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2002–2003 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Philadelphia Marriott Hotel, Philadelphia, Pennsylvania
December 27–30, 2002

A Winter Meeting of the Association for Symbolic Logic was held December 27–30, 2002, in Philadelphia, Pennsylvania, at the Philadelphia Marriott, in conjunction with the annual meeting of the Eastern Division of the American Philosophical Association. The Program Committee consisted of John Burgess, William Ewald, and Scott Weinstein (Chair). The program included a Symposium on Kant's Philosophy of Mathematics chaired by John Burgess, and a Symposium devoted to Perspectives on Logic and the Foundations of Mathematics chaired by William Ewald. Each of the Symposia consisted of three hour-long talks as follows.

Kant's Philosophy of Mathematics.

Michael Friedman (Stanford University), *Kant's philosophy of mathematics in perspective.*

Anja Jauernig (University of Notre Dame), *The many functions of intuition in Kant and what to do with them.*

Lisa Shabel (Ohio State University), *Ostensive construction and diagrammatic reasoning in Kant's theory of geometry.*

Perspectives on Logic and the Foundations of Mathematics.

Donald A. Martin (UCLA), *A Suggestion about the logic of vagueness*.

Yiannis Moschovakis (UCLA), *Intensionality*.

Dana Scott (Carnegie Mellon University), *Social constructivism as a philosophy of mathematics?*

Max Kanovich and Steven Lindell chaired the two sessions of contributed papers.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
SCOTT WEINSTEIN

Abstracts of invited talks in the Symposium on Kant's Philosophy of Mathematics

► MICHAEL FRIEDMAN, *Kant's Philosophy of Mathematics in Perspective*.
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Kant's approach to the philosophy of mathematics is virtually unique in the history of philosophy in that he assigns the capacity for a priori knowledge in this science to the faculty of sensibility rather than the intellect. In this way he arrives at the quite unusual idea of a pure or a priori faculty of sensibility, whose structure is given by the "pure intuitions" of space and time. I explore the variety of factors motivating and sustaining Kant's unique

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conception, including his views on the relationship between pure mathematics and logic, the roles of construction and calculation in geometry and arithmetic, and the relationship between pure mathematics and sense perception (empirical intuition). I conclude with some brief remarks concerning the contemporary relevance of Kant's approach.

► ANJA JAUERNIG, *The many functions of intuition in Kant and what to do with them*.
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It is notoriously difficult to pin down exactly what Kant means by 'intuition' in his critical philosophy in general, and in his philosophy of mathematics in particular. This difficulty is to a large extent due to the fact that there are several distinct, though related functions which intuition is supposed to serve, but which are not sufficiently distinguished by Kant. The first part of this paper tries to clarify the Kantian conception of intuition by identifying and explicitly distinguishing its various functions. The discussion proceeds by looking at some of the pre-critical writings, in which the different roles and functions of intuition are introduced as Kant's transcendental idealism is gradually taking shape. The second part of the paper consists in a critical evaluation of the identified functions from a contemporary point of view, where special emphasis is placed on the functions of intuition allegedly played in mathematics and mathematical reasoning. Is the performance of the function under consideration indeed required and/or part of standard mathematical practice? Does this performance indeed call for a special kind of representation that differs from conceptual representation? The outcome of this evaluation is that for the majority of the considered functions at least one of the two mentioned questions has to be answered in the negative. The paper concludes with a brief discussion of the proposal that one of the functions of Kantian intuition is the provision of quasi-perceptual models for certain mathematical theories, e.g., Euclidean geometry or arithmetic, which proposal is commonly rejected in the more recent literature, both as a reading of Kant and as a view about what intuition can do for us. Contrary to this trend it is suggested that the said function of intuition, if rightly understood, is indeed to be found in Kant, and that with regard to it affirmative answers to both of the above questions seem at least not obviously out of the question.

► LISA SHABEL, *Ostensive construction and diagrammatic reasoning in Kant's theory of geometry*.
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Kant famously claims that our synthetic a priori cognition of mathematical propositions depends upon the construction of magnitudes. Such construction is ostensive when the constructed magnitude is a geometrical object manifestly or directly present to the mind. In this talk, I discuss Kant's account of our cognition of such ostensive constructions and their employment in mathematical reasoning. I argue that Kant's account of such ostensive constructions provides him with a compelling theory of the distinctively diagram-based geometry of the eighteenth century.

Abstracts of invited talks in the Symposium on Perspectives on Logic and the Foundations of Mathematics

► DONALD A. MARTIN, *A suggestion about the logic of vagueness*.
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Three commonly acknowledged desiderata in an account of the logic of vague discourse are the following: (1) to assign the correct meanings to "or," "and," etc., which probably means to give a truth-functional account of these connectives; (2) to allow for so-called *penumbral connections*, precise logical relations holding between vague statements; (3) to avoid introducing sharp boundaries where there should be none. Several known accounts accomplish (1) or (2), but it is doubtful that any of them accomplish both. Perhaps no known account accomplishes (3), lest it be an account that is itself couched in vague language.

The *degree theories* constitute one important class of accounts of the logic of vagueness. Degree theories hold that sentences containing vague terms can take on a large number of truth-values intermediate between truth and falsity. Usually truth-values are taken to range over the unit interval $[0, 1]$. Some degree theories treat sentential connectives truth-functionally; some do not. The former are subject to what seem intuitively to be counterexamples. The latter do not seem to give any real account of the logic of vagueness. All of them have problems with regard to desideratum (3).

Several people have proposed that the unit interval is not the right range for truth-values. I would like to put forward the suggestion that the range of possible truth-values of a vague sentence is a complete Boolean algebra, with different vocabularies usually leading to different algebras. This suggestion, even though accompanied by little or no attempt to characterize the algebras associated with particular vocabularies, does more work than one might think. It accomplishes both (1) and (2), for example. Whether it is compatible with (3) is, however, more problematic.

► DANA S. SCOTT, *Social constructivism as a philosophy of mathematics?*
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No one denies that social constraints, social pressures, and social prejudices influence the development of science, often in very powerful ways. They have done so in the past and certainly do so today. No one denies that to understand mathematics as a scientific activity, it is absolutely necessary to find out what real mathematicians actually do and how they communicate about it. (That is somewhat easier to do for the contemporary scene than it is for past centuries or millennia! Unfortunately there is a lot of made-up history written, or at least misinterpreted history.) But does a catalog of these influences and practices, interesting as they might be, tell us what mathematics is as a subject? It is a debate that both mathematicians and logicians should pursue.

Abstracts of contributed talks

► P. V. ANDREEV AND E. I. GORDON, *On a theory of hyperfinite sets*.
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A new theory in the ϵ-language will be discussed. A natural model for this theory is given by the family of all internal or external subsets of ${}^*V_\omega$—some special nonstandard extension of the set V_ω of all hereditarily finite sets. The necessary nonstandard extensions must satisfy the Nonstandard Classes Theory [1]. The introduced theory is called the Theory of Hyperfinite Sets (THS). It has many features of the P. Vopenka's Alternative Set Theory (AST), but opposite to the AST a huge class of standard mathematics can be interpreted in the THS. For example any locally compact topological space can be constructed as quotient set of a σ-subset of a hypefinite set by some π-equivalence relation. This gives a possibility to formalize a lot of theorems about the locally compact spaces in the THS. Indeed, the THS can be considered in some sense as the general theory of approximation of continuous structures by finite ones.

[1] P. V. ANDREEV and E. I. GORDON, *An axiomatic for nonstandard set theory based on Von Neumann–Bernays–Gödel theory*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1321–1341.

► HIROSHI AOYAMA, *The relation between the dual intuitionistic logic and the quantum logic*.
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G. Takeuti says in his paper [3] "quantum logic is drastically different from the classical logic or the intuitionistic logic." It is, however, unclear how drastically different these logics are from each other. In this talk, I will clarify syntactically the relation among the classical logic, the intuitionistic logic, and the quantum logic (orthomodular logic), using the formalization of sequent calculus.

The dual intuitionistic logic, a kind of paraconsistent logic, plays a crucial role in this talk. The relation between the intuitionistic logic and the dual intuitionistic logic is already known (cf. Urbas [4]). So, I will mainly discuss the relation among the quantum logic, the dual intuitionistic logic, and the classical logic.

[1] N. J. CUTLAND and P. F. GIBBINS, *A regular sequent calculus for quantum logic in which* $\wedge$ *and* $\vee$ *are dual*, ***Logique et Analyse***, vol. 99 (1982), pp. 221–248.

[2] H. NISHIMURA, *Sequential method in quantum logic*, ***The Journal of Symbolic Logic***, vol. 45 (1980), pp. 339–352.

[3] G. TAKEUTI, *Quantum set theory*, ***Current issues in quantum logic*** (E. G. Beltrametti and B. C. van Fraassen, editors), Plenum Press, New York, 1981, pp. 303–322.

[4] I. URBAS, *Dual-intuitionistic logic*, ***Notre Dame Journal of Formal Logic***, vol. 37 (1996), pp. 440–451.

► KATALIN BIMBÓ, *Inequational dual and symmetric combinatory logics*.
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Dual combinators and symmetric λ-calculus were motivated and introduced in [2] and [1]. A dual combinator is similar to a combinator (s. [3]), but applies to its arguments from right to left, while a symmetric combinator applies from both sides. For instance, the permutator $\mathbf{T}$ applied to x and y (in that order) computes to yx (i.e., $\mathbf{T}xy \rhd yx$). The dual and the symmetric analogues ($\mathbf{t}, \overline{\mathbf{T}}$) are characterized by $y(x\mathbf{t}) \rhd xy$, and by the two axioms above (with $\overline{\mathbf{T}}$ in place of $\mathbf{T}$ and $\mathbf{t}$).

Inequality (or reduction) is arguably a better model of computation than equality. We define inequational logics involving dual and symmetric combinators, both non-extensional and extensional ones. These logics do not possess the Church–Rosser property, hence we prove them *consistent* (with arbitrary sets of constants) using a property related to variable occurrences.

We provide *algebraic*, as well as, *set theoretical semantics* (operational and relational ones) for all these logics. Certain modifications of the set theoretical semantics, and the relation between the operational and relational semantics are also investigated. For each pair of a logic and a semantics *soundness* and *completeness* is proven.

[1] F. BARBANERA and S. BERARDI, *A symmetric lambda calculus for classical program extraction*, ***Information and Computation***, vol. 125 (1996), pp. 103–117.

[2] J. M. DUNN and R. K. MEYER, *Combinators and structurally free logic*, ***Logic Journal of the IGPL***, vol. 4 (1997), pp. 505–537.

[3] J. R. HINDLEY and J. P. SELDIN, ***Introduction to combinators and λ-calculus***, Cambridge University Press, Cambridge, 1986.

► OTÁVIO BUENO, *Why the notion of logical consequence cannot by analyzed*.
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In this paper, I discuss a dilemma for the adequacy of any conceptual analysis of logical consequence. Analyses of logical consequence either provide a sharp distinction between logical and non-logical notions, or they don't. If a sharp distinction is not provided, it's not possible to characterize properly the notion of logical consequence—given that, in this case, the latter notion cannot be sharply distinguished from mathematical or physical consequence. If a sharp distinction between logical and non-logical notions is provided, then the account of consequence will end up presupposing the notion of logical consequence—after all, ultimately it is in terms of the latter notion that the distinction is drawn. But in this case, the analysis is clearly circular, since it presupposes the very notion that needs to be characterized. As a result, in either case the analysis would be inadequate. After examining the dilemma, and providing a case for it (both for first- and second-order logics), I discuss the significance of the result for the current debate about the notion of logical consequence (see [1]–[5]).

[1] J. ETCHEMENDY, ***The concept of logical consequence***, Harvard University Press, Cambridge, Mass., 1990.

[2] S. SHAPIRO, *Logical consequence: Models and modality*, ***The Philosophy of Mathematics Today*** (M. Schirn, editor), Clarendon Press, Oxford, 1998, pp. 131–156.

[3] G. SHER, *Did Tarski commit 'Tarski's Fallacy'?*, ***The Journal of Symbolic Logic***, vol. 61 (1996), pp. 653–686.

[4] A. TARSKI, *On the concept of logical consequence*, ***Logic, Semantics, Metamathematics***, Hackett, Indianapolis, 1983, pp. 409–420.

[5] ———, *What are logical notions* (John Corcoran, editor), ***History and Philosophy of Logic***, vol. 7 (1986), pp. 143–154.

► DENNIS F. CUDIA, *Probability and inductive logic*.
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Following *Tractatus* 5.151, for r and s wffs of propositional logic, let $\mathrm{P}(s \mid r)$ denote the *conditional probability of s given r* and define $\mathrm{P}(s \mid r)$ by $\mathrm{T}_{rs} / \mathrm{T}_r$ where $\mathrm{T}_{rs} = \mathrm{P}(s\&r)$ and $\mathrm{T}_r = \mathrm{P}(r)$, the probability of $s\&r$ and r, respectively. Using *Tractatus* 5.152 let p, q be an ordered pair of dependent wffs. Denote the *likelihood table* by LT and define LT by

LT	q	$\sim q$
p	$\mathrm{P}(p \mid q)$	$\mathrm{P}(p \mid \sim q)$
$\sim p$	$\mathrm{P}(\sim p \mid q)$	$\mathrm{P}(\sim p \mid \sim q)$

Note that LT is determined by truth tables. Denote the *joint probability matrix* by JP and define JP by $\mathrm{JP}(1,1) = \mathrm{P}(p\&q)$, $\mathrm{JP}(1,2) = \mathrm{P}(p\&\sim q)$, $\mathrm{JP}(2,1) = \mathrm{P}(\sim p\&q)$, $\mathrm{P}(2,2) = \mathrm{P}(\sim p\&\sim q)$. Denote a diagonal matrix by $\mathrm{diag}(u, v)$. Let $c = \mathrm{P}(q)$ and $d = \mathrm{P}(p)$ and let $\mathrm{C} = \mathrm{diag}(c, 1-c)$, and $\mathrm{D} = \mathrm{diag}(d, 1-d)$. Denote the *dependence ratio* by R and define R by $\mathrm{R} = \mathrm{D}^{-1} \cdot \mathrm{JP} \cdot \mathrm{C}^{-1}$ and the *posterior probability* denoted by PV satisfies $\mathrm{PV} = \mathrm{R} \cdot \mathrm{C}$ and $\mathrm{JP} = \mathrm{LT} \cdot \mathrm{C} = \mathrm{D} \cdot \mathrm{PV}$ where all multiplications are matrix multiplications. The goal of inductive logic is to reverse LT and using Bayes' Theorem this is done by PV. Note that c is the *prior probability* of the cause q and d is the probability of the positive effect p. It is assumed that both c and d are neither zero nor one.

► FERNANDO FERREIRA, *Amending Frege's* Grundgesetze der Arithmetik.
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More than a century ago, Bertrand Russell found an inconsistency in Frege's *Grundgesetze der Arithmetik*. The formal system of the *Grundgesetze* is, except for minor differences, second-order logic together with an abstraction operator governed by Frege's Axiom V. A few years ago, Richard Heck showed that the predicative and the ramified predicative second-order fragments of the *Grundgesetze* are consistent. In other words: On a predicative reading, we may coherently assume that every concept has an extension. It is therefore pertinent to ask how much arithmetic (and analysis) can be developed within Heck's systems. Unfortunately, there are good reasons to believe that primitive recursive arithmetic is not interpretable in Heck's systems.

We show that Heck's ramified predicative fragment augmented with the axiom of reducibility for concepts true of only finitely many individuals is still consistent, and that elementary Peano arithmetic (in fact, the second-order theory of arithmetic **ACA**) is interpretable in this extended system.

[1] RICHARD HECK, *The consistency of predicative fragments of Frege's* Grundgesetze, ***History and Philosophy of Logic***, vol. 17 (1996), pp. 209–220.

► BILLY JOE LUCAS, *A deontic, doxastic variation on Fitch's theorem*.
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Were you to claim a right to believe any truth, and even to claim that the truth of p proves you have the right to believe p, need you be irrational? Let 'uB' be a monotonic modal belief operator, and 'P' a normal deontic permission operator.

DEFINITION. Negative Infallibility: uB(not uBp) proves not uBp.

Negative Infallibility is thus a rationality constraint on what you believe (we never credit you with believing you lack some belief except when you do in fact lack it).

DEFINITION. Right to Believe Truth: p proves P(uBp).

DEFINITION. Omniscience for u: p proves uBp.

By a variation on the proof of Fitch's theorem, Negative Infallibility and Right to Believe Truth prove that you are omniscient. (The proof does not assume that ought implies permission or that you believe any theorems of logic.) It follows that you can consistently maintain your right to believe truth only if you already believe all truths or if you exhibit at least one of two kinds of doxastic irrationality: you believe you lack a belief you do not lack, or, you believe a conjunction but do not believe its left conjunct.

► FRED SEYMOUR MICHAEL, *Intuitionistic mathematics and relevant logic*.
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From the outset, it has been characteristic of intuitionistic logic that it rejects the laws of the excluded middle and double negation. Yet this is required only of constructive proofs in intuitionistic mathematics and these proofs can be embedded in a logic that admits both LEM and DN. The idea is to introduce an intuitionistic truth predicate, represented by, say, 'T*' that holds of a proposition p iff a constructive proof of p is available. Then LEM and DN hold both with respect to p and to T*p. The logic needs no special intuitionistic axioms and logical connectives need be given no special intuitionistic sense.

What is distinctive of intuitionism, so understood, is that T*p cannot be inferred from p. Now if the values of p are the two classical truth values, then it can readily be seen that T*p is not a truth function of p; and if the values of p are intuitionistic truth, falsity and neither truth nor falsity, and entailment is seen as just preservation of intuitionistic truth, then p would entail T*p. So entailment cannot just be truth preservation, even preservation of intuitionistic truth. What we need is a generalized notion of entailment, according to which p entails q iff there is no valuation on which the value of p is greater than the value of q. The logic in which intuitionistic mathematics is embedded is then not classical, but will be shown to be a variant of the relevant system E. The separation of mathematics and logic effected by this approach to intuitionism lends support to Brouwer's view of the independence of mathematics from logic.

► G. MINTS AND T. ZHANG, *A proof of topological completeness for* $S4$ *in* $(0, 1)$.
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The completeness of the modal logic $S4$ for all topological spaces as well as for the real line $\mathbb{R}$, $\mathbb{R}^n$ and the segments $(0, 1)$ etc. (with $\Box$ interpreted as interior) was proved by McKinsey and Tarski [2]. Simplified proofs in [1], Section 5 and [4], Chapter 9 contain gaps. A new proof presented here combines the ideas in [3], [4] and [1] and provides a further simplification.

Let $\mathcal{B}$ be the set of all infinite binary sequences $\vec{b} = b_1 b_2 \dots (b_i \in \{0, 1\})$ except the identical 0^ω and the sequences ending in a tail of 1's; let $\mathcal{B}_1 = \{\vec{b} \in \mathcal{B} \mid (\exists i)(\forall j > i)\vec{b}(j) = 0\}$, $\mathcal{B}_2 = \mathcal{B} \setminus \mathcal{B}_1$. A one-to-one correspondence between $\mathcal{B}$ and real interval $(0, 1)$ is given by $\mathbf{real}(\vec{b}) = \sum_{i=1}^{\infty} \vec{b}(i)2^{-i}$. Define $B(x) =$ the unique $\vec{b} \in \mathcal{B}$ such that $\mathbf{real}(\vec{b}) = x$. Let $\mathbf{K} = \langle W, R\rangle$ be a finite Kripke $S4$-model with root w_0. Let $\mathcal{K}$ be the topological space with the carrier W and the base of open sets $\mathcal{O}_w = \{w' \in W \mid Rww'\}$ for all $w \in W$. Define a labeling $\mathcal{W}$ of finite binary sequences $\overline{b} = b_1 b_2 \dots b_n$, $n \geq 1$ by worlds $w \in W$ as follows. $\mathcal{W}(\emptyset) = w_0$; If $\mathcal{W}(\overline{b}) = w$, no extension of $\overline{b}$ is yet labeled and $w, w_1, \dots, w_m$ are all R-successors of $w \in W$, then let $\mathcal{W}(\overline{b}0^i) = w$ for all $0 < i \leq 2m$ $\mathcal{W}(\overline{b}0^{2i-1}1) = w_i$ for $0 < i \leq m$, and $\mathcal{W}(\overline{b}0^{2i}1) = w$ for $0 \leq i < m$. For $\vec{b} \in \mathcal{B}$ let $\vec{b} \restriction n = \vec{b}(1)\vec{b}(2)\dots\vec{b}(n)$, $\lambda(\vec{b}) =$ the least $n \geq 1$ $[(\forall i, j \geq n)R\mathcal{W}(\vec{b} \restriction i)\mathcal{W}(\vec{b} \restriction j)]$, $\rho(\vec{b}) = max(1, n)$, if $\vec{b} = \vec{b}(1)\vec{b}(2)\dots\vec{b}(n)10^\omega \in \mathcal{B}_1$; $\rho(\vec{b}) = \lambda(\vec{b})$, if $\vec{b} \in \mathcal{B}_2$, and $\pi(x) = \mathcal{W}(B(x) \restriction \rho(B(x)))$ for $x \in (0, 1)$.

THEOREM 1. *The function* $\pi\colon (0, 1) \to \mathcal{K}$ *is a continuous and open mapping.*

PROOF SKETCH. Define $\delta(\vec{b}) = max(1, n)$, if $\vec{b} = \vec{b}(1)\vec{b}(2)\dots\vec{b}(n)10^\omega \in \mathcal{B}_1$; $\delta(\vec{b}) =$ the least $n > \lambda(\vec{b})(\vec{b}(n) = 1$ and $\vec{b}(n-1) = 0)$, if $\vec{b} \in \mathcal{B}_2$. Then for any $x, y \in (0, 1)$, $|x - y| < 2^{-(\delta(B(x))+2)}$ implies $R\pi(x)\pi(y)$. Hence π is continuous. On the other hand for every $x \in (0, 1)$, $w \in W$ with $R\pi(x)w$ and any $\epsilon > 0$, there exists $y \in (0, 1)$ such that $|y - x| < \epsilon$ and $\pi(y) = w$ Hence π is open. ⊣

Since it is well-known that the class of finite rooted Kripke models is complete for $S4$, we have

COROLLARY 2. $S4$ *is complete for topological semantics in* $(0, 1)$.

[1] MARCO AIELLO, JOHAN VAN BENTHEM, and GURAM BEZHANISHVILI, *Reasoning about space: the modal way*, ***Technical Report PP-2001-18***, ILLC, 2001.

[2] J. C. C. MCKINSEY and ALFRED TARSKI, *The algebra of topology*, ***Annals of Mathematics***, vol. 45 (1944), no. 1, pp. 141–191.

[3] GRIGORI MINTS, *A completeness proof for propositional $S4$ in Cantor space*, ***Logic at work: Essays dedicated to the memory of Helena Rasiowa*** (Ewa Orlowska, editor), Physica-Verlag, Heidelberg, 1999, pp. 79–88.

[4] ———, *A short introduction to intuitionistic logic*, Kluwer Academic-Plenum Publishers, 2000.

► SATOKO TITANI AND HARUHIKO KOZAWA, *Quantale valued set theory*.
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The formulation of quantum physics in terms of lattice theory was first introduced by Birkhoff and von Neumann [1]. Projections on a Hilbert space H form a complete orthomodular lattice $P(H)$ called *quantum logic*. G. Takeuti developed a quantum set theory on $P(H)$-valued universe $V^{P(H)}$ in [3] and [4]. He showed that self-adjoint operators on H, interpreted as observables in quantum theory, are represented as real numbers in $V^{P(H)}$. C. J. Mulvey and J. W. Pelletier proposed in [2] to use quantales for studying the foundations of quantum logic and for studying noncommutative C^*-algebras of Hilbert space operators. A quantale is a complete lattice equipped with an associative binary operation & distributive over arbitrary joins. Lattice $Q(P(H))$ of sup-preserving endomorphisms on $P(H)$ is a quantale with & composition of endomorphisms, and quantum logic $P(H)$ is embedded in the quantale $Q(P(H))$.

We consider quantale $Q(P(H))$ as a logic with basic implication $\to$, and develop a set theory on quantale valued universe $V^{Q(P(H))}$, where logical operator & is interpreted as "and then". Quantum set theory on $V^{P(H)}$ are interpreted in the quantale valued set theory. Observables in quantum theory are represented by real numbers defined in the quantale valued set theory. Propositions are represented as idenpotent and self-adjoint real numbers. Introduction of & enables us to define multiplication of real numbers in $V^{Q(P(H))}$.

[1] G. BIRKHOFF and J. VON NEUMANN, *The logic of quantum mechanics*, ***Annals of Mathematics***, vol. 37 (1936), pp. 823.

[2] C. J. MULVEY and J. WICK PELLETIER, *Quantization of the calculus of relations*, ***Canadian Mathematical Society, Conference Proceeding***, vol. 13, 1992.

[3] G. TAKEUTI, ***Two applications of logic to mathematics***, Iwanami and Princeton University Press, Tokyo and Princeton, 1978.

[4] ———, ***Current issues in quantum logic***, (E. Beltrametti and B. C. van Frassen, editors) Plenum, New York, 1981, pp. 303–322.

Abstracts presented by title

► M. J. CRESSWELL, *Situation semantics for intuitionist logic*.
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In standard modellings of intuitionistic logic disjunction is usually understood as classical, while negation is understood non-classically. In this paper I want to look at another way of presenting the semantics of intuitionistic logic. I call it 'situation semantics', because one way of looking at situations is as 'partial' or 'incomplete' indices—and one way of understanding an incomplete index is to think of it in the sense of a proposition, where a proposition is thought of as a class of possible worlds. Without imposing restrictions on which sets of worlds count as 'acceptable' all you get is classical logic. I state restrictions which lead to intuitionistic logic.