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2002–2003 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Baltimore Convention Center, Baltimore, Maryland
January 17–18, 2003

A Winter Meeting of the Association for Symbolic Logic was held on January 17–18, 2003 at the Baltimore Convention Center in conjunction with the annual Joint Mathematics Meetings. The members of the Program Committee were H. Becker (Chair), C. Laskowski, and R. Solomon. The program included eight invited 50-minute talks and five contributed talks. The ASL hosted a welcoming reception on January 17th in the Annapolis Room of the Hyatt Regency Hotel. In addition, on January 15–16, prior to the ASL Meeting, there was a joint AMS-ASL Special Session, **Interactions between Logic, Group Theory and Computer Science**, organized by A. V. Borovik and A. Myasnikov, and an MAA–ASL Special Session, **The Role of Logic in Learning to Write Proofs**, organized by J. L. Hirst and D. Velleman.

The 50-minute invited addresses were:

Arthur W. Apter (Baruch College of City University of New York), *Some results concerning strong compactness and supercompactness*.

Krzysztof Ciesielski (West Virginia University) *Set theoretical aspects of Fubini's theorem and separate continuity*.

Lou van den Dries (University of Illinois, Urbana-Champaign), *Lower bounds in arithmetic complexity*.

Jeff Hirst (Appalachian State University), *Hindman's Theorem, ultrafilters, and reverse mathematics*.

David Marker (University of Illinois at Chicago), *Model theory and differential algebra*.

Timothy H. McNicholl (University of Dallas), *Automorphisms of the c.e. weak truth-table degrees*.

Rahim Moosa (Massachusetts Institute of Technology), *F-structures and semiabelian varieties over finite fields*.

Andrzej Roslanowski (University of Nebraska at Omaha and University of Northern Iowa), *Proper forcing revisited*.

Bjørn Kjos-Hanssen (Ruprecht-Karls Universität Heidelberg) was unable to attend, and his invited talk, *Initial segments of the Turing degrees with a view toward automorphisms* was cancelled.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
HOWARD BECKER

1079-8986/03/0902-0010/$1.60

Abstracts of invited talks

► ARTHUR W. APTER, *Some results concerning strong compactness and supercompactness*.
Department of Mathematics, Baruch College of CUNY, One Bernard Baruch Way, Box B6-230, New York, New York 10010, USA.
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I will discuss the nature of different universes in which strongly compact and supercompact cardinals can exist, surveying known results, and mentioning future directions for research.

► KRZYSZTOF CHRIS CIESIELSKI, *Set theoretical aspects of Fubini's theorem and separate continuity*.
Department of Mathematics, West Virginia University, Morgantown, WV 26506-6310, USA.
E-mail: K_Cies@math.wvu.edu.

The first, main part of the talk will concentrate on a discussion of assumptions of the following version of the Fubini's theorem.

If f is bounded and measurable on the unit square $[0, 1]^2$ then

$$\int_0^1 \left(\int_0^1 f \, dx \right) dy = \int_0^1 \left(\int_0^1 f \, dy \right) dx. \tag{1}$$

By a 1910 theorem of Lichtenstein, (1) holds for every bounded function f for which the sections f_x and f^y are Reimann integrable, even though such a function f need not to be measurable. Furthermore, it follows from the results of W. Sierpiński, H. Friedman, I. Recław, and P. Zakrzewski that the statement that (1) *holds for every bounded f with Lebesgue measurable sections* is independent of ZFC. In the talk we will discuss the exact condition implying that (1) holds for every bounded function with measurable sections. We will also relate this fact to other similar statements considered earlier in the literature.

The new results presented in this part are based on a joint work with M. Laczkovich.

We will also discuss some classical results concerning separately continuous functions and relate them to the new results and open problems around this subject.

► LOU VAN DEN DRIES, *Lower bounds in arithmetic complexity*.
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To what extent is the Euclidean algorithm optimal with respect to the number of arithmetic operations used? In other words, could there exist algorithms that compute the greatest common divisor of two integers in many fewer arithmetic steps? I will discuss a new method to deal with this and similar problems.

This represents joint work with Y. Moschovakis.

► JEFFRY L. HIRST, *Hindman's Theorem, ultrafilters, and reverse mathematics*.
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Hindman's Theorem [3] is a Ramsey-style result which says that if $f : \mathbb{N} \to 2$, then there is an infinite set S such that f is constant on the set of all sums of finite subsets of S. Even before the theorem was proved, its statement had been linked to the existence of certain ultrafilters on the power set of $\mathbb{N}$ (see [2]). This talk will include an overview of the reverse mathematics and computability theory of Hindman's Theorem [1] and conclude with a preliminary report on adaptations of the ultrafilter connections to a countable setting.

[1] Andreas Blass, Jeffry Hirst, and Stephen Simpson, *Logical analysis of some theorems of combinatorics and topological dynamics*, ***Logic and combinatorics*** (Stephen Simpson, editor), Contemporary Mathematics, vol. 65, American Mathematical Society, 1987, pp. 125–156.

[2] Neil Hindman, *The existence of certain ultrafilters on N and a conjecture of Graham and Rothschild*, ***Proceedings of the American Mathematical Society***, vol. 36 (1972), pp. 341–346.

[3] ———, *Finite sums from sequences within cells of a partition of N*, ***Journal of Combinatorial Theory. Series A***, vol. 17 (1974), pp. 1-11.

► BJØRN KJOS-HANSSEN, *Initial segments of the Turing degrees with a view toward automorphisms*.
Ruprecht-Karls Universität Heidelberg, Mathematisches Institut, Im Neuenheimer Feld 294, D-69120, Heidelberg, Germany.
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We discuss our thesis results on initial segments of the Turing degrees and their consequences. First we present a characterization of the upper semilattice isomorphism types of certain kinds of initial segments of the Turning degrees below a fixed degree above $\mathbf{0}''$. This characterization gives a new proof of the rigidity result of Slaman and Woodin that every (jump-preserving) automorphism of the Turing degrees is equal to the identity on the degrees above $\mathbf{0}''$. We then discuss the possibility of replacing $\mathbf{0}''$ by $\mathbf{0}'$. As a step in this direction, we extend results of Lerman and obtain a characterization of the isomorphism types of those initial segments of the degrees below $\mathbf{0}'$ that happen to be lattices.

[This abstract is presented by title; the talk could not be given.]

► DAVID MARKER, *Model theory and differential algebra*.
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The model theoretic analysis of differentially closed fields leads to a number of fascinating problems about the geometry of Kolchin closed sets. I will survey some of the model theoretic background, recent results and open problems in this area.

► TIMOTHY H. MCNICHOLL, *Automorphisms of the c.e. weak truth-table degrees*.
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We discuss work on constructing a non-trivial automorphism of the *c.e.* weak truth-table degrees.

► RAHIM MOOSA, *F-structures and semiabelian varieties over finite fields*.
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Motivated by the problem of determining the structure of integral points on subvarieties of semiabelian varieties defined over finite fields, we prove a quantifier elimination result for finitely generated modules over certain finite simple extensions of the integers given together with predicates for orbits and cycles of the distinguished generator of the ring.

Joint work with Thomas Scanlon.

► ANDRZEJ ROSLANOWSKI, *Proper forcing revisited*.
Department of Mathematics, University of Northern Iowa, Cedar Falls, IA 50614-0506, USA and Department of Mathematics, University of Nebraska at Omaha, Omaha, NE 68182-0243, USA.
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We will discuss some topics from the past, present and future of properness for iterating forcing notions with uncountable supports.

Abstracts of contributed talks

► GREGORY GILLETTE, *Embedded contextual logic*.
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We show how to embed the propositional calculus in a contextual structure, comprised of two Galois-connected, bounded quasi-orderings. The embedding encompasses two distinct aspects. First, the logical relations are expressed solely in terms of the nonlogical symbols of the structure, viz. the ordering relation and the Galois connection. This is an adaptation of Quine's early logic of inclusion and abstraction, with quasi-ordering playing the role of inclusion and the Galois connection playing the role of abstraction. Second, development rules which produce the standard calculus are extended, to encompass as well the rules governing relations between terms in the quasi-orderings. If, in addition, we restrict the quasi-ordering to atomic lattices, the predicate calculus can also be embedded in the structure. This introduces a single logical symbol which we call an **application**. Applications map formulas to general terms of the structure (not just to the extrema)— and therefore form a generalization of universal/existential quantification. The extended development rules entail the standard predicate calculus, but also include all order relations among general application terms. An interpretation of applications consistent with these rules is presented, by means of the construction of augmented lattices which include virtual as well as real atoms.

► L. YU GLEBSKY, E. I. GORDON, C. W. HENSON, *On approximation of topological algebraic systems by finite ones*.
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We introduce a definition of a topological algebraic system A by systems of a class $\mathcal{K}$ and investigate it for a case when the class $\mathcal{K}$ consists of finite systems. For the case when the topology on the system A is discrete this definition is equivalent to the well-known definition of local embedding of A in $\mathcal{K}$, which means that A is a subsystem of an ultraproduct of some systems in the class $\mathcal{K}$. We obtain a similar characterization of approximation of a locally compact system A by systems in $\mathcal{K}$. We consider the bounded formulas in the signature of A and their approximations similar to those, introduced for Banach spaces in the paper [1]. We prove that a positive bounded formula φ holds in A if all precise enough approximations φ hold in all precise enough approximations of A. We prove that a locally compact field cannot be approximated by finite associative rings (not necessary commutative). Finite approximations of the field $\mathbf{R}$ can be interpreted as computer systems for reals. So it is impossible to construct a computer arithmetic for reals that is an associative ring.

[1] C. W. HENSON, *Nonstantard hulls of Banach spaces*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 108–144.

► JEFFREY SCOTT LEANING, *A report from the trenches: The intersection of mathematical logic and patent law*.
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Most mathematicians believe or have heard that pure mathematics is not patentable subject matter, In fact, the situation is far from clear. The courts have gradually loosened the restrictions on patenting mathematical algorithmic inventions, to the point that in 1999 they questioned the very existence of a prohibition.

To the extent it exists, the bar on mathematical subject matter is particularly focused on "mathematical algorithms." Logicians are at least partially concerned with defining what, exactly, mathematical algorithms are. Therefore, mathematical logic should have some bearing on how the courts decide cases involving whether a particular invention is unpatentable mathematical subject matter.

We will address three issues: (1) what the state of the law is in the U.S. and abroad regarding the legality of patenting mathematical subject matter, (2) what the law should be, given technological understanding, past precedent, and current policy, and (3) what formal (and informal) logical definitions have to add to the discussion. We will present a coherent legal theory that is both consistent with legal precedent (to the extent possible), and takes into account what logicians and mathematicians regard as "mathematics per se."

► JOSEPH S. MILLER, *Degrees of unsolvability of continuous functions*.
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We will show that the Turing degrees are not sufficient to measure the complexity of continuous functions on the reals. Computability of real functions was introduced almost simultaneously by Lacombe [2] and Grzegorczyk [1] and is the subject of ongoing study in Computable Analysis. However, no satisfactory theory of degrees of continuous functions exists. We show that the degrees of continuous functions form a proper extension of the Turing degrees and a proper subclass of the enumeration degrees (or partial degrees).

The key to proving these theorems is the reduction of questions about degrees of continuous functions to questions about sequences of reals. The construction of a sequence of reals which can not be computably diagonalized is used to prove that there are continuous functions without a Turing degree. On the other hand, by showing that any sequence of computable real numbers is computably diagonalizable, we show that there are enumeration degrees which contain no continuous functions. As a corollary, it is shown that every non-computable continuous real function computes a non-computable subset of $\mathbb{N}$. Proofs draw from classical and constructive analysis, as well as from computability theory.

[1] ANDRZEJ GRZEGORCZYK, *Computable functionals*, ***Fundamenta Mathematicae***, vol. 42 (1955), pp. 168–202.

[2] DANIEL LACOMBE, *Extension de la notion de fonction récursive aux fonctions d'une ou plusieurs variables réelles. I*, ***Comptes Rendus de l'Académie des Sciences. Paris***, vol. 240 (1955), pp. 2478–2480.

► PAVEL NAUMOV, *An extension of the classical propositional logic by type constructors*.
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The classical propositional logic is known to be sound and complete with respect to the set semantics that interprets conjunction, disjunction, and negation as intersection, union, and complement correspondingly. We study and extension $\mathcal{S}$ of this logic by new connectives $\otimes$

and $\oplus$ that are interpreted as Cartesian product and disjoint union.

In addition to the standard sequential axioms and inference rules of the classical propositional logic, system $\mathcal{S}$ includes the following two rules:

$$\frac{\{\phi_i\}_{i\in I} \Rightarrow \{\psi_j\}_{j\in J} \qquad \{\chi_i\}_{i\in I} \Rightarrow \{\eta_j\}_{j\in J} \qquad |I| > 0}{\{\phi_i \otimes \chi_i\}_{i\in I} \Rightarrow \{\psi_j \otimes \eta_j\}_{j\in J}}$$

$$\frac{\forall J' \subseteq J(\{\phi_i\}_{i\in I} \Rightarrow \{\psi_j\}_{j\in J'} \text{ or } \{\chi_i\}_{i\in I} \Rightarrow \{\eta_j\}_{j\in J\setminus J'}) \qquad |I| > 0}{\{\phi_i \otimes \chi_i\}_{i\in I} \Rightarrow \{\psi_j \otimes \eta_j\}_{j\in J}}.$$

For any set of atoms $\mathcal{A}$ let $U(\mathcal{A})$ be the set of all terms built from $\mathcal{A}$ using operators *pair*, *inl*, and *inr*. By valuation * over $\mathcal{A}$ we mean an arbitrary mapping of propositional variables into $2^{U(\mathcal{A})}$. Any valuation * could be extended to a mapping from formulas of logic $\mathcal{S}$ into $2^{U(\mathcal{A})}$, based on the described above interpretation of the connectives.

THEOREM 1.

$$\vdash_{\mathcal{S}} \Gamma \Rightarrow \Delta \Longleftrightarrow \forall \mathcal{A} \forall * \bigcap_{\gamma\in\Gamma} \gamma^* \subseteq \bigcup_{\delta\in\Delta} \delta^*.$$

THEOREM 2. *Any provable in $\mathcal{S}$ sequent could be derived in $\mathcal{S}$ without use of the Cut rule.*

COROLLARY 1. *Logic $\mathcal{S}$ is decidable.*

Abstract presented by title

► BILLY JOE LUCAS, *A pragmatically viable alternative to negative omniscience (negative introspection) for the logic of belief*.
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Let ‘iB’ (i > 0) be normal modal belief operators (akin to knowledge operators ‘Ki’ in [1]). Schema R: iB(not(iBp)) $\vee$ iB(iBp). (The basic facts about KR model logics are in [2]; KR frames are characterized: (x) (y) (z) (v) [(xRy and yRz and xRv) implies vRz]). Negative Omniscience (O−): not iBp implies iB(not iBp). Doxastic logics with O−, such as KD45, have substantial pragmatic value since the class of distinct modal functions of a finite number of variables is finite, and (for KD45) the satisfiability problem is NP-complete. Yet O− has less intuitive appeal than other common principles, and could well produce paradox when not restricted to ideas expressible by propositional modal logics. A belief logic in which R replaces O− must be at least as correct about belief as the logic with O−, yet retains many of the pragmatic advantages of O− logics, since KR also proves a modal distribution normal form theorem. Thus, for example, in KDR4 the class of distinct modal functions of a finite number of variables is finite, and all belief modalities reduce to one of eight: iB, not(iBnot), iB(not(iBnot)), not(iB(not(iB))), and their negations. When belief is distinguished from knowledge (a distinction not always properly made in doxastic logic), applications of doxastic logic that use R logics might well produce interesting results that compare favorably with similar results based on O− logics. However, when knowledge (not belief) is the concept under investigation, R logics become O− logics, since KRX proves O− (X: iB(iBp) implies iBp). Should KDR4 be NP-complete (open question), the combined theoretical and pragmatic value of (future) applications of R logics might match (or surpass) those of O− logics.

[1] RONALD FAGIN, JOSEPH HALPERN, YORAM MOSES, and MOSHE VARDI, ***Reasoning about knowledge***, MIT Press, Cambridge, 1995.

[2] BILLY JOE LUCAS, *Systems in which MLA implies LLA and deontic logics with no morally contingent obligations*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 366–367.