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NEW PROGRAMS AND OPEN PROBLEMS IN THE FOUNDATION OF MATHEMATICS

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This special issue of the Bulletin of Symbolic Logic contains the refereed versions of some of the lectures delivered at the conference "New programs and open problems in the foundation of mathematics and of its applications, in the year 2000" held at the École Normale Supérieure, Paris, November 13 and 14, 2000. The talks were organized as follows:

1. J.-Y. Girard (CNRS, Marseille): "Locus solum"
   Respondent: P.-L. Curien (CNRS-Univ. Paris VII)
2. G. Parisi (Univ. Roma I): "Six spaces looking for a geometer"
   Respondent: J.-P. Nadal (CNRS-ENS, Paris)
3. A. Macintyre (Univ. Edinburgh): "Prospects in logic"
   Respondent: A. Marcja (Univ. Firenze)
4. R. Milner (Univ. Cambridge): "The flux of computation"
   Respondent: G. Berry (Ecole des Mines, Sophia-Antipolis)
5. F. W. Lawvere (SUNY, Buffalo): "Dialectical foundations of, by, and for mathematics"
   Respondent: A. Kock (Aarhus Univ.)
6. A. Connes (College de France, Paris): "Non-commutative Geometry"
   Respondent: C. Soul (IHES, Paris);
7. Panel discussion: "Geometric Structures in Logic, Physics and Computing" with the invitees and the members of the working group "Géométrie et Cognition".

The conference and these proceedings witness the vitality of foundational research and the renewal of well-established paradigms from the past century.

The geometric structuring of proofs is at the core of Girard's approach. Both in his paper and in the applications to computing presented in Curien's paper, some key regularities originating in spatial phenomena (symmetries, connectivities which allow interactions, various forms of duality ...) organize deduction and root it in our cognitive experience of space.

Parisi's paper describes a familiar theme (since the time of Riemann): the modern connections of Geometry to Physics. In particular, he surveys spaces commonly used by physicists yet lacking rigorous mathematical foundations.

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Such spaces lie at the heart of recent geometric research in Quantum Field Theory and statistical mechanics. As might be expected, symmetries (and "symmetry-breaking") play a key rôle.

Macintyre describes the progressive shift of Model Theory from the traditional set-theoretic approach to a more geometric or category-theoretic framework. This arises from renewed interest of model theorists in algebraic geometry and the work of Grothendieck. The use of sheaf-theoretic and functorial tools, largely based on adjunctions and dualities, are at the core of these new foundational approaches.

Similarly, the geometrical/categorical mode of analysis is applied by Lawvere to sketch a foundation of analysis based on continuum physics, and also to interpret some original insights of Cantor in a way that suggests a conceptual solution to the continuum problem. Kock's paper gives a specific example of this in synthetic geometry, which compels us to view Mathematics as an interaction of categories, and meaning as given by this interaction. According to this view, formal consistency says nothing or little about "existence" of mathematical objects or structures.

Unfortunately, A. Connes, R. Milner and a few more respondents could not send us their contributions. Yet, their talks also stressed the foundational role of the organization of space in Physics and Computing. In the former, the structure of "access" to space (quantum measurement) turns out to be the key foundational issue (how do we "act on" and measure physical phenomena?). But also in concurrent computing, the key and novel issue is the spatial distribution of computations, in contrast to spaceless sequential computations (and deductions).

We warmly thank the authors and the referees for their work and all the speakers for the success of the Conference.

The Conference was dedicated to H. Poincaré, 100 years after his remarks on the infeasibility of the reduction of the foundation of Mathematics to the "potential mechanization" of deduction and his stress on "meaning and structures" also in the analysis of proofs. And to H. Weyl, in the 50th anniversary of his lectures on Symmetry, the swan song, as he says, of a lifelong understanding of the foundation of Mathematics as an integral part of a Philosophy of Natural Sciences and of their interactions.

The Conference was organized by the "Atelier de Recherche" Géométrie et Cognition and financed by the "Action Cognitique" of the French Ministry of Research (see `http://www.di.ens.fr/users/longo/geocogni.html`).

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