and prove facts about various systems of paraconsistent logic. Notably, five of these papers deal with *nonmonotonic* paraconsistent logics, i.e., paraconsistent logics in which a valid inference can be rendered invalid by expanding its premise set. Most of the papers in Part I will be of interest primarily to those already well-immersed in the study of paraconsistent (or perhaps nonmonotonic) logics.

An exception to this last statement is the contribution by Walter Carnielli and João Marcos, *A taxonomy of C-systems*. This long but lively article charts the territory for a very large class of paraconsistent logics. Along the way, it provides a clear and rigorous introduction to many of the concepts central to paraconsistent logics in general, such as the important relations between the principles of *non-contradiction*, *non-triviality*, and *explosion*. A *C-system*, according to the authors, is a paraconsistent logic which preserves the positive fragment of some non-paraconsistent logic (e.g., classical logic), and which contains a consistency operator ($\circ$) or an inconsistency operator ($\bullet$) at the object language level. ('$\circ A$' can be read intuitively as 'A is consistent' or 'A is either completely true or completely false'.) The C-systems are a subclass of those paraconsistent logics in which the principle of *gentle explosion* $(\circ A, A, {\sim}A \vdash B)$ holds. The authors dub such systems 'logics of formal inconsistency', or LFIs. They show that any paraconsistent logic which preserves the positive fragment of classical logic and has a bottom particle (i.e., a formula from which everything follows) can be characterized as an LFI. (This includes, e.g., Jaśkowski's $\mathsf{D_2}$ and D'Ottaviano and da Costa's $\mathsf{J_3}$.) LFIs are arguably superior to certain well-known paraconsistent logics such as Priest's LP ("logic of paradox"), which cannot express the consistency of a formula in the object language. The value of "internalizing" the notion of consistency is that it allows us to preserve *qualified* principles of classical (or intuitionistic) logic—in particular, those involving negation. For example, while disjunctive syllogism $(A \vee B, {\sim}A \vdash B)$ does not hold in any of the C-systems discussed by the authors, a certain *qualified* form of same, namely $\circ A, A \vee B, {\sim}A \vdash B$, holds in all of them.

The twelve papers in Part II explore various combinations of, and convergences between, paraconsistent logics and other kinds of logics (modal, relevant, substructural, many-valued, fuzzy, nonmonotonic, etc.). Like the papers in Part I, these papers are aimed primarily at the professional paraconsistentist, though they may also be of interest to those who specialize in the related fields of logic which are discussed. As in Part I, combinations of paraconsistency and nonmonotonicity are prominent in many of these articles.

Diderik Batens, in his contribution, shows how certain paraconsistent logics and modal logics can be defined within the ambiguity logic AL. Roughly, AL is just a version of classical first-order logic where all non-logical constants and individual variables are indexed to a natural number, and where quantifiers may range over such indices. Thus, for example, $(\forall x)(\exists i)F^1x^2a^i$ is a formula of AL. Batens shows how to define the modal logic $\mathsf{S5}$ within AL. (The converse, he notes, is impossible.) The main idea is that we interpret the indices as referring to *possible worlds*, and define $\Box A$ and $\Diamond A$ as $(\forall i)A^{(i)}$ and $(\exists i)A^{(i)}$, respectively (where $A^{(i)}$ is the result of adding the index i to all letters in A). He goes on to show that first-order LP without identity can be defined within $\mathsf{S5}$ (and thus also within AL), and that first-order LP with identity can be defined within AL (but not within $\mathsf{S5}$). He also provides reason for suspecting that many paraconsistent logics which (unlike LP) include a detachable material conditional are not definable within AL.

The eight papers in Part III explore the philosophical foundations of paraconsistent logics. Some of these papers provide philosophical justifications for certain formal approaches to paraconsistent logic. Others bring out philosophical affinities between paraconsistent logic and other kinds of logic. Otávio Bueno, in his contribution, discusses and clarifies the philosophical position that has come to be known as *logical pluralism*, and argues that the paraconsistent theorist must embrace this position in order to make sense of his or her own practice. Somewhat ironically, two of the papers in Part III (those by Slater and Béziau) express deep skepticism about the very *possibility* of paraconsistent logic.

Jean-Yves Béziau's contribution will be of particular interest to anyone who has wondered whether so-called paraconsistent "logics" are really *logics*. Given that paraconsistent logics differ from non-paraconsistent logics primarily in their treatment of negation, this basically reduces to the question that forms the title of Béziau's paper: *Are paraconsistent negations negations? This* question, in turn, can be reduced to the following two questions: (a) What conditions must a unary operator satisfy in order to be considered a genuine (or "nice") negation? and (b) Are there any *paraconsistent* "negations" that satisfy all (or enough) of these conditions? The author, after taking us on "a guided tour in the land of paraconsistency" during the course of which he highlights some important properties of negations and sketches a rough answer to question (a), answers question (b) with a tentative 'no'. ("But we can still hope," he adds.)

Logicians specializing in paraconsistent logics (especially nonmonotonic varieties) will find much to interest them in Parts I and II. Philosophers of logic will appreciate the bold and insightful articles in Part III. Enthusiastic newcomers to the field will find the collection a useful (and, importantly, up-to-date) starting point for further research. Those with a more casual interest in the subject, who merely want to acquaint themselves with the fundamentals, may find some of the papers in this collection helpful, but are advised to first see Graham Priest's accessible survey of the subject in D. M. Gabbay and F. Guenthner (eds.), ***Handbook of philosophical logic***, 2nd ed., vol. 6 (Kluwer 2002).

CASEY N. MCGINNIS

Department of Philosophy, University of Minnesota, Twin Cities, Minneapolis, MN 55455, USA. mcgin017@umn.edu.

PETER BERNHARD. ***Euler-Diagramme.*** Mentis, Paderborn, 2001, 162 pp.

In recent years there has been growing interest in the logical properties and uses of diagrams, and in the role that visualisation plays in reasoning. This interest is reflected, most notably, in the logical work of Jon Barwise and Eric Hammer, and more widely, in research on reasoning in cognitive psychology. But this interest is not new. Diagrams have always played a role in logic, and that role has to a greater or lesser extent been conceptualised and discussed. In this book Peter Bernhard takes the particular case of Euler diagrams to illustrate the role that diagrams have played in the history of logic, and the varying conceptualisations that they have received. His book makes a fascinating and instructive contribution both to the historical study of logic and to the ongoing philosophical debate about the role of diagrams in reasoning.

The book has eight chapters, the first of which is a six-page introduction and the last of which is a two-page conclusion. Chapter 2 provides an exposition of syllogistic theory, explaining both the method of proof by reduction to the four main first figure syllogisms, by means of which the validity of syllogistic schemata can be demonstrated, and the (medieval) theory of distribution, by means of which the invalidity of syllogistic schemata can be demonstrated. Chapter 3 offers an account of Euler diagrams, showing how they can be used both in proving that a given syllogistic schema is generally valid and in working out what conclusion follows from a given pair of syllogistic premises. Chapter 4 discusses the interpretation and use of Euler diagrams in the history of logic. Bernhard shows how their use accords with (and requires) an extensionalist rather than intensionalist interpretation of syllogistic theory—Euler circles representing the extensions of concepts; and he explains what is involved in their being used to model syllogistic theory. Here what is required is an isomorphism between the diagrams and the corresponding categorical judgements: as Bernhard puts it, "a Euler diagram show s what a categorical judgement says" (p. 72). He elaborates on this by elucidating the similarities and differences between grammatical (linguistic), diagrammatic and graphical (pictorial) representations of categorical judgements. In this fourth chapter, Bernhard also comments on the significance of Euler diagrams in the history of logic. By representing the relations between the extensions of concepts geometrically, he suggests, Euler diagrams allow anyone to simply perceive or read off the validity of a syllogistic schema without having to rely on the authority, say, of Aristotle. This is perhaps particularly useful in the case of the fundamental ('complete') first figure moods, whose validity is the basis on which syllogistic proof proceeds, although attempts were made to ground that validity on the so-called *dictum de omni et nullo*. "Diagrammatisation", Bernhard writes, "was thereby a democratisation of logic" (p. 81), the use of Euler diagrams offering a further example of the Galilean revolution in science as a result of which geometrical figures were employed in representing more than just spatial relations.

Chapter 5 outlines the three traditional problems in the interpretation of syllogistic theory, concerning existential import, the fourth figure, and the subaltern moods. Bernhard here compares the traditional view on these matters with the views of Ockham, Mill and modern logicians. Chapter 6 discusses the inadequacies of Euler diagrams and various strategies for overcoming them. Bernhard here distinguishes between segment-semantic (*segmentsemantisch*) and area-semantic (*flächensemantisch*) approaches. According to the first, additions or modifications to the Euler circles are made to indicate which of the constituent segments carry existential import. The second constitutes the more traditional approach, as systematised on the basis of the five Gergonne relations. Bernhard explains the problems that arise on each approach, but through the use of flow diagrams, and an emphasis on falsification rather than verification (considering the contradictory of a given conclusion), develops the area-semantic approach to provide a decision procedure for the general validity of syllogistic schemata. This procedure, he claims, is simple, intuitive (*anschaulich*) and adequate (pp. 122–3). Although adequate (and indeed, ingenious), however, it did not strike the present reviewer to be quite as simple and intuitive as the more traditional (non-diagrammatic) methods of proof in syllogistic theory. Chapter 7 considers the question of how Euler diagrams might be used or developed for an expanded syllogistic system, covering singular judgements, the indirect moods, and judgements with negative terms, in particular. Bernhard shows, however, that further problems are generated here, though he suggests that Euler diagrams can still serve a heuristic function in the development of new representational systems. Bernhard's focus in this book is on the use of Euler diagrams to exhibit the validity or invalidity of syllogistic schemata in a more intuitive and immediately apprehensible form. But as he shows, as the issues become more complex, as they do, for example, when questions of existential import arise, the use of diagrams becomes progressively more problematic. Bernhard may be right that Euler diagrams can serve a useful heuristic purpose, but at a certain point the disadvantages of diagrams start to outweigh their advantages, and one finds oneself having to rely more and more on traditional symbolic or linguistic representations. Their interpretation too requires more (conceptual) understanding than Bernhard sometimes seems to admit. However, this should not detract from the value of the book. Bernhard offers a fine case-study of the role of diagrams in logic. The book is elegantly organised, beautifully produced, with clear diagrams and tables, and above all, lucidly written. I spotted just one typo (on p. 63), and found only one sentence ambiguous (the third sentence on p. 58). The book exemplifies German-language analytic philosophy at its very best, combining clarity and precision with detailed knowledge of the historical and philosophical literature.

MICHAEL BEANEY
Department of Philosophy, Open University, Milton Keynes, MK7 6AA, England
m.a.beaney@open.ac.uk.

YURII ROGOZHIN. *Small universal Turing machines*. ***Theoretical Computer Science***, vol. 168 (1996), pp. 215–240.

In this paper, the author gives an account of the machines which he constructed in the paper *Sem' universal'nykh machin Tjuringa* (*Seven universal Turing machines*) in *Systemy i teoretichesloie programmirovanie*, ***Matematicheskie Issledovanija***, vol. 69, (1982) pp. 76–90. The paper deals with deterministic Turing machines with one head and one tape which is infinite in both directions.

The machines which are constructed in the paper are universal and they are very small in the size of their program. Let k be the number of letters of the machine program, blank being included. Let n be the number of the states of the machine, possible halting states not taken into account. Then, an $n \times k$ machine is a machine which needs n states and k letters. The paper gives the program of seven machines: 24×2, 10×3, 7×4, 5×5, 4×6, 3×10 and 2×18. Two of them improve the corresponding machines of the 1982-paper which were 11×3 and 2×21 machines.

The paper provides the reader with the explicit programs of the machine and a very detailed description of the encoding of the data. It also indicates very clearly how the machines work.

The paper uses known techniques of previous authors, for instance, M. Minsky and S. Watanabe, in order to simulate Post's tag-systems. Yurii Rogozhin's technique is very clever and it succeeds in combining several ingredients of his predecessors. The paper introduces something new with the machines 5×5 and 4×6. The new idea is a kind of distorted simulation of the given tag-system. We could also call that a simulation "with noise" and the encoding of the author allows him to control the "noise" so that it is still possible to simulate the tag-system computation.

The most recent progress is a machine 19×2 by Claudio Baiocchi, *Three small universal Turing machines*, LNCS 2055, 2001, pp. 1–10.

MAURICE MARGENSTERN
LITA, UFR MIM, Université de Metz, Île du Saulcy, 57 045 Metz Cédex, France.
margens@lita.univ-metz.fr.

WILLIAM MITCHELL, ERNEST SCHIMMERLING, and JOHN STEEL. *The covering lemma up to a Woodin cardinal*. ***Annals of Pure and Applied Logic***, vol. 84 (1997), pp. 219–255.

The paper contains a great pinnacle of inner model theory, the covering lemma for Steel's core model K. In reviewing it I will first discuss some of the relevant background, then discuss the result in the paper, and finally give a very rough index to the proof.

A strong **covering lemma** for a transitive class M asserts that any set X in V can be covered by some set X^* in M of cardinality not more than $\mathrm{card}(X) + \aleph_1$. Of course the covering lemma in the case $M = \mathrm{V}$ is trivial. But the point of covering is that M should be some canonical model, more robust than V and not so easily influenced by forcing.

In one of the most remarkable developments in set theory Jensen showed that either $0^\sharp$ exists, or else the strong covering lemma for L holds true. This can be read as a dichotomy: either $0^\sharp$ exists whence the universe is drastically different from L; or else covering holds for L whence L is a reasonable approximation for the actual universe. The result was pushed further by Dodd–Jensen who isolated a canonical model for which covering holds in the context of large cardinals below a measurable. This is as far as one could possibly expect to go with strong covering. Once measurable cardinals are allowed it is easy to construct counterexamples to strong covering, between V and generic extensions even, using Prikry forcing.

Notice that the strong covering lemma for M immediately implies the following assertion, known as **weak** covering: for every singular limit cardinal κ, the successor of κ in the sense of M is precisely equal to the true successor of κ. Weak covering distills the cardinal essence of strong covering and can be used in many applications as a substitute for strong covering. Moreover weak covering *can* be proved in the presence of measurable cardinals. This was seen by Mitchell who isolated a canonical model for sequences of measures, and proved weak covering for that model.

The first difficulty in proving weak covering in the presence of large cardinals is isolating a rich enough, yet canonical, inner model M. The most general result to date achieves this in cases where a Woodin cardinal can be avoided. The canonical inner model in such cases is the **core model** K of Steel (***The core model iterability problem***, Lecture Notes in Logic 8, Springer, 1996). It is a definable model, canonical in several ways: it cannot be changed by set forcing; it admits an internal structure theory in the spirit of the fine structure theory of L; and it takes in as much of the large cardinal structure of V as possible. This core model is the natural replacement for L in the context of large cardinals, at least below a Woodin cardinal.

The paper under review establishes a variant of weak covering for Steel's core model. Specifically the authors show that K computes the successors of countably closed singular cardinals correctly. (This is a special case of Theorem 1.1 in the paper.) The restriction of countable closure is later eliminated by Mitchell–Schimmerling (*Weak covering without countable closure*, ***Mathematical Research Letters***, vol. 2 (1995), pp. 595–609). Thus it is shown that weak covering holds for K. All this is done under a largeness assumption stipulating the existence of a measurable, and a smallness assumption forbidding the existence of class models with a Woodin cardinal. Some restriction of the latter kind is necessary, since Woodin's work on stationary tower forcing shows that even weak covering (for a definable model invariant under set forcing) cannot hold in the presence of actual Woodin cardinals.

Coverings of various kinds are pivotal in equiconsistency proofs, hence the importance of the paper. Among its many applications the most central involves the combinatorial principle square. Weak covering for K at κ allows reflecting $\square_\kappa$ from K, where it is known to hold, to V. Thus a failure in V of $\square_\kappa$ at a singular cardinal κ requires large cardinals strong enough to violate weak covering for Steel's K. For a precise argument along these lines see Theorem 4.2 of Schimmerling–Steel (*The maximality of the core model*, ***Transactions of the American Mathematical Society***, vol. 351 (1999) pp. 3119–3141).

I want to now say a few words about the proof of weak covering. Fix a singular cardinal κ. Let λ be the successor of κ in K. Suppose for contradiction that λ is not the true successor of κ. Since κ is singular it follows that $\mathrm{cof}^{\mathrm{V}}(\lambda) < \kappa$. Fix then a set X in V which is cofinal in λ and has cardinality less than κ. Enlarging X (but not increasing its cardinality) we may assume that it is an elementary substructure of V_θ for some large regular cardinal θ, with $\kappa, \lambda, \mathrm{K}\restriction\lambda \in X$. Let H be the transitive collapse of X and let $\pi\colon H \to X$ be the anticollapse embedding. Let $\bar{\kappa}$, $\bar{\lambda}$, and $\bar{K}$ be the preimages of κ, λ, and $\mathrm{K}\restriction\lambda$; so that $\pi(\bar{\lambda}) = \lambda$, $\pi(\bar{\kappa}) = \kappa$, and $\pi(\bar{K}) = \mathrm{K}\restriction\lambda$. π is elementary from H to V_θ. The critical point of π is smaller than κ, since X has cardinality less than κ. Most importantly $\pi''\bar{\lambda}$ is cofinal in $\pi(\bar{\lambda})$, in other words π is *continuous* at $\bar{\lambda}$, since X was chosen cofinal in λ.

Let $\kappa_0 = \mathrm{crit}(\pi)$ and let λ_0 be the successor of κ_0 in $\bar{K}$. The fact that $\bar{K}$ embeds into K with critical point κ_0 allows proving that $\bar{K}$ is iterable when backed-up by K with exchange ordinal κ_0. From this it follows that K absorbs $\bar{K} \parallel \lambda_0$. Thus if $\bar{K}$ and K disagree on subsets of κ_0 it must be that K has more of them. The situation therefore separates into the following two cases:

Case 1: If K has more subsets of κ_0 than $\bar{K}$. Let P_0 be the first level of K which defines a subset of κ_0 not present in $\bar{K}$. Let R_0 be the ultrapower of P_0 by the long extender derived from $\pi \restriction \mathcal{P}(\kappa_0)$. This ultrapower makes sense because all the subsets of κ_0 in P_0 belong to $\bar{K}$, and therefore belong to the domain of π. Lemma 3.10 in the paper uses a comparison to show that R_0 is absorbed by K. Now P_0 defines a function from κ_0 cofinally into λ_0. (This is because it defines a subset of κ_0 not present in $\bar{K}$.) So R_0 defines a function from $\pi(\kappa_0)$ cofinally into $\sup \pi''\lambda_0$. From this and the fact that R_0 belongs to K it follows that $\sup \pi''\lambda_0$ is smaller than $(\pi(\kappa_0)^+)^{\mathrm{K}} = \pi(\lambda_0)$. So π is not continuous at λ_0.

Case 2: If K and $\bar{K}$ agree on subsets of κ_0. In this case the long extender derived from $\pi \restriction \mathcal{P}(\kappa_0)$ can be applied to the whole of K. Let R_0 be the resulting ultrapower. Note that R_0 is then a class inner model. Using a comparison argument and the hull and definability properties for K, it is possible to see that the extender leading from K to R_0 in fact belongs to K. (This is a very simple instance of the proof of Lemma 3.11 in the paper.) But that extender is enough to collapse $\sup \pi''\lambda_0$ to $\pi(\kappa_0)$. So again it follows that $\sup \pi''\lambda_0 < (\pi(\kappa_0)^+)^{\mathrm{K}} = \pi(\lambda_0)$, and π is not continuous at λ_0.

The sketch given above is lacking and imprecise in many ways. But its greatest deficiency is that it does not lead to a contradiction. It merely shows that π is not continuous at λ_0. To reach a contradiction the sketch must be iterated, establishing discontinuity not only at the successor of κ_0 but at *all* successors in $\bar{K}$ above κ_0. The contradiction is obtained when the argument reaches $\bar{\lambda}$, where it is known through the original choice of X that π is continuous.

At the heart of this progression is a list of iterability conditions. The starting point is the iterability of $\bar{K}$ when backed-up by K with exchange ordinal κ_0, used above just before case 1. But to progress beyond λ_0 it is necessary (among other things) to slowly increase the exchange ordinal. This is done through an intricate argument involving a whole array of iterability conditions, listed as $(2)_\alpha$ through $(6)_\alpha$ on page 236 of the paper. It is the interplay between these iterability conditions that's the most beautiful aspect of the proof.

ITAY NEEMAN

Department of Mathematics, University of California Los Angeles, Los Angeles, CA 90095-1555, USA. ineeman@math.ucla.edu.