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2002 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '02

Münster, Germany, August 3–9, 2002

Logic Colloquium'02, the 2002 European Summer Meeting of the Association for Symbolic Logic, took place on August 3–9, 2002, at the Institut für Mathematische Logik und Grundlagenforschung of the Westfälische Wilhelms-Universität in Münster (Germany). The conference was organized as a collaborative project of logicians in Münster and Bonn. It was mainly funded by the Association for Symbolic Logic, the Deutsche Forschungsgemeinschaft (DFG, reference number 4851/50/02), the Westfälische Wilhelms-Universität, and the Deutsche Vereinigung für Mathematische Logik und für Grundlagen der Exakten Wissenschaften (DVMLG). Travel Grants for students and researchers from Eastern European countries were funded by the Association for Symbolic Logic, the Deutsche Forschungsgemeinschaft (reference number 436 114/104/02), and the National Science Foundation.

The Program Committee consisted of K. Ambos-Spies, S. Buss, Z. Chatzidakis, A. Kechris (Chair), P. Koepke, P. Komjath, M. Lerman, V. McGee, W. Pohlers, M. Rathjen, K. Segerberg, and B. Zilber. The Organizing Committee consisted of M. Burghardt, J. Diller, P. Koepke, B. Löwe, M. Möllerfeld, W. Pohlers (Chair) and A. Weiermann. In addition to the official Organizing Committee members, there were a number of helpers from Bonn and Münster.

The program included four tutorials, fourteen invited plenary lectures, and twenty-two talks in special sessions. The topics of the special sessions were (with organizers in parentheses): *Computability Theory* (S. Lempp, M. Lerman, and A. Sorbi); *Non-monotonic logic* (K. Schlechta and K. Segerberg); and, *Set theory* (A. Andretta and S. Friedman). Logic Colloquium 2002 had almost 200 participants from thirty countries representing the continents of Asia, Australia, Europe, and North and South America. Altogether, 107 talks were given, 5 posters were presented, and 17 additional papers were presented by title.
The following tutorials were presented:

Lev D. Beklemishev (Utrecht), *Reflection principles and provability algebras*.
Stephen Cook (Toronto), *Resolution Theorem Proving*.
Olivier Lessmann (Oxford), *Homogeneous model theory*.
Simon Thomas (Piscataway, NJ), *Borel equivalence relations and classification problems for countable structures*.

The following invited plenary lectures were given:

Jeremy Avigad (Pittsburgh), *Forcing in proof theory*.
Arnold Beckmann (Vienna), *Bounded arithmetic, propositional proof complexity and dynamic ordinals*.
Tim Carlson (Columbus), *Patterns of resemblance and upper bounds in ordinal analysis*.
Robert L. Constable (Ithaca), *Logical foundations for a digital library of formal mathematics*.
Kosta Došen (Belgrade), *Identity of Proofs*.

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Moti Gitik (Tel Aviv), *Some aspects of the singular cardinals problem*.

Volker Halbach (Konstanz), *Paradox and possible worlds*.

Bakhadyr Khoussainov (Auckland), *Computably enumerable algebras, abstract data types, and algebraic specifications*.

Steffen Lempp (Heidelberg/Madison WI), *Trivial, strongly minimal models and computable model theory*.

Toniann Pitassi (Toronto), *The complexity of weak pigeonhole principles*.

Thomas Scanlon (Berkeley), *Model theory of enriched valued fields*.

Ralf Schindler (Vienna), *Transfer theorems*.

Patrick Speissegger (Madison WI), *O-minimality for Quasianalytic Denjoy-Carleman classes*.

Katrin Tent (Würzburg), *BN-pairs in model theory*.

Directly after the Logic Colloquium, the biannual German logic meeting **Colloquium Logicum** (organized by the DVMLG) took place on August 9–11, 2002 in the same locale as a satellite meeting of the ASL meeting. It was coordinated by J. Diller, P. Koepke, B. Löwe, W. Pohlers (Chair of the Organizing & Scientific Committee), C. Thiel, W. Thomas, and A. Weiermann. The meeting featured the following invited talks: T. Arai (Kobe), *Epsilon substitution methods for impredicative theories*; J. Bagaria (Barcelona), *The Absoluteness of $\boldsymbol{L}(\mathbb{R})$*; A. Nies (Chicago), *Effectively closed sets of reals and randomness*; M. Otto (Swansea), *Invariances, games and models*; C. Parsons (Cambridge MA), *The concept of object in mathematics*; A. Pillay (Urbana-Champaign), *Model theory of compact complex manifolds*; M. Rathjen (Leeds) *Metamathematical properties of constructive Zermelo-Fraenkel Set Theory*; and, J. van Benthem (Amsterdam/Stanford CA), *What logical systems do: from reasoning to communication*.

The proceedings of both conferences, edited by Z. Chatzidakis, P. Koepke, and W. Pohlers, will contain many of the invited talks and will appear in the book series, *Lecture Notes in Logic*.

For more information on the meeting, the reader is referred to the conference webpage, `http://www.math.uni-muenster.de/LC2002/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
ALEKOS KECHRIS

Abstracts of invited tutorials

▶ LEV D. BEKLEMISHEV, *Reflection principles and provability algebras*.

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In the study of formal theories one is often interested in questions of their (relative) consistency, conservativity and characterizations of the corresponding classes of provably total computable functions. Various approaches to such questions, proof-theoretic as well as model-theoretic, are known.

The main focus of this tutorial is to present an new approach of a more algebraic flavour to proof-theoretic analysis. It based on the notion of *graded provability algebra*, that is, the Lindenbaum boolean algebra of a formal theory enriched by additional operators which allow the structure to capture proof-theoretic (essentially syntax-sensitive) information. We use this method to analyze Peano arithmetic and show how an ordinal notation system up to ϵ_0 can be recovered from the corresponding algebra in a canonical way. This provides a new short proof of old results of Gentzen, Ackermann and Kreisel. Because of its technical clarity, we hope that it will also be of interest for non-prooftheorists.

This method is based on the work which has been done in the last two decades on provability logic and reflection principles. We survey some results in this area and present other applications of these techniques in formal arithmetic and its fragments.

► STEPHEN COOK, *How to associate propositional proof systems and theories with complexity classes*.
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Many propositional proof systems are naturally associated with complexity classes. For example, the lines in a "Frege" proof are propositional formulas, and a problem in the associated complexity class $\mathbf{NC}^1$ is defined by a polynomial size family of propositional formulas. For bounded-depth Frege systems the corresponding class is $\mathbf{AC}^0$, and for Extended Frege systems the class is $\mathbf{P}$ (polynomial time). Each such proof system S and complexity class C is also associated with a logical theory T, where the Σ_1-definable functions in T are C-functions, and where the Π_1 theorems of T translate into tautology families with polynomial size S proofs.

We present in detail the second-order theories V^0 and V^1 (where V^1 corresponds to Buss's first-order theory S^1_2) and demonstrate their relations with complexity classes $\mathbf{AC}^0$ and $\mathbf{P}$, and with appropriate propositional proof systems. The main proof-theoretic tool used is free-cut-free proofs in Gentzen's system **LK**. We present other examples of such triples $\langle S, C, T\rangle$, and present interesting combinatorial principles which are candidates for separating various theories and their associated proof systems.

[1] STEPHEN COOK, *Proof complexity and bounded arithmetic*, Course notes for CSC 2429H, Department of Computer Science, University of Toronto, Winter, 2002.
www.cs.toronto.edu/~sacook/csc2429h/

► OLIVIER LESSMANN, *Homogeneous model theory*.
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Initiated in the early 1970s, homogeneous model theory is concerned with the class of (elementary) submodels of a large homogeneous — rather than saturated — universal domain. This context is quite general and includes much of current model theory; the first order case, Robinson theories, existentially closed models, Banach space model theory, many generic constructions, various logics with amalgamation (n-variable, infinitary), as well as concrete examples like Hilbert spaces and some operator algebras. At this level of generality, the compactness theorem fails. However, homogeneous model theory is still well-behaved; the overall picture is similar to the first order case, and the reliance on set-theoretic methods minimal. In these three lectures, I will survey various aspects of dimension theory inside a homogeneous model: (1) Categoricity; Under the assumption of uncountable categoricity, all models are prime and minimal over a type-definable pregeometry whose dimension determines the isomorphism-type of the model. (2) Stability, superstability, and the Main Gap; I will discuss these natural extensions of the categorical case, where more than one type-definable pregeometry is at work. (3) Simplicity and the theory of dividing; Dividing is a dependence relation defined inside any homogeneous model using its group of automorphisms — in a vector space it becomes linear dependence, in an algebraically closed field, algebraic dependence, and non-orthogonality in a Hilbert space. In simplicity, one assumes that dividing satisfies some basic properties (here local character, extension, and finite character) and derives the others (symmetry, transitivity, amalgamation for Lascar strong types).

Important examples will also be described. These talks will not assume any prior knowledge in model theory.

► SIMON THOMAS, *Borel equivalence relations and classification problems for countable structures*.
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Suppose that $\mathcal{L}$ is a countable first-order language and that $\mathcal{K}$ is the class of countable models of some $\mathcal{L}_{\omega_1,\omega}$-sentence. In this tutorial, following Friedman-Stanley and Hjorth-Kechris, I will explain how to use the theory of Borel equivalence relations to analyze the isomorphism relation on $\mathcal{K}$ and develop a framework for measuring the complexity of possible complete invariants.

Throughout, I will concentrate on classes $\mathcal{K}$ of structures which are "finitely generated" in some broad sense, such as the class of finitely generated groups, the class of torsion-free abelian groups of finite rank, the class of fields of finite transcendence dimension, etc. As we shall see, these are the classes $\mathcal{K}$ whose isomorphism relations correspond to countable Borel equivalence relations; i.e., Borel equivalence relations E such that every E-equivalence class is countable. The tutorial will include an account of the recently discovered connections between countable Borel equivalence relations and Zimmer's superrigidity theory, as well as discussions of some of the many easily stated open problems in this very exciting area.

Abstracts of invited plenary talks

► JEREMY AVIGAD, *Forcing in proof theory*.
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Paul Cohen's method of forcing, together with Saul Kripke's related semantics for modal and intuitionistic logic, has had profound effects on a number of branches of mathematical logic, from set theory and model theory to constructive and categorical logic. In this talk, I will argue that forcing also has a place in traditional Hilbert-style proof theory, where the goal is to formalize portions of ordinary mathematics in restricted axiomatic theories, and study those theories in constructive or combinatorial terms. I will discuss the aspects of forcing that are useful in this respect, as well as applications, including constructivizing model-theoretic arguments, translating classical theories to constructive ones, and obtaining conservation results for subsystems of second-order arithmetic.

► ARNOLD BECKMANN, *Bounded arithmetic, propositional proof complexity and dynamic ordinals*.
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Bounded arithmetic theories, as introduced by Sam Buss [2], are sub-theories of Peano arithmetic which are closely related to computational complexity classes like the polynomial hierarchy. Many questions about complexity classes, e.g., "Does the polynomial hierarchy collapse?", have a natural correspondence in terms of bounded arithmetic theories, like "Does the hierarchy of bounded arithmetic theories collapse, i.e., do higher level bounded arithmetic theories prove the same sentences as lower level?".

Cook and Reckhow [3] have observed that proving superpolynomial lower bounds for all feasible proof systems for TAUT is equivalent to separate **NP** from **coNP**. This has been the starting point of an area of *propositional proof complexity* where one tries to prove lower bounds for all kinds of propositional proof systems.

Proof-theoretic ordinals measure the amount of order-types of well-orderings that can be recognized by a theory. They often give good characterizations of provability, consistency, and computability strength of theories. *Dynamic ordinals* [1] are an adaption of proof-theoretic ordinals to bounded arithmetic.

We will survey the role of dynamic ordinals in bounded arithmetic, the relationship of bounded arithmetic to propositional proof complexity, and how this relation can be used to determine the provability strength of bounded arithmetic theories in terms of dynamic ordinals.

[1] ARNOLD BECKMANN, *Dynamic ordinal analysis*, submitted for publication, 2001.

[2] SAMUEL R. BUSS, ***Bounded arithmetic***, Stud. Proof Theory, Lecture Notes, vol. 3, Bibliopolis, 1986.

[3] STEPHEN A. COOK and ROBERT A. RECKHOW, *The relative efficiency of propositional proof systems*, ***The Journal of Symbolic Logic***, vol. 44 (1979), pp. 36–50.

▶ TIM CARLSON, *Patterns of resemblance and upper bounds in ordinal analysis*.
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Patterns of resemblance are finite structures derived from the manner in which degrees of absoluteness arise between levels in a continuous hierarchy of structures. They provide an alternative to ordinal notations generated by collections of operations. We will discuss the extent to which patterns of resemblance have been used to calculate ordinals of theories of sets.

▶ ROBERT L. CONSTABLE, *Logical foundations for a digital library of formal mathematics*.
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A world-wide community of logicians, computer scientists and information scientists is creating a distributed on-line digital library of formalized mathematics. The process is loosely coordinated. It is driven by the increasing use of automated theorem provers and interactive proof development systems. These provers are employed in a variety of ways, say in designing and validating software systems. A major by-product of this work is the creation of on-line libraries of mathematics that contain hundreds of definitions and theorems of general interest in addition to the hundreds of facts about specific artifacts.

This talk will illustrate the activity and frame new logical problems that arise from it. One is the problem of consistently combining results from different theories, another is the problem of accounting for truth in the context of an applied proof technology in which programs that build parts of proofs are considered as part of the proof object. A consistency theorem is established for one such mechanism — tactic-tree proofs.

▶ KOSTA DOŠEN, *Identity of Proofs*.
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Some thirty years ago, two proposals were made concerning criteria for identity of proofs. Prawitz proposed to analyze identity of proofs in terms of the equivalence relation based on reduction to normal form in natural deduction. Lambek worked on a normalization proposal analogous to Prawitz's, based on reduction to cut-free form in sequent systems, but he also suggested to understand identity of proofs in terms of an equivalence relation based on generality, two derivations having the same generality if after generalizing maximally the rules involved in them they yield the same premises and conclusions up to renaming of variables. These two proposals proved to be extensionally equivalent only for limited fragments of logic.

The normalization proposal stands behind very successful applications of the typed lambda calculus and of category theory in the proof theory of intuitionistic logic. In classical logic, however, it didn't fare well.

The generality proposal was rather neglected in logic, though related matters were much studied in pure category theory in connection with coherence problems, and there are also links to low-dimensional topology and linear algebra. This proposal seems more promissing than the other one for the general proof theory of classical logic.

▶ MOTI GITIK, *Some aspects of the singular cardinals problem*.
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We will discuss some recent results concerning the problem. In particular determining the least singular cardinal which power can be made arbitrary large and its strength.

▶ VOLKER HALBACH, *Paradox and possible worlds*.
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We look at the semantical paradoxes and several limitative results in metamathematics (Liar paradox, Montague's Theorem [4], Löb's Theorem, McGee's theorem [3] on the ω-inconsistency of truth theories) from the viewpoint of possible-worlds semantics.

It is our aim to trace the common source of all these paradoxes and to exhibit the limits of syntactical treatments of $\Box$ as a predicate.

Let $\mathcal{L}$ be the language of arithmetic and $\mathcal{L}_\Box$ its expansion by the new unary *predicate* symbol $\Box$. If S is a set of sentences of $\mathcal{L}_\Box$, we write $(\boldsymbol{N}, S) \models A$ iff $A \in \mathcal{L}_\Box$ holds in the standard model $\boldsymbol{N}$ of arithmetic and if $\Box$ is interpreted by S.

A frame (W, R) is a pair with W a nonempty set and R a binary relation on W. The arithmetical vocabulary is interpreted by the standard model at any world. A valuation V assigns a suitable extension of $\Box$ to any world. More precisely, a function V assigning to every $w \in W$ a set $V(w)$ of $\mathcal{L}_\Box$-sentences is a valuation iff $V(w) = \{A\colon \forall v(wRv \Rightarrow (\boldsymbol{N}, V(v)) \models A\}$. Here $[A]$ is the numeral of the code of the sentence A. Therefore $\Box[A]$ holds at w (i.e., $(\boldsymbol{N}, V(w)) \models \Box[A]$) iff A holds at all worlds v accessible to w.

Since, on this approach, $\Box$ is treated as a predicate rather than an operator of sentences and therefore diagonalization is available, there is no recursive definition of validity as in operator modal logic and there are frames for which no valuation can be found. In particular, the paradoxes translate into limitative results on the existence of valuations. According to Montague's Theorem, for instance, the scheme $\Box[A] \to A$ (for all sentences $A \in \mathcal{L}_\Box$) and necessitation are inconsistent, which in turn implies that reflexive frames do not admit valuations.

We shall report among others the following results obtained jointly with Hannes Leitgeb and Philip Welch [1, 2]: partial characterizations of the class of frames that admit valuations, non-uniqueness of valuations, completeness theorems for the possible-worlds semantics for predicates, and new proofs, e.g., of McGee's ω-inconsistency via possible-worlds semantics.

[1] Volker Halbach, Hannes Leitgeb, and Philip Welch, *Possible worlds semantics for modal notions conceived as predicates*, submitted.

[2] ———, *Possible worlds semantics for predicates*, ***Intensionality*** (Reinhard Kahle, editor), Lecture Notes in Logic, to appear.

[3] Vann McGee, *How truthlike can a predicate be? a negative result*, ***Journal of Philosophical Logic***, vol. 14 (1985), pp. 399–410.

[4] Richard Montague, *Syntactic treatments of modality, with corollaries on reflexion principles and finite axiomatizability*, ***Acta Philosophica Fennica***, vol. 16 (1963), pp. 153–167.

▶ BAKHADYR KHOUSSAINOV, *Computably enumerable algebras, abstract data types, and algebraic specifications*.
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Basic components of a program consist of declarations of variables, definitions of classes, methods, and their interactions. On an abstract level, the program defines the structure consisting of the set of objects and operations on the set. From a programming point of view, this structure is the abstract data type (ADT) implemented by the program. From an algebraic point of view, this structure is an algebra. A specification of an ADT is a formula of a formal system that defines the ADT in a precise sense. Based on the thesis that ADTs are finitely generated c.e. algebras, in this talk we discuss some issues related to specifications of ADTs. In particular, we adress a long standing open problem of Jan Bergtsra and John Tucker about specifiability of ADTs by equations and conditional equations. The talk includes foundational aspects of the area and presents recent algebraic and computability-theoretic techniques developed towards the study of the problem. The work is joint with S.Goncharov from Novosibirsk University.

▶ STEFFEN LEMPP, *Trivial, strongly minimal models and computable model theory*.
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A trivial, strongly minimal theory is model complete after expansion by constants, naming all elements of one model of the theory. This result arose from and answers a question in computable model theory: If a trivial, strongly minimal theory has a computable model, then the theory, and thus all its countable models, are $\mathbf{0}''$-decidable. We also obtain a bound on the complexity of the spectrum of computable models of a trivial, strongly minimal theory.

This is a joint result with Goncharov, Harizanov, Laskowski, and McCoy.

▶ TONIANN PITASSI, *The complexity of weak pigeonhole principles*.
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The propositional pigeonhole principle asserts that m pigeons cannot be placed in n holes with at most one pigeon per hole whenever m is larger than n. It is an exceptionally simple fact that underlies many theorems in mathematics, and is the most important combinatorial principle in proof complexity. It can be formalized as a propositional formula, denoted PHP_n^m. A major achievement of proof complexity was to prove exponential lower bounds for Resolution and bounded-depth Frege proofs of PHP_n^m when $m = n + 1$.

The proof complexity of the pigeonhole principle depends in a crucial way on the number of pigeons, m, as a function of the number of holes, n. As m increases, the principle becomes weaker (easier to prove) and in turn the proof complexity question becomes more difficult. In the case of larger m (say $m = 2n$), the complexity of PHP_n^m has deep connections with other longstanding problems. First, lower bounds for bounded-depth Frege proofs of the weak pigeonhole principle suffice to show unprovability results for the $\boldsymbol{P}$ versus $\boldsymbol{NP}$ statement. Secondly, the long-standing question of whether or not the existence of infinitely many primes has an $I\Delta_0$ proof is closely related to the complexity of WPHP. Thirdly, the question is closely related to the complexity of approximate counting.

In this talk, we first review the rich and interleaved lower and upper bounds that have been recently proven for the weak pigeonhole principle in various natural proof settings. This will include the new lower bounds for Resolution proofs of the WPHP, as well as new upper bounds on bounded-depth Frege proofs of the WPHP. We will try to give intuition for these results, and mention several key open problems in this area.

Secondly, we present a new lower bound for bounded-depth Frege proofs of the weak pigeonhole principle whenever $m \leq n + n/\operatorname{polylog} n$. More precisely, we show that given integers c and h such that c is sufficiently large compared to h, there exists an integer $a > 1$ such that any depth-h proof of PHP_n^m, where $m \leq n + n/\log^c n$, requires size $2^{\log^a n}$. This is a substantial improvement over previous lower bounds. Our proof technique applies a switching lemma to a weaker tautology based on certain bipartite graphs, where both the tautologies themselves and the restrictions we consider remain random throughout most of the argument.

The result presented in the second part of the talk is joint work with Josh Buresh-Oppenheim, Paul Beame, Ran Raz and Ashish Sabharwal.

▶ THOMAS SCANLON, *Model theory of enriched valued fields*.
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We survey some recent work on the model theory of valued fields given with extra structure.

▶ RALF SCHINDLER, *Transfer theorems*.
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We take "transfer theorem" to mean a result of the form $\alpha \Rightarrow \beta$ which is proven by showing $\alpha \Rightarrow \gamma$ as well as $\gamma \Rightarrow \beta$ for some γ. We shall focus on the case in which γ is a statement of inner model theory, β is a statement of descriptive set theory, and α is — for instance — a statement of cardinal arithmetic. This leads us to survey some of the central methods, results, problems, and obstacles of contemporary inner model theory.

▶ PATRICK SPEISSEGGER, *O-minimality for Quasianalytic Denjoy-Carleman classes*.
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We show that the expansion of the real field by the functions of a quasianalytic Denjoy-Carleman class is model complete and o-minimal, provided that the class satisfies certain closure conditions. Some of these structures do not admit analytic cell decomposition, and they show that there is no largest o-minimal expansion of the real field.

This is joint work with J.-P. Rolin and A. J. Wilkie.

▶ KATRIN TENT, *BN-pairs in model theory*.
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I will survey recent results concerning the Cherlin-Zilber Conjecture that any simple group of finite Morley rank is an algebraic group over an algebraically closed field. In particular, I will explain the use of Tits-buildings in this context and will give a new application of model theory to affine buildings and differential geometry.

Abstracts of invited talks in the Special Session on Computability Theory

▶ VASCO BRATTKA, *Computable subsets of Banach spaces and their application in computable functional analysis*.
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We introduce some natural definitions of r.e. and recursive subsets of metric spaces and we demonstrate how these notions can be applied in computable functional analysis. The open mapping theorem, Banach's inverse mapping theorem and the closed graph theorem are basic theorems of functional analysis whose classical proofs rely on the Baire category theorem. Therefore these theorems count as non-constructive. We discuss several computable and partially computable versions of these theorems which cannot be expressed in the same way in constructive analysis. For instance, the effective version of Banach's inverse mapping theorem states that on Banach spaces the inverse T^{-1} of any computable linear and bijective operator T is computable as well, whereas the inversion mapping $T \mapsto T^{-1}$ itself is computable only in the finite dimensional case. As an application of the effective version of the theorem we discuss the initial value problem of linear differential equations.

► ELIAS F. COMBARRO, *Automorphisms of the universal computable function*.
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We study the symmetry of the universal computable function by means of the group of automorphisms of its graph, that is, the group of permutations f such that

$$\varphi_n(x) = y \iff \varphi_{f(n)}(f(x)) = f(y)$$

where φ_n is the nth computable function.

We prove that all these functions are computable. In fact, we show that all the homomorphisms of the universal computable function (defined in the obvious way) are computable.

We also study some functions strongly related to the automorphisms of the universal computable function, including the permutations f such that

$$x \in W_n \iff f(x) \in W_{f(n)}$$

where W_n is the domain of φ_n.

► ISKANDER KALIMULLLIN, *Definability of the jump operator in the enumeration degrees*.
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We show that the jump operator in the enumeration degrees is order-theoretic definable in the enumeration degrees. This solves a question posed by S. B. Cooper.

Namely, for all e-degrees u and x the following are equivalent:

1. $x \geq u'$,
2. $x \geq u$ and for all e-degrees $a > u$, $b > u$ and $c > u$ if each of pairs (a, b) and (a, c) is u-e-ideal, then $b \cup x = c \cup x$.

(We say that a pair of e-degrees (a, b) is u-e-ideal iff every e-degree $z \geq u$ is the greatest lower bound of e-degrees $a \cup z$ and $b \cup z$.)

Thus, the class of e-degrees above u' is definable by an $\forall\exists\forall$-formula (in the language of ordering).

Furthermore, we show that for all e-degrees u and x the following are equivalent:

1. $x \leq u'$,
2. there exists a triple of e-degrees $a > u$, $b > u$ and $c > u$, such that each of pairs (a, b), (a, c), (b, c) is u-e-ideal, and $x \leq a \cup b \cup c$.

Thus, the class of e-degrees below u' is definable by an $\exists\forall\exists$-formula (in the language of ordering).

Hence, the map $u \mapsto u'$ is definable in the e-degrees by a formula which is a conjunction of an $\forall\exists\forall$-formula and an $\exists\forall\exists$-formula. Also, by Friedberg Completeness Criterion we can define the class of all total e-degrees above $\boldsymbol{0}'$ as the image of the jump operator.

▶ CHARLES MCCOY, JULIA KNIGHT, STEFFEN LEMPP, AND REED SOLOMON, *Higher-level categoricity in Boolean algebras*.
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We examine issues of effectiveness, particularly those related to categoricity, in Boolean algebras. We are especially interested in establishing structural characterizations.

[This abstract is presented by title; the talk could not be given].

▶ TIMOTHY MCNICHOLL, *Automorphisms of the computably enumerable wtt degrees*.
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We will discuss current work on constructing a non-trivial automorphism of the computably enumerable weak truth-table degrees.

▶ WOLFRAM MENZEL AND FRANK STEPHAN, *Inductive versus approximative learning*.
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It is investigated to which extent it is possible to combine concepts like Ex and BC from inductive inference with constraints on the probably approximate correctness for intermediate hypotheses. While in the case of BC-learning, adding this constraint does not restrict learning at all, this is different in the case of Ex-learning: the constraints on the intermediate hypotheses weaken the learning power significantly and enforce reliability. The exact type of restriction depends on the nature of the constraints. The main result is that a class S has an Ex-learner which at the same time approximates all functions (including the non-recursive ones) iff the class S is Ex-learnable by a weakly consistent learner. Here, roughly spoken, a learner is weakly consistent if it outputs for every x only hypotheses correct at x after having seen all values $f(y)$ with y below some bound b_x depending on x.

▶ IVAN SOSKOV, *Degree Spectra and Co-spectra of structures*.
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Given an abstract structure A we define the Degree Spectrum of A to be the set of all enumeration degrees generated by the presentations of A on the natural numbers. The Co-spectrum of A consists of all lower bounds of the Degree Spectrum. We show that the elements of the Degree Spectrum are related to those of the Co-spectrum like the degrees of a cone based on some degree **a** are related to the degrees less than or equal to **a**.

▶ SEBASTIAAN TERWIJN, *Computably enumerable reals and presentations*.
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We study the relationship between a computably enumerable real and its presentations. A set A presents a computably enumerable real α if A is a computably enumerable prefix-free set of strings such that $\alpha = \sum_{\sigma\in A} 2^{-|\sigma|}$. Note that $\sum_{\sigma\in A} 2^{-|\sigma|}$ is precisely the measure of the set of reals that have a string in A as an initial segment. So we will simply abbreviate $\sum_{\sigma\in A} 2^{-|\sigma|}$ by $\mu(A)$. It is known that whenever A so presents α then $A \leq_{\mathrm{wtt}} \alpha$, where $\leq_{\mathrm{wtt}}$ denotes weak truth table reducibility, and that the wtt-degrees of presentations form an ideal $\mathcal{I}(\alpha)$ in the computably enumerable wtt-degrees. We prove that any such ideal is Σ^0_3, and conversely that if $\mathcal{I}$ is any nonempty Σ^0_3 ideal in the computably enumerable wtt-degrees then there is a computable enumerable real α such that $\mathcal{I} = \mathcal{I}(\alpha)$. We also prove a kind of Rice Theorem for these ideals, namely that if such a Σ^0_3 ideal is not empty or equal to ω then it is Σ^0_3-complete.

This is joint work with Rod Downey.

Abstracts of invited talks in the Special Session on Non Monotonic Logic

► NICK ASHER, *Nonmonotonic logic and its applications*.
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Nonmonotonic logic, discovered by computer scientists who were worrying about formalizing common sense knowledge, is now rapidly becoming a tool in formal semantics and philosophy. I'll review some of the recent developments in that area and argue that these constitute useful data for those looking for appropriate formalizations of common sense knowledge.

[This abstract is presented by title; the talk could not be given].

► ALEXANDER BOCHMAN, *Nonmonotonic inference and belief change. New faces of the coin*.
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The notion of an epistemic state is suggested (in Bochman 2001) as a general semantic framework covering both nonmonotonic reasoning and belief change. An epistemic state is defined as a triple (S, L, R) consisting of a set of admissible states S ordered by a preference relation R, and a labeling function L assigning each admissible state a deductively closed theory. Nonmonotonic reasoning and belief change are considered, respectively, as static and dynamic (constructive) kinds of reasoning in epistemic states.

For nonmonotonic reasoning, epistemic states allow to define a number of nonmonotonic inference relations, including traditional sceptical ones, different kinds of credulous (brave) inference, and contraction inference (corresponding to contractions in the theory of belief change). Completeness results are established for most of them.

For belief change, epistemic states allow to give an exact description of the main belief change operations, such as contraction, merge, expansion and revision. The theory covers both non-prioritized and prioritized belief change. The suggested operations preserve categorial matching, and hence create a framework of a fully iterative theory of belief change. The theory could be viewed also as a constructive theory of epistemic states, since it determines, in effect, the ways of constructing epistemic states based on a sequence of information inputs.

[1] A. Bochman, *A logical theory of nonmonotonic inference and belief change*, (***Artificial Intelligence*** series) Springer, 2001.

▶ DOV GABBAY, *Interpolation by translation*.
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Interpolation is a central notion in the traditional study of logical systems. In recent years many non-classical logics have found increasing use in AI and computer science. In such applications, interpolation has special practical significance. This has put pressure on researchers in the area of logic and computation to provide algorithms for finding interpolants for various logics, in a way that the majority of applied consumers of logic will find familiar, and which can use existing automated deduction tools available in the community. This talk describes a package for interpolation by translation. Interpolation by translation is part of a program entitled "Property π by translation". Take any property π of a non-classical logic which is well studied in classical logic (such as Revision, Interpolation, Abduction, etc.), and define and investigate π for the non-classical logic by translation. We apply our package to quantified modal S5 and propositiola modal S4.3.

Sample Theorems:

1. An extension of quantified S5 has interpolation if and only if it is capable of naming its possible worlds uniquely.
2. An extension of propositional K4.3 has interpolation if and only if it is functionally fully expressive.

▶ DANIEL LEHMANN, *Connectives in nonmonotonic logics*.
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In [2], three main classes of nonmonotonic inferences were defined: cumulative, loop-cumulative and preferential. The first two can easily be characterized by properties that do not involve connectives. A similar characterization of cumulative operations was proposed in [3].

Possible semantics and connectives for those classes are studied. Preferential inferences support classical connectives.

An original family of nonmonotonic inferences stemming from Quantum Logic has been defined by [1]. These inferences are loop-cumulative. The proof-theoretic properties of the connectives generally associated with Quantum Logic are considered.

[1] K. Engesser and D. M. Gabbay, *Quantum Logic, Hilbert space, Revision Theory*, ***Artificial Intelligence***, vol. 136, no. 1, pp. 61–100, March 2002.

[2] S. Kraus, D. Lehmann, and M. Magidor, *Nonmonotonic reasoning, preferential models and cumulative logics*, ***Artificial Intelligence***, vol. 44, nos. 1–2, pp. 167–207, July 1990.

[3] D. Lehmann, *Nonmonotonic logics and semantics*, ***Journal of Logic and Computation***, vol. 11, no. 2, (2001), pp. 229–256.

▶ DAVID MAKINSON, *Bridges between classical and nonmonotonic logic*.
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Those coming to nomonotonic reasoning from a background in classical logic often find the subject rather perplexing. The purpose of this talk is to show how some of the mystery may be taken out of the subject for the student and newcomer. Nonmonotonic reasoning arises naturally out of classical logic, provided certain misunderstandings are avoided and an old habit is put aside.

As we show, there are logics that act as natural bridges between classical consequence and the main systems of nonmonotonic inference to be found in the literature. Like the latter, these bridge logics are supraclassical, but like classical logic they are perfectly monotonic. Moreover, as well as providing easy passage from one to the other, they have an interest of their own.

► RICHMOND THOMASON, *Nonmonotonic reasoning about agents' attitudes*.
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This talk is concerned with the problem of formalizing beliefs for communities with many agents, in a way that provides an adequate account of the reasoning that is required in order to form reasonable hypotheses about other agents' beliefs. (The theory should apply to other attitudes as well, but belief illustrates the problem well.)

A belief is *mutual* for agents a and b in case a and b both have the belief, and also the belief that they each have the belief, and also the belief that they each have the belief that they each have the belief, and so forth. Achieving something like mutuality provides a major challenge to the of formalization task. Mutuality is crucial for microeconomics, the theory of communication protocols, and the analysis of human communication, but I know of no plausible account of how it can be achieved or approximated. (In the talk I will criticize the most important proposal, which goes back to David Lewis, who used mutuality in his account of social conventions.)

The theory I present has two original features. (1) It uses multimodal logic to formalize the beliefs of a *single* agent. The enriched structure that this logic imposes makes it much easier to formulate appropriate hypotheses about portions of other agents' beliefs. (2) It is based on nonmonotonic logic, and rules of the sort "If my belief has these features then this person will share the belief" are nonmonotonic. This enables us to write plausible rules about belief sharing, and to reason about the exceptions when they occur.

The main technical results have to do with cases in which beliefs about mutuality can be obtained from finite, plausible sets of hypotheses.

► RENATA WASSERMANN, *Generalized change and the meaning of rationality postulates*.
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The standard theory of belief revision was developed to describe how a rational agent should change his beliefs in the presence of new information. Many interesting tools were created, but the concept of rationality was usually assumed to be related to classical logics.

In this talk, we explore the fact that the logical tools used can be extended to other sorts of logics, to describe models that are closer to the rationality of a real agent. This is proved in [1].

[1] Sven Ove Hansson and Renata Wassermann, *Local Change*, Research Report PP-1999-17 in the ILLC Series (to appear in ***Studia Logica***)

Abstracts of invited talks in the Special Session on Set Theory

► DAVID ASPERÓ, *Generic absoluteness for H_{ω_2} and the continuum problem*.
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Let Γ be the class of all posets $\mathbb{P}$ such that every stationary subset of ω_1 remains stationary after forcing with $\mathbb{P}$. Bounded Martin's Maximum (BMM) can be characterized as the statement that for every Σ_1 formula $\varphi(x)$ and every $a \in H_{\omega_2}$,

$$H_{\omega_2} \models \varphi(a) \text{ iff there is some } \mathbb{P} \in \Gamma \text{ such that } \Vdash_{\mathbb{P}} H_{\omega_2} \models \varphi(a)$$

Given this natural characterization, due to J. Bagaria, a central question in set theory is whether BMM—or some other nice principle of generic absoluteness involving only parameters in H_{ω_2}—decides the size of the continuum.

In this talk, I will present proofs of the following recent developments, due to myself, on this question:

THEOREM 1. BMM *implies that the dominating number is* $\aleph_2$.

Given a formula $\varphi(x)$, say that $\varphi(x)$ is persistent under forcing in Γ if, whenever $\varphi(x)$ is true (for some a), $\varphi(a)$ remains true after forcing with any poset $\mathbb{P}$ in Γ.

THEOREM 2. *Suppose that the following strengthening of* BMM *holds*:
If $\varphi(x)$ *is a* Σ_2 *formula such that '*$H_{\omega_2} \models \varphi(x)$*' is persistent under forcing in* Γ *and* $a \in H_{\omega_2}$, *then* $H_{\omega_2} \models \varphi(a)$ *iff there is some* $\mathbb{P} \in \Gamma$ *such that* $\Vdash_{\mathbb{P}} H_{\omega_2} \models \varphi(a)$.
Then, $2^{\aleph_0} = \aleph_2$. *Actually there is a well-order of* H_{ω_2} *which is* Σ_2 *definable over* H_{ω_2}.

The proof of Theorem 2 uses Theorem 1 and Todorčević's oscillation theory as used in the proof that PFA implies that the bounding number is equal to the size of the continuum.

► DOUGLAS BURKE, *Splitting stationary sets*.
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A subset S of $\mathcal{P}(X)$ (the set of all subsets of X) is stationary iff for all functions f from finite sequences from X to X, there is an element m in S such that f applied to any finite sequence from m is in m. In this talk we discuss the problem of splitting stationary sets into disjoint (or almost disjoint) stationary subsets. For example, if we define crit(S) to be the least k such that the collection of m in S with m intersect k in k is stationary (this is also the least critical point in the associated generic embedding), then S splits into at least k many disjoint stationary sets.

[This abstract is presented by title; the talk could not be given].

► ILIJAS FARAH, *Some consequences of the Proper Forcing Axiom*.
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I am going to discuss some consequences of PFA.

► SU GAO, *Actions of the unitary group*.
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We analyze the complexity of various equivalence relations induced by Borel actions of the unitary group, that is, the group of unitary transformations of an infinite dimensional separable complex Hilbert space. Among the most interesting actions are the ones realizing the universal equivalence relations and those arising from functional analysis and operator theory.

► JAMES HIRSCHORN, *Random gaps*.
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Previous investigations into the "random gap" problem are completed, by proving that a destructible (ω_1, ω_1) gap in $\mathcal{P}(\mathbb{N})$ / Fin can always be constructed from $\aleph_1$ co-generic random reals. It is also proved that in this situation, there exists an indestructible (ω_1, ω_1) gap which is not equivalent to a gap with the Hausdorff property.

▶ PAUL LARSON, *Iterations of countable models*.
Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada.
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We discuss iterated embeddings of countable models of ZFC by generic ultrapowers derived from the nonstationary ideal on ω_1. We show that for each such model and each real number there is a unique shortest iteration of the model picking up the given real, if any. We also discuss some open problems in this area.

▶ RENÉ SCHIPPERUS, *Some simple games on cardinals*.
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We look at a pick and chose game played on an infinite cardinal. One player choses a finite set and then the players alternatively remove and add elements to and from the set. The goal of the remove player is to have as few stable elements as possible, and his strategies can depend only upon the set at the current stage. How well can he play?

These games were formulated by Boris Model, in a context outside of set theory.

Our work is joint with Uri Abraham.

Abstracts of contributed talks

▶ ZOFIA ADAMOWICZ, *A version of bounded arithmetic*.
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For every integer n we define a theory $\mathsf{Sk}(\Sigma_n^b)$ which is related to S_n^2 and is a finitely axiomatizable fragment of S_2. We prove that $\mathsf{Sk}(\Sigma_n^b)$ proves the counterpart of $\mathsf{Sk}(\Sigma_{n+1}^b)$, where the cut of $\log s$ is replaced by the cut of $\log\log s$. This is an analogue of the $\forall\Sigma_{n+1}^b$ conservativity of $\mathsf{s}\Sigma_{n+1}^b\mathsf{LLIND}$ (in Beckmann's notation – [1]) over S_2^n. Conservativity is strengthened in our case to provability.

[1] ARNOLD BECKMANN, *Dynamic ordinal analysis*, submitted.

▶ PAVEL ALAEV, *Computable homogeneous Boolean algebras*.
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We propose a criterion saying when a homogeneous Boolean algebra has a computable copy. Andrei Morozov proved that such an algebra can be described by an invariant which is a subset of the set of natural numbers.

We introduce a hierarchy of Δ_ω^0-sets which generalizes Feiner's one. A countable homogeneous Boolean algebra has a computable copy if and only if its invariant belongs to a class of this hierarchy.

▶ MOHAMMAD ARDESHIR AND FARZAD DIDEHVAR, *Rapid elimination of quantifiers*.
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B. Poizat based on the work of Blum, Shub, Smale extends the concept of computations to a concept of computationa usual complexity of models. His aim is, among other things, to find a structure which satisfies the generalizad form of problem $\boldsymbol{P} = \boldsymbol{NP}$. The relation between rapid elimination of quantifiers in a language and the generalized $\boldsymbol{P} = \boldsymbol{NP}$ problem is elaborated in [1]. In this talk we answer this question positively.

[1] BRUNO POIZAT, ***Les petits cailloux, Une approche modèle-théorique de l'algorithmie***, Nur al-Mantiq wal-Ma'rifah [Light of Logic and Knowledge], volume 3, Aléas, Lyon, 1995

▶ JASON AUBREY, *The unsplitting number and its relatives*.
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The results presented in this talk are a contribution to the study of cardinal characteristics of the continuum, in particular the dominating number $\mathfrak{d}$, the bounding number $\mathfrak{b}$, and the unsplitting numbers $\mathfrak{r}$ and $\mathfrak{r}_\sigma$. It is known that $\mathrm{cf}(\mathfrak{d})$ is at least as large as $\mathfrak{b}$, and thanks to Mildenberger and Blass it is also known to be at least as large as the groupwise density number $\mathfrak{g}$, and the splitting number $\mathfrak{s}$ (Mildenberger). Furthermore, it is known that that if one partitions a dominating family into less than $\mathfrak{b}$ pieces, then one of pieces is actually dominating (not just of cardinality $\mathfrak{d}$), and if one partitions a dominating family into less than $\mathfrak{g}$ pieces, then (as noted by Blass) one of the pieces is 2-dominating. I will give a similar characterization for partitions of dominating families into less than $\mathfrak{s}$ pieces, and apply this characterization to obtain results on the unsplittting and σ-unsplitting numbers. The results presented here are part of my thesis, which is being written with Andreas Blass.

▶ STANISLAV BAROV, *Constructive logic*.
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In late 19th century, G. Frege introduced three fundamental concepts of semantics: extensionals of language expressions, them truth-values, and them meaning. The meaning of the sentense can be defined as function which provide construction of value (e.g., truth-value), object for simple name or some set for predicate. Here we intend to show in which way this noton can be represented within newest constructive (e.g., BHK-like) semantics where it play same role as realizability of sentences. Also this notion have conection with partial type of constructive truth and falsity in sence of Shanin and notion of restorability of formulas in sense of Nagorniy can be derived from this by simple factorization procedure.

▶ ANATOLY BELTIUKOV, *Theories with strongly ordered quantification*.
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Diagonal proofs in the algorithm theory are critically considered by S. S. Lavrov in[1]. Here we are constructing an understanding of predicate formulas that does not allow traditional diagonal proofs. Let us consider a first order languages. Let the universe of an interpretation of this language be the set of all words over some finite alphabet. Let some strong ordering R be given for this set. The sense of the formula xRy is "x is EARLIER than y". We think of this ordering as of an order of some natural "constructing" or "arising" of these words. For any formula of the language we assume that all variables under quantifiers of this formula are bounded by the ordering R: each quantifier is bounded by the variable of the nearest outer quantifier. Exceptions are most outer quantifiers, that are not bounded. For example, for the formula $Q_1x_1Q_2x_2\dots Q_nx_nA$ we assume that $x_2Rx_1, \dots, x_nRx_{n-1}$.

Ways of expressing programming task in this theory are investigated. It is possible to express common bounds for several quantifiers by using appropriate codings of objects by words. Ways of axiomatization of this theory are investigated. Traditional diagonal proofs of algorithms and complexity theories are not valid for this interpretation of formulas.

Therefore some results of these theories remains open for new understanding.

This work is supported by the Intas grant No 2000-447.

[1] S. S. LAVROV, *On algorithmic unsolvability*, ***4th international Andrei Ershov memorial conference, PSI 2001*** (Akademgorodok, Novosibirsk, Russia) (Dines Bjørner, Manfred Broy, and Alexandre V. Zamulin, editors), Perspectives of System Informatics, Revised Papers, Lecture Notes in Computer Science, vol. 2244, Springer, 2001, pp. 25–32.

► MARTA BILKOVA, *Feasible disjunction and interpolation properties in propositional modal logic* S4.

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We consider propositional modal logic S4 with respect to complexity of proofs, namely to the feasibility of disjunction and interpolation properties. Our ideas come from the same problems solved in intuitionistic logic by reason that intuitionistic logic is interpretable in modal logic S4. It is important to get the feasible interpolation theorem for modal logic S4 because it often enables to prove exponential lower bounds or absence of polynomial simulation between proof systems.

The disjunction property for modal logics is the following proposition: if $\Box A \vee \Box B$ is a modal tautology then one of $\Box A$, $\Box B$ is a modal tautology. In modal logic S4 the disjunction property holds. We work with naturally defined sequent calculus for $S4$ which is equivalent to often used Hilbert style calculus. We prove our main theorem, the feasible disjunction property for modal logic S4: Theorem: Let P be a proof of the sequent $\Rightarrow \Box A_0 \vee \Box A_1$. Then we can construct in polynomial time a proof Q of $\Rightarrow \Box A_i$ for some $i \in \{0, 1\}$. We generalize this result for P a proof of $\Gamma \Rightarrow \Box A_0 \vee \Box A_1$ where Γ is a set of so called S4-Harrop formulas which are defined by analogy with Harrop formulas in intuitionistic logic. As a corollary we obtain feasible interpolation theorem for modal logic S4: Theorem: Let P be a proof of

$$\Box x_1 \vee \Box\neg x_1, \dots, \Box x_n \vee \Box\neg x_n \Rightarrow \Box A_0 \vee \Box A_1$$

Then it is possible to construct a circuit $C(x)$ whose size is polynomial in the size of P such that for every input $a \in \{0, 1\}^n$, if $C(a) = i$, then A_i where we substitute for variables x_j $\perp$, if $a_j = 0$, and $\top$, if $a_j = 1$, is an S4 tautology.

► BRANISLAV BORICIC, *On the logic of reliability*.

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The concept of reliability is usually developed in the context of probability theory in case when it is needed to define the reliability function and the risk function. In this paper we present a system of reliability logic based on the propositional language extended by a family of *unary degree operatos* r, for $r \in B$, where $\mathbf{B} = \langle B, \dots \rangle$ is a complete Boolean algebra, and formulae set of the form A^r, making possible to express the fact that "the degree of reliability of A is equal to r". We introduce two cut free sequent systems **LKrlb** and **LJrlb** for the classical and the intuitionistic versions of the reliability logic extending Gentzen's calculi **LK** and **LJ**. For instance, the reliability monotonicity rules and the cut rule have the following forms:

$$\frac{\Gamma A^r \vdash \Delta}{\Gamma A^s \vdash \Delta}\ (M^r \vdash) \text{ where } r \leq s \qquad \frac{\Gamma \vdash A^r \Delta}{\Gamma \vdash A^s \Delta}\ (\vdash M^r) \text{ where } r \geq s$$

$$\frac{\Gamma \vdash A^r \Delta \quad \Pi A^s \vdash \Lambda}{\Gamma\Pi \vdash \Delta\Lambda}\ (\text{cut}^r) \text{ where } r \geq s$$

▶ DOUGLAS BRIDGES, *A proof-technique in apartness space theory*.
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The theory of apartness spaces, a counterpart of the classical theory of proximity spaces, appears promising as a foundation for constructive topology. In several papers dealing with metric and uniform apartness spaces (for example, [1, 2]), we have used *ad hoc* variants of what appears to be a general proof–technique, along the following lines.

We have two sequences $(x_n)_{n=1}^{\infty}$ and $(x_n')_{n=1}^{\infty}$ in a uniform space $(X, \mathcal{U})$, and two properties $P(x, x')$ and $Q(x, x')$ of ordered pairs in $X \times X$. We want to prove that $P(x_n, x_n')$ holds eventually—that is for all sufficiently large n. We first use additional information, such as the strong continuity of some mapping from X into a uniform space Y, to establish a constructive surprise: either $P(x_n, x_n')$ holds eventually or else $Q(x_n, x_n')$ holds infinitely often. In order to rule out the second alternative, we then show that it implies a weak form of the law of excluded middle, and that if this weak form of excluded middle is added to intuitionistic logic, then it is contradictory that $Q(x_n, x_n')$ holds infinitely often. It follows from all this that $P(x_n, x_n')$ holds eventually.

In this paper we extract the essence of this argument and then show how the general technique can be applied. In particular, we clarify the connection between certain continuity properties for functions between uniform spaces; show that the natural notion of Cauchy sequence in an apartness space is equivalent, in a metric space, to the usual notion; produce a constructive version of the classical theorem that a compact apartness space has a unique associated uniform structure; and establish the connection between certain types of convergence of sequences of functions.

This is joint work of Douglas Bridges and Luminiţa Simona Vîţă.

[1] Douglas Bridges and Luminiţa Vîţă, *Strong continuity implies uniform continuity: the uniform space case*, preprint, University of Canterbury, Christchurch, New Zealand, November 2001.

[2] Luminiţa Vîţă, *Proximal and uniform convergence*, preprint, University of Canterbury, Christchurch, New Zealand, November 2001.

▶ LEV BUKOVSKY, *Bases and towers of trigonometric thin sets*.
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In our recent paper [1] we have shown that every basis of the family of trigonometric N_0- or A-sets is of cardinality at least continuum. Moreover, every such family contains an upward directed tower of cardinality $\mathfrak{t}$. Using some infinite combinatorics, we show similar result for the family of Dirichlet sets.

[1] Z. Bukovská and L. Bukovský, *Comparing families of thin sets*, ***Real Analysis Exchange***, vol. 27 (2001/2002) pp. 1–18.

▶ DUMITRU BUSNEAG, *On the lattice of ideals of an MV-algebra*.
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MV-algebras were introduced by C. C. Chang in 1958 as an algebraic counterpart of the Lukasiewicz infinite valued propositional logic. In this talk I put in evidence new characterisations for the meet-irreducible elements on the lattice of the ideals of an MV-algebra.

[1] R. Cognoli, I. M. L. D'Ottaviano, and D. Mundici, ***Algebraic foundations of many-valued reasoning***, Kluwer Academic Publ., Dordrecht, 2000.

[2] D. Busneag and D. Piciu, *On the lattice of ideals of an MV-algebra*, to appear in ***Scientiae Mathematicae Japonicae (SCMJ)***, 2002.
[3] D. Busneag and D. Piciu, *Meet-irreducible ideals in an MV-algebra*, submitted to ***Glasgow Mathematical Journal***, 2002.

▶ LORENZO CARLUCCI, *Provably total functions and the Hydra Game*.
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The usual characterization of provably total functions of **PA** in terms of the Hardy Hierarchy (see [1], [4], [5]) can be established in the context of Gentzen-Takeuti Proof Theory in a quite simple way by using the following result from [2].

Theorem. *A translation $\mathcal{H}$ from **PA**-derivations to hydras can be defined such that if derivation D' is obtained by derivation D by one step of Gentzen's Reduction Strategy, then $\mathcal{H}(D')$ can be obtained (recursively) from $\mathcal{H}(D)$ by a finite number of steps of Kirby-Paris' Hydra Game.*

The proof of the Theorem in [2] is entirely diagrammatic and implies the independence of the Hydra Theorem from **PA** (see [3]) by Gödel's Second Incompleteness Theorem. Since the **PA**-provably total functions can be characterized by a simple modification of Gentzen's Reduction Strategy (see [4]), a simple adaptation of the Theorem above implies that these functions are bounded by the lenghts of the Hydra battles. The latter are *naturally* measured by the Hardy Hierarchy up to ε_0, so the result follows. This method bypasses the proof of the relation between ordinal recursive functions and the Hardy Hierarchy.

[1] W. Buchholz and S. Wainer *Provably computable functions and the Fast-growing Hierarchy*, ***Logic and combinatorics*** (S. G. Simpson, editor), Advances in Mathematics, American Mathematical Society, 1987, pp. 179–198.
[2] L. Carlucci *A new proof-theoretic proof of the independence of Kirby-Paris' Hydra Theorem*, ***Theoretical Computer Science***, (to appear).
[3] L. Kirby and J. Paris *Accessible independence results for Peano Arithmetic*, ***Bulletin of the London Mathematical Society***, vol. 14 (1982), no. 3, pp. 285–293.
[4] G. Takeuti ***Proof theory***, second ed., North-Holland, Amsterdam, 1987.
[5] A. Weiermann *How to characterize provably total functions by local predicativity*, ***The Journal of Symbolic Logic***, vol. 61 (1996), pp. 52–69.

▶ YURY CHERNOSKUTOV, *Truth and consequence in perspective of nomonotonicity*.
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The usual way of dealing with consequence relations for nonmonotonic reasoning is based on Tarskian approaches modified by replacing some of his rules with weakened ones. In semantic his notion of consequence (a formula p is deducible from a set of formulae X) as satisfaction of a formula p by all models satisfying the set of formulae X as a rule is being replaced by a notion of satisfaction of p by some designated or preferred models (minimal with respect to some order) satisfying X. The question of the nature of truth in perspective of nonmonotonicity remains open. The presentation proposed attempts to fill the gap by examining appropriateness for this end of iterated theories of truth, developed by H. G. Herzberger, A. Gupta and S. Kripke. Kripke's approach constructs extension of truth predicate in progressive fashion until it reaches fixpoint. Gupta-Herzberger act in revisional fashion until the process of truth representation reaches stability. The problem of relevance the concept of truth as such for nonmonotonic reasoning is discussed.

► ANAHIT A. CHUBARYAN AND ARMINE A. CHUBARYAN, *On the proofs complexity in* ***CPL***.
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Two systems of ***CPL***, described by Anahit Chubaryan earlier, are the following: the system $\mathfrak{C}$, which is a variant of Hilbert style cut-free system and the system $\mathcal{E}$, directed on the establishment of deducibility of the formulas, given in the disjunctive normal forms. A condition of tautologies was specified, at presence of which the formulae require the exponential complexity (lines, size) of proofs in these two systems. The sets of tautologies of the length n were described, the proofs complexities of which in these two systems are accordingly $O(n), O(n^2), O(n^3), \dots, O(2^{\frac{n}{2}})$. It's interesting to find out, where are the places of these two systems in the hierarchy of proofs systems of ***CPL***.

It is proved, that the systems $\mathfrak{C}$, $\mathcal{E}$, resolution system $\mathfrak{R}$ and the propositional cut-free sequent calculus $P\mathcal{K}^-$ are p-equivalent.

Therefore the following statements are true:

(1) the bounds of the proofs complexities of above-mentioned sets of tautologies are valid also for the systems $\mathfrak{R}$ and $P\mathcal{K}^-$;

(2) the set of all tautologies of the length n is devided into classes of the formulas, which upper bounds of the proofs complexity don't exceed accordingly $n, n^2, n^3, \dots, 2^{O(n)}$ in a system Φ, which p-simulates one of the systems $\mathfrak{C}$, $\mathcal{E}$, $\mathfrak{R}$ or $P\mathcal{K}^-$;

(3) for sufficiently large n and every t $\left(1 \leqslant t \leqslant \left[\frac{n}{2}\log_n 2\right]\right)$ there are tautologies φ_n^t of the length n, requiring the proofs complexity $O(n^t)$ in every system Φ, which is p-simulated by one of the systems $\mathfrak{C}$, $\mathcal{E}$, $\mathfrak{R}$ or $P\mathcal{K}^-$.

► PHILIP EHRLICH, *Surreal numbers: an alternative construction*.
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J. H. Conway introduced an ordered field ***No*** of surreal numbers [1] which contains (in a suitable sense that can be made precise [3]) "all numbers great and small." However, in addition to its distinguished structure as an ordered field ***No*** has a rich algebraico-tree-theoretic structure that depends upon ***No***'s structure as a lexicographically ordered binary tree [3]. We examine the relative merits of the standard constructions of surreal numbers with the following terminological variant of the construction of surreal numbers introduced by the author in [2, p. 256] which brings ***No***'s ordered-tree theoretic structure to the fore.

CONSTRUCTION OF GAMES. If L and R are any two sets of games, then there is a game (L, R). All games are constructed in this way.

A game x is *simpler than* a game $y = (L_y, R_y)$, written $x <_s y$, if $x \in L_y$ or $x \in R_y$; a chain of games (ordered by $<_s$) is *ancestral* if it is closed under the simpler than relation; and a partition L, R of an ancestral chain of games is *orderly*, if $L_x \subseteq L$ and $R_x \subseteq R$ for each element $x = (L_x, R_x)$ of the chain.

CONSTRUCTION OF NUMBERS. If L, R is an orderly partition of an ancestral chain of surreal numbers, then there is a surreal number (L, R). All surreal numbers are constructed in this way.

DEFINITION OF ORDER. For all surreal numbers x and y, $x < y$ if and only if $x \in L_y$ or $y \in R_x$ or $R_x \cap L_y \neq \emptyset$.

[1] J. H. Conway, ***On Numbers and games***, Academic Press, 1976.

[2] P. Ehrlich, ***All number great and small***, in ***Real numbers, generalizations of the reals, and theories of continua***, Kluwer Academic Publishers, 1994, pp. 239–258.

[3] ________, *Number systems with simplicity hierarchies: a generalization of Conway's theory of surreal numbers*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1231–1258.

▶ ANDREY FROLOV, *Set-theoretical properties of classes of computable sets*.
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In my talk I study set-theoretical properties of computable sets, classifying these sets in respect to primitive recursive sets. For instance, we say that a set D is left (right) primitive recursive in limit if for any primitive recursive set $P \subset_\infty D$ there is a primitive recursive set R such that $P \subset_\infty R \subset_\infty D$ (accordingly $D \subset_\infty P \to D \subset_\infty R \subset_\infty P$). We say that a set is primitive recursive in limit if it is left and right primitive recursive in limit.

It is proved that any computable set can be obtained from finitely many primitive recursive in limit sets by operations of union and intersection. Further, we introduce the following special set-theoretical reducibility of sets: $A \leq B$ iff $A = (B \cup P) \cap R$, where P and R are primitive recursive sets. This relation is reflexive and transitive and, therefore, we can consider the set $\mathcal{A}$ of appropriate equivalency classes. We show that $\mathcal{A}$ is not upper semilattice, have maximal elements. A characterization of maximal elements of $\mathcal{A}$ is obtained. Among other results we prove lower density of maximal elements of $\mathcal{A}$.

▶ MARTA GARCIA-MATOS, *Abstract model theory without negation*.
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Lindström's Theorem [3] characterizes first-order logic in terms of the Downward Löwenheim-Skolem Theorem (DLS) and the Compactness Theorem (COMP). It would be interesting to have similar model-theoretic characterizations for other logics than first-order logic, whether they be classical or non-classical logics. One question in this respect is what closure properties must a logic have in order for Lindström's Theorem to hold in one form or other. We are particularly interested in the question of closure under (classical) negation. In [1] we presented a generalized form of Lindström's Theorem that characterizes the fragment (obviously not closed under negation) of first-order sentences in which a particular predicate occurs only positively. This basic example was very useful in displaying the connections between Craig Interpolation Theorem and model-theoretic characterization of logics, but also lead to the question of analyzing in detail the role of negation in the usual proof of Lindström's Theorem. It turns out that the assumption that the logic to be characterized is closed under negation cannot always be dropped. In this paper, we present a counterexample to this effect. We show that no countable logic can be maximal with respect to DLS and COMP. In particular, if we consider the logic of *PC* classes over first-order logic, this counterexample shows that so called IF logic [2], which does have DLS and COMP, is not maximal with respect to these properties.

[1] M. García-Matos, *On interpolation and model-theoretic characterization of logics*, ***Proceedings of the sixth ESSLLI student session***, pp. 107–116, European Summer School of Logic, Language, and Information, Helsinki, August, 2001.

[2] J. Hintikka, ***The principles of mathematics revisited***, Cambridge University Press, New York, 1996.

[3] P. Lindström, *On extensions of elementary logic*, ***Theoria***, vol. 35 (1969), pp. 1–11.

▶ MILAN GRULOVIC, *Reduced products of forcing systems*.
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We discuss the conditions under which the Łoś theorem holds for ultraproducts of forcing systems and prove that in general in the case of ultraproducts of finite forcing systems the ultraproduct of generic models (when they exist) do not have to satisfy the conditions of the definition of generic models with respect to the ultraproduct of corresponding theories and ultraproduct forcing relation. We give as well an example of the family of theories, each of which has a (finitely) generic model, while the ultraproduct of that family is without generic models.

The ultraproducts of infinite forcing systems (which can be treated informally as a special case of the ultraproducts of forcing systems) raise a lot of questions considering the mutual relations of ultraproducts of models.

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The logic ***BL*** (basic fuzzy logic) was introduced in [5] and inspired by continuous t-norms as possible truth functions of conjunction and by their residua as truth functions of implication. They form the standard semantics of the logic ***BL***. A general semantics is given by the variety of ***BL***-algebras: each ***BL***-algebra serves as an algebra of truth functions for connectives of fuzzy logic. ***BL*** is complete with respect to interpretations in ***BL***-algebras, in linearly ordered ***BL***-algebras and in standard ***BL***-algebras. We shall present three generalizations of the logic ***BL***: First, Esteva's and Godo's logic ***MTL*** (monoidal t-norm logic) [3] inspired by left-continuous t-norms and their residua. Second, falsity-free fragment of ***BL*** related to the variety of basic hoops [4]. Finally, a logic psBL (pseudo-basic fuzzy logic) with possibly non-commutative conjunction complete with respect to pseudo-***BL*** algebras by diNola, Georgescu and Iorgulecsu [2]. Details are in [6].

[1] R. Cignoli, F. Esteva, L. Godo, and A. Torrens. *Basic logic is the logic of continuous t-norms and their residua*, ***Soft Comp.***, vol. 4 (2000), pp. 106–112.

[2] A. di Nola, G. Georgescu, and A. Iorgulescu, *Pseudo-**BL**-algebras, Part I and II*, to appear in ***Many-valued logic***.

[3] F. Esteva and L. Godo. *Monoidal t-norm based logic: Towards an axiomatization of the logic of left continuous t-norms*, ***Fuzzy Sets and Systems***, vol. 124 (2001), pp. 171–188.

[4] F. Esteva, L. Godo, P. Hájek, and F. Montagna, *Hoops and fuzzy logic*, submitted.

[5] P. Hájek, ***Metamathematics of fuzzy logic***, Kluwer, 1998.

[6] P. Hájek, ***Fuzzy logics with non-commutative conjunctions***, submitted.

▶ KAI HAUSER, *Is choice self-evident?*
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Nowadays acceptance of the axiom of choice is mainly justified on extrinsic grounds by its fruitfulness in consequences. In this talk, I will examine whether and in what sense choice is intrinsicly plausible. Arguments in that direction typically appeal to a particular concept of set or some form of maximality principle. I will describe a few of the difficulties involved and explore how choice might fail by taking into account strong axioms of infinity which refute it.

▶ ALBERT HOOGEWIJS AND PIETER AUDENAERT, *Implementing undefinedness in a two-valued prooftool through a four-valued Kleene logic*.
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Undefinedness arises in a natural way in computer science. Several three-valued logics were devised to handle questions concerning recursive partial functions and to deal with correctness in computational programs and hardware specifications. We introduced such a three-valued logic called ***PPC*** (see [1]) based on the connectives $\neg$, Δ, $\wedge$, $\vee$, $\rightarrow$, $\forall$ and sequents of the form $L \vdash \alpha$. The connectives (except for the Δ) correspond to the Kleene connectives. $\Delta(\alpha)$ is true (T) if α is true or false, else it is false(F), and $\forall x \alpha(x) \equiv \bigwedge \alpha(x_s)$. The interpretation of $L \vdash \alpha$ is special, since formulas $\beta \in L$ on the left of the turnstile are considered to be true whereas the formula α on the right is considered to be "not false" (T or U).

Seeing that proofs about program and hardware correctness involve hundreds of prooflines, the use of a prooftool is inevitable. In stead of implementing a new three-valued prooftool (see [2]) we now propose an embedding of the three-valued logic in the two-valued system ***PVS*** (see also [3]).

We show that this can be done using a four-valued extension of the Kleene connectives using the following implementation: $T \equiv (1, 1)$, $U_1 \equiv (1, 0)$, $U_2 \equiv (0, 0)$, $F \equiv (0, 1)$ and identifying $U_1 \equiv U_2 \equiv U$ using Hilbert's epsilon operator: $U = (\varepsilon x : x \in \{0, 1\}, 0)$.

[1] A. Hoogewijs, *On a formalization of the non-definedness notion*, ***Zeitschrift für Math. Logik und Grundlagen der Math.***, Band 25 (1979), pp. 213–217.

[2] D. Van Heule and A. Hoogewijs, *Tableaux in automated theorem proving: Isabelle and PPC*, ***The Bulletin of Symbolic Logic***, vol. 5 (1999), pp. 151–152.

[3] P. Audenaert and A. Hoogewijs, *A PVS-proof for a memory modelling problem is a proof!*, ***Annals of the Kurt-Gödel-Society***, Collegium Logicum, vol. 4, 2001, p. 51.

► SHAMIL ISHMUKHAMETOV, *On distribution of associated degrees*.
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Let $D = A - B$ be a difference of computably enumerable sets, briefly a d.c.e. set, $s_A(y)$ for $y \in A$ be the stage when y enters A. Then the associated set is set $B[D] = \{s_A(y) : y \in B\}$. It is known that

(1) $B[D]$ is c.e.

(2) D is relatively enumerable in $B[D]$,

(3) $deg(B[D])$ is the least degree in which D is enumerable. Set D is called proper if its degree does not contain a c.e. set. The existence of properly d.c.e. degrees was proved by S. B. Cooper in 1971.

Let for a degree $\mathbf{d}$ $R[\mathbf{d}]$ denote the class of c.e. degrees in which $\mathbf{d}$ is relatively enumerable. Clearly, for any d.c.e. degree $\mathbf{d}$ class $R[\mathbf{d}]$ contains at least one element. Classes $R[\mathbf{d}]$ for d.c.e. $\mathbf{d}$ have been studied in [1, 2]. It was proved, in particular, that there exist d.c.e. degrees with $R[\mathbf{d}]$ containing a minimal element and d.c.e. degrees with $R[\boldsymbol{d}]$ consisting of one element.

Let ACE be the class of degrees containing associated sets for propery d.c.e. sets. From the Weak Density Theorem [3] we know that ACE degrees are distributed widely in the c.e. degrees.In particular, they are located in any non-trivial intervals with c.e.end-points. This raises the problem of a distribution of ACE degrees. In [4], we showed that not every intermediate c.e. degree is ACE. Now we present a new result in this direction:

Theorem. *There exists a non-zero c.e. degree* $\boldsymbol{a}$ *such that every c.e. degree* $\boldsymbol{b}$, $\boldsymbol{0} < \boldsymbol{b} \leq \boldsymbol{a}$, *is* ACE.

[1] S. Ishmukhametov, *On the r.e. predecessors of d.r.e.degrees*, ***Archive for Mathematical Logic***, vol. 38 (1999), pp. 373–386.

[2] S. Ishmukhametov, *On relative enumerability of Turing degrees*, ***Archive for Mathematical Logic***, vol. 39 (2000), pp. 145–154.

[3] S. B. Cooper, S. Lempp, and P. Watson, *Weak Density and cupping in the r.e.degrees*, ***Israel Journal of Mathematics***, vol. 67 (1989), pp. 137–152.

[4] S. Ishmukhametov and M. Mukhin, *D.c.e. sets and the associated degrees*, to appear.

▶ ALEKSANDER IVANOV, *Group actions on pretrees and definability*.
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A ternary structure (T, B) is a *pretree* if it satisfies the standard axioms of the betweenness relation in a tree: $B(x; y, z)$ means that x is between y and z. A point $x \in T$ is *terminal*, if $(\forall y, z \in T)\neg B(x; y, z)$. The pretree T can be naturally decomposed $T = T_0 \cup P$, where P is the set of all terminal points. It is clear that every simplicial or real tree can be considered as a pretree with $P \neq \emptyset$ by adding ends as terminal points and taking the reduct to the natural betweenness relation.

We define an axiomatizable class K_{tr} of structures $((G, \cdot, ^{-1}), (T, B), *)$, where $*$ satisfies the axioms of an action of a group G by automorphisms of a pretree. All structures of isometric actions on **R**-(and **Z**-)trees (which are not first-order axiomatizable) belong to the class K_{tr} and can be considered as typical representatives of K_{tr}. Then the basic theory of actions on **R**-trees can be carried out in K_{tr} in a model theoretic form (for example for a neutral G-fixed end $\mathbf{e}$ the relation $x <_{\mathbf{e}} y$, the half-line (x, ∞) representing $\mathbf{e}$ contains y, becomes first-order definable).

The subclass $K_{\mathrm{nnest}} \subseteq K_{\mathrm{tr}}$ of non-nesting actions is defined by the following axiom: $\forall g \forall t, t' \in T_0 \neg([g * t, g * t'] \subset [t, t'])$. It can be considered as a first-order version of the condition of isometric actions. In the class K_{nnest} we develop some results of Tits about isometric actions of algebraic groups on **R**-trees.

For example, let $((G, \cdot, ^{-1}), (T, B), *) \in K_{\mathrm{nnest}}$ where $G = SL_2(D)$ and G is considered with the structure of the division ring D. Assume that G does not fix any point and any end of T_0. Then there exists a definable ordered line $L \subseteq T_0$ and a definable map $\phi : UT_2(D) \to (L, <)$ such that $\phi(I) = \infty$ and for any $t \in L$ the set $\phi^{-1}([t, +\infty])$ is a subgroup of $UT_2(D)$. In the situation of the existence of a valuation v from D to a totally ordered abelian group Λ there is an action of $SL_2(D)$ on the Λ-tree of lattices over the corresponding valuation ring. Then the line L can be interpreted as Λ and $\phi(a) = v(a_{1,2})$, $a \in SL_2(D)$.

▶ REINHARD KAHLE AND THOMAS STUDER, *Least fixed points in applicative theories*.
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In applicative theories the recursion theorem provides a term rec which solves recursive equations. However, it is not provable that a solution obtained by rec is minimal. Here, we introduce an applicative theory in which it is possible to define a *least* fixed point operator, [1, 2]. Still, our theory has a standard recursion theoretic interpretation. This work is an important ingredient for the recent modeling of Featherweight Java in a system of explicit mathematics, [3, 4].

[1] Reinhard Kahle, *Einbettung des Beweissystems Lambda in eine Theorie von Operationen und Zahlen*, ***Diplomarbeit***, Mathematisches Institut der Universität München, 1992.

[2] Reinhard Kahle and Thomas Studer, *Formalizing non-termination of recursive programs*, ***The Journal of Logic and Algebraic Programming***, vol. 49 (2001), pp. 1–14.

[3] Thomas Studer, *Constructive foundations for featherweight Java*, ***Proof theory in computer science*** (R. Kahle, P. Schroeder-Heister, and R. Stärk, editors), Lecture Notes in Computer Science, vol. 2183, Springer, 2001, pp. 202–238.

[4] ______________, *Object-oriented programming in explicit mathematics: Towards the mathematics of objects*, ***Ph.D. thesis***, Institut für Informatik und angewandte Mathematik, Universität Bern, 2001.

▶ VLADIMIR KANOVEI AND MICHAEL REEKEN, *Borel and countably determined reducibility in nonstandard domain*.
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We consider, in a nonstandard domain, reducibility of equivalence relations in terms of the Borel reducibility $\leq_{\mathrm{B}}$ and the countably determined (CD, for brevity) reducibility $\leq_{\mathrm{CD}}$. This reveals phenomena partially analogous to those discovered in modern "standard" descriptive set theory of Polish spaces. The $\leq_{\mathrm{CD}}$-structure of CD sets (partially) and the $\leq_{\mathrm{B}}$-structure of Borel sets (completely) in ${}^*\mathbb{N}$ is described. We prove that all "countable" (those with countable equivalence classes) CD equivalence relations are CD-smooth, but not all are B-smooth: the relation $x \mathsf{M}_{\mathbb{N}}\, y$ iff $|x - y| \in \mathbb{N}$ is a counterexample. Similarly to the Silver dichotomy theorem in Polish spaces, any CD equivalence relation on ${}^*\mathbb{N}$ either has at most continuum-many classes (and this can be witnessed, in some manner, by a countably determined function) or there is an infinite internal set of pairwise inequivalent elements. Our study of *monadic* equivalence relations, i.e., those of the form $x \mathsf{M}_U\, y$ iff $|x - y| \in U$, where U is an additive countably determined cut (initial segment) in ${}^*\mathbb{N}$ demonstrates that these equivalence relations split in two linearly $\leq_{\mathrm{B}}$-(pre)ordered families, associated with countably cofinal and countably coinitial cuts, and the equivalence $u \mathsf{E}_0\, v$ iff $u \,\Delta\, v$ is finite, on the set of all hyperfinite subsets of ${}^*\mathbb{N}$, $\leq_{\mathrm{B}}$-reduces all "countably cofinal" equivalence relations but does not $\leq_{\mathrm{CD}}$-reduce any of "countably coinitial" equivalence relations.

The collaboration over this project has been supported by DFG.

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We construct an intuitionistic system for Quine's "New Foundations".

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We will consider countable algebraic systems of finite signature. L. Richter has introduced in [1] the Recursive Embeddability Condition and has obtained the result: if a noncomputable system $\mathfrak{A}$ satisfies the recursive embeddability condition then $\mathfrak{A}$ has no degree.

In the given work we show:

– Any countable Abelian p-group (Ershov algebra) $\mathfrak{A}$ satisfies the Recursive Embeddability Condition.

– Let $\mathfrak{B}$ be a countable Boolean algebra and $\mathfrak{A}$ be an antisymmetric connected system Σ-definable in $\mathbb{HF}(\mathfrak{B})$. Then $\mathfrak{A}$ is computable.

From these results we obtain

– Any countable not computable Abelian p-group (Ershov algebra) has no degree.

– If L is a linear ordering Σ-definable in $\mathbb{HF}(\mathfrak{B})$ over a countable Boolean algebra $\mathfrak{B}$ then L is computable.

The work is partially supported by INTAS-00-483.

[1] L. J. Richter, *Degrees of structures*, ***The Journal of Symbolic Logic***, vol. 46 (1981) pp. 723–731.

▶ LESZEK ALEKSANDER KOLODZIEJCZYK, *The expressive power of Henkin quantifiers with dualization*.
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We consider an extension of the logic with Henkin (branched) quantifiers (general information on Henkin quantifiers may be found in [2]). The extension, known as ***LBD*** (logic with dualization), differs from the usual logic with Henkin quantifiers in that the set of available quantifiers is additionally closed on a dualization operator (where the dualization of a quantifier Q is defined as $\neg Q \neg$).

It is known that ***LBD*** is semantically equivalent to full second order logic. We develop this result and show that introducing a naturally defined notion of dualization depth of a quantifier makes it possible to find in ***LBD*** a counterpart of the Σ^1_n hierarchy of second order logic. Namely, let $\boldsymbol{LBD_n}$ be the restriction of ***LBD*** to formulae without quantifiers of dualization depth greater than n. We prove that the class of all positive formulae of $\boldsymbol{LBD_n}$ has the same expressive power as the class Σ^1_{n+1}.

It follows that the class of simple positive formulae of $\boldsymbol{LBD_n}$ captures the class Σ^P_{n+1} of the polynomial hierarchy of complexity theory. Thanks to results of Georg Gottlob (see [1]), this also shows that full $\boldsymbol{LBD_n}$ captures $\boldsymbol{LOGSPACE}^{\Sigma^P_{n+1}}$, which in turn yields a normal form theorem for ***LBD*** on finite ordered models.

[1] GEORG GOTTLOB, *Relativized logspace and generalized quantifiers over finite ordered structures*, ***The Journal of Symbolic Logic***, vol. 62(1997) pp. 545–574.

[2] MICHAL KRYNICKI, and MARCIN MOSTOWSKI, *Henkin quantifiers*, (M. Krynicki, M. Mostowski, and L. W. Szczerba, editors) ***Quantifiers: Logics, models, and computation, Volume I***, Kluwer Academic Publishers, 1995, pp. 193–262.

▶ SREĆKO KOVAČ, *Weakened Gödelian ontological systems*.
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Gödelian ontological systems are in this talk systems similar to the modal second-order S5 system (including five axioms managing the ultrafilter-like concept of "positiveness") that Gödel most probably assumed while establishing his ontological proof (one reconstruction is e.g., in [2]). Hájek [3] showed that Magari's reductive system [4] with two Gödel's axioms excluded does not suffice to prove $\Box\exists x Gx$ (Gx for 'x is God'). He proposes a weakened system KD45.

Magari's system suffices only to prove $\Diamond\exists x Gx$. Besides KD45, other weakened Gödelian systems such as KB also suffice for proving $\Box\exists x Gx$ (and $\exists x Gx$)(cf. [1]). The completeness proof for some of such systems (with restricted λ-abstract rules) is established. The consequences of introducing world-relative domains are examined.

A comparison of Gödel's "moral aesthetic" and "attributive" interpretations of Gödelian ontological systems is outlined and the philosophical consequences of the weakening of systems briefly sketched.

[1] ROBERT MERRIHEW ADAMS, *The logical structure of Anselm's arguments*, ***The Philosophical Review***, vol. 80 (1971), pp. 28–54.

[2] MELVIN FITTING, ***Types, tableaus and Gödel's God***, forthcoming in Kluwer.

[3] PETR HÁJEK, *Magari and others on Gödel's ontological proof*, ***Logic and algebra*** (A. Ursini and P. Aglianó, editors) Dekker, New York, 1996, pp. 125–135.

[4] ROBERTO MAGARI, *Logica e teofilia*, ***Notizie di logica***, vol. 7 (1988), pp. 11–20.

▶ KRZYSZTOF KRUPINSKI, *Profinite structures interpretable in separably closed fields*.
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Profinite groups are inverse limits of finite groups. Likewise profinite structures are inverse limits of finite permutational structures. We consider the question, which profinite structures are interpretable in separably closed fields. We introduce the notion of profinite structure weakly interpretable in a first order theory. Using some model theory of separably closed fields (and showing some new results) we show that profinite structures interpretable in separably closed fields are the same as profinite structures weakly interpretable in ACF_p. Then we find a strong connection between the last notion and the main problem of Galois theory.

The next part consists of field theoretic constructions of profinite structures weakly interpretable in ACF_p and satisfying some model theoretic properties (like smallnes, m-normality, non-triviality, being $\mathcal{M}$-rank 1). For example we inerpret in this way a profinite structure having all of the above properties, which is a profinite group close to Z_p^ω.

▶ MICHAŁ KRYNICKI, DOMINIKA MAJSAK AND ŁUKASZ WOJTYNIAK, *Decidability of logics with Henkin quantifiers*.
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The Henkin quantifier was defined by Henkin in [1]. The simplest one denoted by H has the form:

$$\begin{matrix}\forall x_1 \exists y_1\\ \forall x_2 \exists y_2.\end{matrix}$$

The sentence

$$\begin{matrix}\forall x_1 \exists y_1\\ \forall x_2 \exists y_2\end{matrix}\ \phi(x_1, y_1, x_2, y_2)$$

is equivalent to the second order sentence:

$$\exists f \exists g \forall x_1 \forall x_2\ \phi(x_1, f(x_1), x_2, g(x_2)).$$

It was proven in [2] that $L^1(H)$—the monadic logic with simple Henkin quantifiers—is decidable. In [3] it was shown that the logic with Henkin quantifiers having two variables in the scope of the general quantifiers—$L(H_2^2)$—is undecidable.

We consider so called *narrow* Henkin quantifiers which are Henkin quantifiers of the form

$$\begin{matrix}\forall \bar{x} \exists \alpha\\ \forall \bar{y} \exists \beta,\end{matrix}$$

where α, β are boolean variables. Such quantifiers were introduced in [4]. Monadic logic with *narrow* Henkin quantifiers which have only one general variable on each level is decidable as a case of the simplest Henkin quantifier. We discuss a decidability problem for monadic logic with arbitrary narrow Henkin quantifiers (i.e., with an arbitrary number of variables in the scope of the general quantifiers).

[1] L. Henkin, *Some remarks on infinitely long formulas* in ***Infinitistic methods***, Pergamon Press, New York, and Polish Scientific Publishers, Warsaw, 1961, pp. 167–183.

[2] M. Krynicki and A. H. Lachlan, *On the semantics of the Henkin quantifier*, in ***The Journal of Symbolic Logic*** vol. 44 (1979), pp. 184–200.

[3] M. Krynicki and M. Mostowski, *Decidability problems in languages with Henkin quantifiers*, in ***Annals of Pure and Applied Logic*** vol. 58 (1992), pp. 149–172.

[4] A. Blass and Y. Gurevich, *Henkin quantifiers and complete problems*, in ***Annals of Pure and Applied Logic*** vol. 32 (1986), pp. 1–16.

▶ MILOŠ KURILIĆ, *Boolean algebras and forcing*.
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Some forcing-related properties of Boolean algebras are considered. For example,firstly, if κ is an infinite cardinal, then a complete Boolean algebra B is called κ-independent iff in some generic extension by B an independent subset X of κ appears (that is $A \cap X$ and $A \setminus X$ are nonempty sets for each set $A \subset \kappa$ of size κ belonging to the ground model) or, equivalently, iff there are $C \in [B]^\kappa$ and $q \in B^+$ such that each positive $p \leq q$ is split by all except $< \kappa$ many elements of C. Then $\mathrm{cf}(\kappa) \leq |B|$ and, if κ is a regular cardinal $\kappa \leq \pi(B) =$ the algebraic density of B. Secondly, killing of mad families in generic extensions is considered. For example, forcing by B preserves all mad families on each cardinal $\kappa \geq \mathrm{cc}(B)$. These and similar properties are studied and compared with other forcing-related properties of complete Boolean algebras.

► TADEUSZ LITAK, *Modal incompleteness revisited.*
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I am going to analyze the phenomenon of modal incompleteness from an algebraic point of view. This will be an attempt to provide an analysis complementary to the one contained in a famous paper of van Benthem [1]. The usual method of showing that a given logic L is incomplete is to show that for some $\Sigma \subseteq L$ and some $\varphi \notin L$, φ cannot be separated from Σ by a suitably wide class of complete algebras — usually Kripke algebras. I am going to show that classical examples of incomplete logics, e.g., those constructed by Fine or Shehtman, cannot be approximated by any class of complete boolean algebras with operators; this kind of incompleteness may be called, as de Jongh suggested to me, *complete incompleteness*. Even above **Grz** it is possible to find a continuum of such logics, which immediately implies the existence of a continuum of neighbourhood-incomplete **Grz** logics. Similar results can be proved for Löb logics. Proofs are conducted by means of *complete frames* — Thomason's general frames whose set of possible values is complete as a Boolean algebra. Thus, they combine the power of algebraic methods with the intuitive, model-theoretic character of Kripke-frames techniques.

[1] J. F. A. K. van Benthem, *Syntactic aspects of modal incompleteness theorems*, ***Theoria***, vol. 45 (1979) pp. 63–77.

[2] A. V. Chagrov and M. V. Zakharyaschev, ***Modal logic***. Clarendon Press, Oxford 1997.

[3] K. Fine, *An incomplete logic containing S4*, ***Theoria***, vol. 40 (1974) pp. 23–29.

[4] V. B. Shehtman, *Topological models of propositional logics*, ***Semiotics and Information Science***, vol. 15 (1980) pp. 74–98 (Russian).

[5] S. K. Thomason, *Semantic analysis of tense logics*, ***Journal of Symbolic Logic***, vol. 37 (1972) pp. 150–158.

► DOMINIKA MAJSAK AND MARCIN MOSTOWSKI, *Classes of finite models definable by means of Krynicki quantifiers for monadic vocabularies*.
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In the paper by M. Krynicki and M. Mostowski ([1]) spectra for Krynicki (function) quantifiers in empty vocabulary, or equivalently definable classes of finite models of empty vocabulary, were characterised. They are exactly regular sets of positive integers, called also eventually periodic sets. A natural question arises about characterising of definable sets of finite models in this logic in monadic vocabulary (with unary predicates). The method invented by Loescher [2], and applied in [3] for solving slightly different problem, supplies relatively easy solution to our problem.

Finite models for the vocabulary with k-unary predicates can be described as sets of

2^k-dimensional vectors of natural numbers. We prove that definable sets of finite models correspond exactly to semi–linear sets of such vectors, or equivalently classes of 2^k-ary relations definable in Presburger arithmetic.

[1] M. KRYNICKI and M. MOSTOWSKI, *Decidability problems in languages with Henkin quantifiers*, ***Annals of Pure and Applied Logic***, vol. 58 (1992) pp. 149–172.

[2] B. LOESCHER, *One unary function says less than two in existential second orded logic*, ***Information Processing Letters***, vol. 61 (1997) pp. 69–75.

[3] A. DURAND, R. FAGIN, and B. LOESCHER, *Spectra with only unary function symbols*, ***Annual Conference of the 1997 European Association for Computer Science Logic***, (CSL'97) (M. Nielsen and W. Thomas, editors), Lecture Notes in Computer Science, vol. 1414, Springer-Verlag, 1998, pp. 189–202.

► LARISA MAKSIMOVA, *Implicit definability over Grzegorczyk modal logic*.
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We consider normal modal logics extending the Grzegorczyk logic Grz. We find all normal extensions of Grz possessing the projective Beth property and give their full description.

Let P, Q, Q' be disjoint lists of propositional variables not containing the variables y and z, and $G(P, Q, x)$ a set of formulas. We say that a logic L has the projective Beth property PBP if $G(P, Q, y), G(P, Q', z) \vdash_L y \leftrightarrow z$ implies $G(P, Q, y) \vdash_L y \leftrightarrow C(P)$ for some formula $C(P)$.

It is known [1] that PBP follows from the interpolation property CIP in all normal modal logics. On the other hand, we prove

THEOREM 1. *There exist exactly twelve consistent logics with* PBP *in the family* NE(Grz) *of normal extensions of the Grzegorczyk logic. Six of them possess, and other do not possess* CIP.

All these logics are finitely axiomatizable and have the finite model property. For each of these logics we obtain its axiomatization and a characterization by an appropriate class of finite partially ordered frames. Moreover, we state

THEOREM 2. *The projective Beth property is decidable over* Grz, *i.e., there exists an algorithm which, for each finite set Ax of additional axiom schemes, decides whether the logic* Grz $+ Ax$ *has the* PBP *or not*.

THEOREM 3. *The* PBP *problem over* Grz *is in **CONEXP** and **PSPACE**-hard.*

In [2], the exhaustive list of superintuitionistic logics (SIL) with PBP was found: there exist exactly sixteen SIL with PBP. We note that only thirteen logics in that list have a modal companion with PBP in NE(Grz).

[1] L. MAKSIMOVA, *Projective Beth's properties in modal and superintuitionistic logics*, ***Algebra and Logic***, vol. 38 (1999), pp. 316–333.

[2] L. MAKSIMOVA, *Intuitionistic logic and implicit definability*, ***Annals of Pure and Applied Logic***, vol. 105 (2000), pp. 83–102.

► FREDERICK SEYMOUR MICHAEL, *Relevance and substructure*.
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In Anderson and Belnap, ***Entailment*** I, (Princeton: Princeton University Press, 1975), the claim is made that the structural rule of weakening does not hold in the relevant systems, ***T***, ***E*** and ***R***. According to Arnon Avron, this rejection of the weakening rule is the most characteristic feature of relevant logic. Yet weakening has never been a central concern in relevant logic. Since the concept of substructural logic was first advanced about 10 years ago, relevant logic has been widely seen as being a principal example of substructural logic. My principal objective in this paper is to show that relevant logic need not be substructural.

Without weakening, irrelevant premises cannot be introduced. But we can allow weakening and prevent irrelevant conclusions by modifying the introduction rules for the relevant conditional and negation. The usual $\Gamma, p \vdash q$ iff $\Gamma \vdash p \rightarrow q$ is replaced by the simpler $p \vdash q$ iff $\vdash p \rightarrow q$; and parametric constituents are similarly removed from contraposition and negation introduction. Some weaker relevant logics such as Routley's ***B*** reject the structural rule of contraction because of Curry's paradox. I will show that adding a formula that generates this paradox to even the weakest relevant logic has some very strange consequences, most notably, the logical equivalence between p and $\neg p$. We can only conclude that, in a logic with negation, the formula that gives rise to Curry's paradox is not admissible and the case against contraction collapses. Finally, I will propose a new approach to relevance, one that is semantically based and that results in a logic weaker than ***T*** but stronger than ***B***.

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We prove some independence results for subsystems of Heyting arithmetic by using Kripke models. We present a necessary condition for linear Kripke models of the arithmetical theories which are closed under the negative translation. It implies that the union of the worlds in any linear Kripke model of HA satisfies PA. Let us denote the intuitionistic counterparts of the fragments of the form $\mathsf{I}\Gamma$ and $\mathsf{L}\Gamma$ of Peano arithmetic $\mathsf{i}\Gamma$ and $\ell\Gamma$, respectively. It is already known that $\mathsf{i}\Pi_1 \nvdash \mathsf{i}\Sigma_1$ and $\mathsf{i}\Sigma_1 \nvdash \mathsf{i}\Pi_1$, see [3]. Using the above mentioned necessary condition, we show $\mathsf{i}\exists_1 \nvdash \mathsf{i}\forall_1$ (here the matrix of formulas are quantifier-free formulas). It is also proved that $\mathsf{i}\forall_1 \nvdash \mathsf{i}\exists_1$. We also show that $\mathsf{i}\Pi_2 \nvdash \mathsf{i}\Sigma_2$ and $\mathsf{i}\Sigma_2 \nvdash \mathsf{i}\Pi_2$. We obtain $i\Sigma_2 \nvdash i\Pi_2$ as a direct consequence of the results in [B] about $\mathsf{i}\Pi_2$ and $\mathsf{i}\Sigma_2$. Smorynski's operation $\Sigma^{'}$ is used to show $\mathsf{HA} \nvdash \ell\Pi_1$ and $\mathsf{HA} \nvdash \ell\Sigma_1$. We also examine our methods in the context of intuitionistic bounded arithmetic.

[1] W. BURR, *Fragments of Heyting arithmetic*, ***The Journal of Symbolic Logic***, vol. 63 (2000), pp. 1223–1240.

[2] MORTEZA MONIRI, *Independence results for weak systems of intuitionistic arithmetic*, submitted.

[3] K. F. WEHMEIER, *Fragments of* HA *based on* Σ_1*-induction*, ***Archive for Mathematical Logic***, vol. 37 (1997), pp. 37–49.

► GEORG MOSER, *On generalizations of semi-terms of particularly simple form*.
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It is well-known that we can elementarily bound the maximal depth of terms occurring in cut-free proofs in Gentzen's sequent calculus in the length of the given proof.

In this work we are especially interested in the question, whether it is possible to generalize not only terms, but also occurring semi-terms, which may include bound variables as well. More consisely

Does LK admit generalizations of simple type for semi-terms?

The answer to this question is negative, if the logical form of the endformula is kept. We will present a counter-example. We therefore revise the notion of a more general form of the endformula. In addition to the change in the term-structure, we allow the replacement of single quantors by *blocks* of quantifiers. More precisely, the *extension* A' of the formula A is obtained by replacing the strong quantifier occurrence Qx in A by $Qx, \overline{z}$ for a (suitable defined) string of bound variables $\overline{z}$. Notice that A' is logical stronger than A.

Now we can prove the following result: Any proof Π of $A(t_1, \dots, t_n)$, where the t_i are either terms or semi-terms, i.e., terms containing bound variables, can be transformed to a proof of $A'(r_1, \ldots, r_n)$ such that (i) the t_i are substitution instances of the r_i and (ii) the maximal depth of the (semi-)terms r_i is elementary bounded in the number of steps of the proof Π and the endformula $A(t_1, \ldots, t_n)$, without considering the depth of the (semi-)terms t_i.

This theorem extends one of the main results of [1] to languages with function with arbitrary arity and the central result in [2] to semi-terms.

[1] M. BAAZ and G. SALZER, *Semi-unification and generalization of a particularly simple form*, (L. Pacholski and J. Tiuryn, editors), ***Proceedings of the*** 8^{th} ***workshop CSL'94***, Lecture Notes in Computer Science, vol. 933, Springer Verlag, 1995, pp. 106–120.

[2] J. KRAJÍČEK and P. PUDLÁK, *The number of proof lines and the size of proofs in first-order logic*, ***Archive for Mathematical Logic***, vol. 27 (1988) pp. 69–84.

▶ MARCIN MOSTOWSKI, *Interpretations, first order reducibilities, and defining Henkin quantifiers*.
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We consider interpretations in the sense of Tarski (as codified by Szczerba [4]) as a general method of proving definability theorems for generalized quantifiers. This method is known in finite model theory under the name first order reductions (see e.g., in [3] and [1]). However it can be applied to finite as well as to infinite models.

We give a few applications of the method. Particularly we give essential improvements of normal form theorems for Henkin quantifiers (in comparison to those already known, see [2]).

[1] N. IMMERMAN, ***Descriptive complexity***, Springer Verlag, 1998.

[2] M. KRYNICKI and M. MOSTOWSKI, *Henkin quantifiers*, (M. Krynicki, M. Mostowski, and L. W. Szczerba, editors) ***Quantifiers: Logics, models and computation***, vol. I, pp. 193–262. Kluwer Academic Publishers, 1995.

[3] J. A. MAKOWSKY and Y. B. PNUELI, *Computable quantifiers and logics over finite structures*, (M. Krynicki, M. Mostowski, and L. W. Szczerba, editors) ***Quantifiers: Logics, models and computation***, vol. I, pp. 313–357. Kluwer Academic Publishers, 1995.

[4] L. W. SZCZERBA, *Interpretability of elementary theories*, ***Logic Foundations of Mathematics and Computability Theory***, pp. 129–145, Reidel, 1977.

▶ HRIPSIME G. MOVSESYAN AND ANAHIT A. CHUBARYAN, *On the proofs complexity in intuitionistic and minimal propositional logic*.
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The notions of φ-determinative conjunction and φ-determinative DNF for any tautology (taut) φ were defined by Anahit Chubaryan earlier. Basing on these notions also the proofs system $\mathcal{E}$ of $\boldsymbol{CPL}$ has been described. The $\mathcal{E}$-proofs can be transformed into the proofs in some other system of $\boldsymbol{CPL}$, basing on Kalmar's proofs of the deducibility of tauts inside $\boldsymbol{CPL}$. But this method is unacceptable in the intuitionstic and minimal logic.

We describe some modifications of cut-free sequent calculus for intuitionistic (minimal) propositional logic $\boldsymbol{PI}^-$ ($\boldsymbol{PM}^-$).

For every taut φ of the length ℓ we define the intuitionistic (minimal) image φ^I (φ^M) such, that

(1) the length of φ^I (φ^M) is no more, that $c \cdot \ell$ for some constant c,
(2) if complexity (size, lines) of $\mathcal{E}$-proof of φ is n, then $\boldsymbol{PI}^-$-proofs ($\boldsymbol{PM}^-$-proofs) complexity of φ^I (φ^M) is no more, than $c_I n^3$ ($c_M n^3$) for some constant c_I (c_M).

It is proved, that the described systems $\boldsymbol{PI}^-$ and $\boldsymbol{PM}^-$ are p-simulated by resolution system $\mathfrak{R}$, which is p-simulated by the system $\mathcal{E}$, therefore we can describe the sets of tauts, of the length ℓ, the proofs complexities of which in the both systems $\boldsymbol{PI}^-$ and $\boldsymbol{PM}^-$ are accordingly $o(\ell), o(\ell^2), o(\ell^3), \dots, o(2^{\frac{\ell}{2}})$ (the similar result for the system $\mathcal{E}$ is proved by Anahit Chubaryan earlier).

▶ ANDREJ MUCHNIK AND ALEXEI SEMENOV, *Frequency and universal approaches to randomness of finite sequences*.
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Kolmogorov in [1] formalized von Mises's frequency approach to randomness of finite sequences. A sequence is called random if the frequency of ones is almost the same in all subsequences chosen by simple rules of choice. More formally, a sequence of length L is called (n, ε, p)*-random* with respect to the set of rules $\mathcal{R}_L$ if the difference between the frequency of ones and p is $< \varepsilon$ in any subsequence of length $> n$ chosen by a rule belonging to $\mathcal{R}_L$. Kolmogorov showed that

1. for any L, $\varepsilon > 0$, p, and $n > \varepsilon^{-4}$ if cardinality of $\mathcal{R}_L$ is $< \mathrm{e}^{2n\varepsilon^2(1-\varepsilon)}$, then there exists (n, ε, p)-random sequence;
2. there exists $\mathcal{R}_L$ of cardinality $< 2^{4n\varepsilon(1+5\varepsilon)}$ such that there are no $(n, \varepsilon, 0.5)$-random sequences.

We remove the difference between these bounds (this problem was stated in [1]): there exists a set $\mathcal{R}_L$ of cardinality $< \mathrm{e}^{\frac{2n\varepsilon^2(1+\varepsilon)}{1-n/L}}$ such that any sequence is non-random with respect to $\mathcal{R}_L$.

Another problem under consideration is a correlation between universal (belonging to small simple set) and frequency randomness. *A specific deficiency* of a set $S \subseteq \{0, 1\}^L$ is $(1 - \log_2 |S|/L)$. A set of great deficiency is small (and consists of "universally non-random" sequences). A rule r is *normal* if r chooses subsequences of the same length from all sequences of length L; a set is *regular* if for some ε and a normal rule r it consists of all sequences such that r chooses subsequences with deviation of frequency from $1/2$ greater than ε (elements of regular sets are "frequency non-random"). We prove that if $\delta > 0$ is small enough, then every set of specific deficiency $> \delta$ can be covered by $L(1/\delta)^{1/\delta}$ regular sets of specific deficiency $> \frac{\delta}{\ln(1/\delta)}(1 - \frac{2\ln\ln(1/\delta)}{\ln(1/\delta)})$. This bound for possible deficiency of covering regular sets is sharp, since there exists a set of specific deficiency $> \delta$ that cannot be covered by $2^{L\delta^5/3}$ regular sets of specific deficiency $> \frac{\delta}{\ln(1/\delta)}(2 + \frac{\ln\ln(1/\delta)}{\ln(1/\delta)})$.

Also we formulate and prove algorithmic analogs of all previous results in terms of Kolmogorov complexity.

[1] A. N. Kolmogorov, *On tables of random numbers*, ***Sankhyā: The Indian Journal of Statistics, Series A***, vol. 25, part 4 (1963). (Reprinted in ***Theoretical Computer Science***, 207(1998), pp. 387–395).

▶ NIKOLAJ NIKITCHENKO, *Logics of equitone predicates and their applications*.
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Let $\langle D, \leq\rangle$ be a preordered set. A partial predicate $p \in P$ $(= D \to \mathrm{Bool})$ is *equitone*, if for any $d, d' \in D$ such that $d \leq d'$ and $p(d)$ is defined, follows that $p(d')$ is defined and $p(d) = p(d')$. The class of all equitone predicates over D is denoted by PE.

Much of the significance of equitone predicates comes from the large number of important examples, which arise in logic and computer science. This is especially clear, when $\leq$ is treated as a knowledge (information) relation.

The aim of this talk is to give a comprehensive study of the class of equitone predicates. Our investigations are based on an algebra $\mathrm{AK}(D) = \langle P, \vee, \wedge, \neg\rangle$, where operations $\vee, \wedge \colon 2^P \to P$ and $\neg \colon P \to P$ are infinitary analogues of Kleene's strong connectives. The following statements are proved.

THEOREM 1. $\mathrm{APE}(D) = \langle PE, \vee, \wedge, \neg\rangle$ *is a sub-algebra of* $\mathrm{AK}(D)$.

THEOREM 2. *A set of predicates* $GS = \{\bot\} \cup \{\eta_d \mid d \in D\}$ *is the set of generators of* $\mathrm{APE}(D)$ (*here* $\bot$ *is a nowhere defined predicate and* η_d *is an equicharacteristic predicate of* $d \in D$).

The set of all term over GS is denoted by $\mathrm{TE}(D)$. We define special infinitary equational logic and axioms schemes E1–E9 to describe properties of $\mathrm{APE}(D)$.

THEOREM 3. (*Soundness and completeness*) *For any* $t, t' \in \mathrm{TE}(D)$

$$(\mathrm{E1{-}E9}) \vdash t = t' \Leftrightarrow \mathrm{APE}(D) \models t = t'.$$

THEOREM 4. *Axiom schemes* $E1$*–*$E9$ *are mutually independent*.

These theorems are used to construct complete infinitary equational axiomatization for classes of predicate compositions of different types (multi-ary, quasi-ary, n-ary) and abstraction levels (propositional, singular, nominative). The obtained results can be applied for program specification.

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NC is usually defined as the union of an infinite family of classes of complexity: $\boldsymbol{NC} = \cup_k \boldsymbol{NC}^k$. These classes $\boldsymbol{NC}^k$, for $k \in \boldsymbol{N}$, were firstly described based on circuits — $\boldsymbol{NC}^k$ is the class of languages (i.e. boolean functions on integers) accepted by uniform boolean circuit families of depth $O(\log^k n)$ and size $2^{O(\log n)}$ with bounded gates, [1]. However, here we consider an alternative definition of ***NC*** due to Ruzzo. In [4], Ruzzo identifies $\boldsymbol{NC}_k$ with the classes of languages recognized by alternating Turing machines (abbreviatedly ATMs) in time $O(\log^k n)$ and space $O(\log n)$. We write $\boldsymbol{NC}_k = \mathrm{ATM}(O(\log^k n), O(\log n))$. Thus, $\boldsymbol{N} = \cup_{k \in \boldsymbol{NC}} \mathrm{ATM}(O(\log^k n), O(\log n))$.

The characterization of ***NC*** described here was established in collaboration with Bellantoni and it is a reformulation of the unsorted characterization of ***NC*** given in [2].

We describe a sorted term system for ***NC***, based on the tree algebra $\mathbb{T}$ — $\boldsymbol{ST}_{\mathbb{T}}$. The term system $\boldsymbol{ST}_{\mathbb{T}}$ is defined by analogy with the term system $\boldsymbol{ST}_{\mathbb{W}}$ which characterizes the polytime functions — see [3]. In particular, this work proposes the δ axis as a measure to describe classes within ***NC***. The δ measure counts the numbers of loops or recursions used to define a $\boldsymbol{ST}_{\mathbb{T}}$ term. The resulting framework, contrast with the time axis used to define the levels $\boldsymbol{NC}^k$ and, provides much of the same mathematical tractability as is usually associated with the $\boldsymbol{Dtime}(n^k)$ hierarchy within $\boldsymbol{P}$.

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▶ MAURICIO OSORIO, JUAN ANTONIO NAVARRO, AND JOSE ARRAZOLA, *Consistent neg-extensions of superintuitionistic theories*.
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We consider only finite propositional theories defined over a signature sig. We use I to denote intuitionistic logic and G_3 to denote Gödel 3-valued logic. A si-logic is any logic stronger or equal to I logic and weaker or equal to G_3. A consistent neg-extension of a si-logic theory T, is a consistent extension of T obtained by adding only negated formulas (i.e., formulas of the form $\neg\alpha$). Of particular interest is the case when these extensions are also complete. For a given theory T, we write $MS(T)$ to denote the set of such extended theories that are complete, under a background si-logic. Consider $T := \{\neg a \Rightarrow b, \neg\neg c \Rightarrow c\}$. Then T has exactly two consistent and complete neg-extended theories, that is $MS(T) = \{\{\neg a, b, c, \dots\}, \{\neg a, b, \neg c, \dots\}\}$.

We present several interesting and useful results about such extensions that have important impact in logic programming and non-monotonic reasoning. Some of them are the following.

We have proved that for a given set T, $MS(T)$ is invariant with respect to the background si-logic. We define $T_1 \equiv_{MS} T_2$, iff for every theory T, $MS(T_1 \cup T) = MS(T_2 \cup T)$. We have shown that $T_1 \equiv_{MS} T_2$ iff $T_1 \equiv_{G_3} T_2$. We say that $MS(T)$ is c-minimal if the set of atoms in each neg-extended theory in $MS(T)$ is a minimal model (in classical logic). We have found some sufficient conditions on T, to ensure that $MS(T)$ is c-minimal. Our ultimate goal is to show that si-logics based on our approach provides a solid framework to model non-monotonic reasoning.

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▶ SERGEI PODZOROV, *Ideals and intervals in the Rogers semilattices of* Σ^0_n*-computable numberings*.
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A *numbering* of a set S is an arbitrary surjective mapping from $\mathbb{N}$ onto S. For two numberings ν and μ we say that ν is *reducible* to μ if there exists a computable function f such that $\nu = \mu f$. All numberings of S with reducibility relation form a preordered structure; it's quotient modulo the induced equivalense forms an upper semilattice $\mathcal{R}(S)$.

Let S be the family of subsets of $\mathbb{N}$. A numbering ν of S is called Σ^0_n- *computable* if the set $\{\langle x, y\rangle : x \in \nu y\}$ is in the class Σ^0_n of arithmetical hierarchy. Subsemilattice of $\mathcal{R}(S)$ defined by all Σ^0_n-computable numberings we call *Rogers semilattice* and denote through $\mathcal{R}^0_n(S)$.

We investigate isomorphism types of initial segments and intervals in $\mathcal{R}^0_n(S)$ for $n \geqslant 2$. For c.e. set A let $\mathcal{E}^*_A$ be a lattice of all supersets of A under inclusion modulo finite sets. We call Σ^0_n-computable numberings ν (and the corresponding element of $\mathcal{R}^0_n(S)$) $0'$*-principal*, if for any Σ^0_n-computable μ we have $\mu = \nu f$ for some total $0'$-computable function f. Now the next propositions are proved (all of them are true for $n \geqslant 2$ and more than one element S).

1. $\mathcal{R}^0_n(S)$ contains an ideal isomorphic to $\mathcal{E}^*_\emptyset$.
2. For any c.e. A if $\mathcal{R}^0_n(S)$ has a $0'$-nonprincipal element than it contains an ideal isomorphic to $\mathcal{E}^*_A$.
3. For any c.e. A and $0'$-nonprincipal $\nu \in \mathcal{R}^0_n(S)$ there exist $\mu \in \mathcal{R}^0_n(S)$ such that the interval $[\nu, \mu]$ is isomorphic to $\mathcal{E}^*_A$.

4. If S is finite or contains the least element under inclusion than for any nongreatest $\nu \in \mathcal{R}_n^0(S)$ there exist $\mu \in \mathcal{R}_n^0(S)$ such that the interval $[\nu, \mu]$ contains only two elements.

▶ GIUSEPPINA RONZITTI, *On the concept of countable spread.*
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We deal with the problem of defining the notion of countable set, as restricted to those subsets of the Baire space $\mathcal{N}$ (the set of all number-theoretic functions) that in intuitionistic analysis are called spreads. Intuitionistic mathematical reasonings about spreads are ruled by the so called *principles of continuity*. We discuss the difficulty of defining the concept of countable spread by analyzing some of the possible definitions available in the literature. We consider, as an instructive example, a spread technically called $\sigma_{2\text{mon}}$. This is the set of all monotone binary sequences of natural numbers. Classically the set $\mathbb{N}$ of natural numbers may be bijectively mapped onto $\sigma_{2\text{mon}}$ while intuitionistically one proves that this is not the case and that $\mathbb{N}$ is smaller than $\sigma_{2\text{mon}}$, in two different senses to be defined. The analysis of this case naturally suggests a possible solution to the finding of a working definition of countable spread. We construct, following Veldman [5], a growing hierarchy of spreads and, taking the inductive closure, we define a class which may be compared with the classical set $\aleph_1$ of all countable ordinals. We then suggest a new definition of countable spread.

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▶ MICHAL RÖSSLER, *Nonstandard rational arithmetic*.
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We introduce the nonstandard arithmetic NQA based on the theory presented by R. Chuaqui, P. Suppes in [2] (we will denote it by NQA^+), with weakened external open minimization scheme. The finitary consistency proof for NQA formalizable in PRA is presented. We also show some interesting facts about the strength of the theories NQA and NQA^+; in NQA can be interpreted $\mathsf{I}\Delta_0 + \mathsf{EXP}$ and simultaneously NQA can be interpreted in $\mathsf{I}\Delta_0 + \mathsf{EXP}$. On the other hand, in NQA^+ can be interpreted $\mathsf{I}\Sigma_1$ and WKL_0.

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[8] R. ZACH, *The practice of finitism: Epsilon calculus and consistency proofs in Hilbert's Program*, (second draft).

▶ WIM RUITENBURG, *Intuitionistic theories with quantifier elimination*.
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We present intuitionistic theories T which satisfy quantifier elimination: For each formula A there exist quantifier-free formulas B such that $T \vdash A \leftrightarrow B$. Examples include theories T outside the categories indicated by the 1973 paper of Craig Smoryński.

[1] C. SMORYNSKI. *Elementary intuitionistic theories*, ***The Journal of Symbolic Logic***, vol. 38 (1973), pp. 102–134.

▶ PETER SCHUSTER, *A constructive Bolzano-Weierstraß principle*.
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Among the various classically equivalent concepts of compactness for metric spaces, the poorest in constructive content is sequential compactness—every sequence has a convergent subsequence—which fails constructively even for a two-element space. A constructively meaningful equivalent, however, is that of being totally bounded and complete; this is also chosen as the definition of compactness in Bishop-style constructive analysis, which is the mathematical framework for our considerations.

Moreover, the Bolzano-Weierstraß principle—that sequential compactness follows from total boundedness plus completeness—is essentially nonconstructive: it is known to be equivalent to the limited principle of omniscience (LPO), a weak form of the law of excluded middle that conflicts with Church's thesis and with Brouwer's continuity principle.

By requiring a certain classically trivial property from the sequences under consideration, we produce a reasonable modification of the Bolzano-Weierstraß principle that is both constructively valid and classically equivalent to the original version. This also clarifies the reason underlying the constructive inapplicability of the latter: namely, that arbitrary sequences constructively lack the aforementioned additional property, which is owned by every sequence in any metric space precisely when LPO is present.

This work has jointly been done with D. Bridges and H. Ishihara, and profited from hints due to R. Taschner.

▶ VOLODYA YU. SHAVRUKOV, *Effectively dense Boolean algebras in lattices of sentences*.
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We show that the 1st order theory of the lattice of Π_1 sentences modulo provability in Peano arithmetic has the same m-degree as the true 1st order arithmetic.

The proof utilises the recently published results by Andre Nies on the 1st order theory of the lattice of r.e. filters on any r.e. presented Boolean algebras, which in their turn build on the study of the lattice of r.e. sets.

We also show how to apply a similar technique to degrees of interpretability and diagonalizable algebras.

▶ ALEXANDRA SOSKOVA, *Cospectra of joint spectra of structures*.
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Given a partial structure A, define the spectrum of A to be the set

$$\mathrm{Sp}(A) = \{d_e(f^{-1}(A)) : f \text{ is an enumeration of } A\}.$$

Suppose that $A_0, \dots, A_n$, are partial structures. Then the *joint spectrum* of $A_0, \dots, A_n$ is the set

$$\mathrm{Sp}(A_0, \dots, A_n) = \{a : a \in \mathrm{Sp}(A_0), a' \in \mathrm{Sp}(A_1), \dots, a^{(n)} \in \mathrm{Sp}(A_n)\}.$$

Finally if $k \leq n$, then the k-th *co-spectrum* of the joint spectrum of $A_0, \dots, A_n$ is the set

$$\mathrm{CS}_k(A_0, \dots, A_n) = \{a : a \in D_e \ \& \ (\forall b \in \mathrm{Sp}(A_0, \dots, A_n))(a \leq b^{(k)})\}.$$

In the talk we present a characterization and a normal form of the elements of $\mathrm{CS}_k(A_0, \dots, A_n)$ based on the following observation:

Let $B \subseteq N$. Then $d_e(B) \in \mathrm{CS}_k(A_0, \dots, A_n)$ iff for all enumerations $f_0, f_1, \dots, f_k$ of $A_0, A_1, \dots, A_k$, respectively,

$$B \leq_e (\dots (f_0^{-1}(A_0)' \oplus f_1^{-1}(A_1))' \oplus \dots \oplus f_{k-1}^{-1}(A_{k-1}))' \oplus f_k^{-1}(A_k).$$

► THOMAS STREICHER, STEVE AWODEY, CARSTEN BUTZ, AND ALEX SIMPSON,
A set theory corresponding to higher order arithmetic.
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From work by Cole, Mitchell and Osius in the early 1970ies one knows that well-pointed toposes with natural numbers object (nno) are equiconsistent with bounded Zermelo set theory (bZ). The aim of our work is to go beyond a mere equiconsistency result by exhibiting a first-order set theory, bIZFA, such that every elementary topos with nno (whether well-pointed or not) appears (up to equivalence) as the category of sets associated with a model of bIZFA.

To obtain a set theory with such properties we are forced to modify ZF in three ways: (i) restrict separation to bounded formulas; (ii) use intuitionistic logic; and (ii) allow urelements/atoms.

It turns out that these are the only essential modifications we need to make to ZF, hence bIZFA: bounded Intuitionistic Zermelo-Fraenkel with Atoms. It is noteworthy that bIZFA retains the full (unbounded) collection axiom of ZF (in a strong form appropriate in the presence of bounded separation). This is made possible by the use of intuitionistic logic. (As is well known, under classical logic, collection implies full separation.)

As our main result we show that every elementary topos with nno provides a model of bIZFA.

The motivation for our work is to justify, in a mathematically precise way, the admissibility of set-theoretic figures of speech in situations where a universe of sets is not a priori available, i.e., we demonstrate "how to fake a universe". Such a programme has been suggested and advertised by S. MacLane in various talks over the recent years, with the aim of bringing together topos logic and the practice of the working mathematician.

► NIKOLAUS THIEL, *Definable ordinals in intuitionistic set theories*.
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The notion of a good Σ_1-definable ordinal plays an important role in admissible set theory. Turning to systems of set theory one is led to study their provably definable ordinals. Proof-theoretic analyses of systems of classical set theory in terms of ordinal representation systems often allow one to characterize their definable ordinals.

For a system of set theory T, the spectrum of T is defined as the set of ordinals having a good Σ_1-definition in T. It has been shown that the spectrum of a classical set theory always constitutes an initial segment of ordinals.

I have investigated the question for intuitionistic set theories. For some intuitionistic systems I showed that their spectrum forms an initial segment as well. However, the answer may depend on the notion of ordinal one employs.

► SERGEI TUPAILO, *Set-theoretic model for the Elementary Theory of Operations and Numbers*.
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Working in Constructive Set Theory, we develop a set-theoretic model for EON. The main task here was to define partial set-recursive application $\{e\}(x)$. For this, we follow an approach of partial set-recursive functions presented in [3]. The following theorem is easily proved.

Theorem. *The class of triples $\langle e, x, y\rangle$ such that $\{e\}(x) = y$ is a Σ_1 class and for each e, $\lambda x.\{e\}(x)$ is a partial function on* V.

This is a joint work with Michael Rathjen which constitutes a part of the paper [2].

[1] S. Feferman, *Notes on operational set theory, I. Generalisation of "small" large cardinals in classical and admissible set theory*. Draft, 2001.
[2] M. Rathjen and S. Tupailo, *Completeness of Aczel's realizability*, In preparation.
[3] G. E. Sacks, ***Higher recursion theory***, Springer, 1990.

► MONICA VANDIEREN, *Categoricity in abstract elementary classes with no maximal models*.
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My work is in a context idenitified by Shelah and Villaveces in which the amalgamation property does not necessarily hold. The longterm goal is to solve Shelah's Categoricity Conjecture in this context. One of the first steps is to isolate a suitable notion of saturation. I have solved a conjecture of Shelah and Villaveces by proving the uniqueness of limit models, which will serve as our notion of saturation.

In this talk I will highlight the history of Shelah's Categoricity Conjecture, describe the context of Shelah and Villaveces and explain how their conjecture can impact work towards the categoricity conjecture. I will end the talk with a description of a few of the features of my solution.

► MATTEO VIALE, *A binary modal logic for the intersection types of Lambda-calculus*.
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It is well known that the intersection types discipline allows to define a wide variety of models for Lambda-calculus and in general for functional programming languages. It is also known that the Curry-Howard isomorphism breaks down for this kind of type systems. In this talk we show that to a reasonable extent the correspondance between Lambda-calculus and suitable logical systems might be recovered, appealing to the fact that there is a very strict relation between the semantics for Lambda calculus induced by the intersection types and the Kripke style semantics for modal and relevant logics (see in particular (2)). We will briefly present BK, a "modal" logical system suggested by the analysis of the subtyping relation for intersection types introduced in (1); for such a logical system, we propose a Kripke style semantic and a suitable sequent calculi and we state the main results so far obtained:

A) BK is a conservative extension of many natural subtyping relation over the intersection types; B) Nice properties of BK : completeness, the cut elimination property, decidability, the finite models property.

[1] H. Barendregt, M. Dezani, and M. Coppo, *A filter lambda-model and the completeness of lambda-calculus*, ***The Journal of Symbolic Logic***, vol. 48 (1983), pp. 931–940.

[2] R. K. Meyer and R. Routley, *The semantics of entailment III*, ***Journal of Philosophical Logic***, vol. 1 (1972), pp. 192–208.

▶ ANDREAS WEIERMANN, *Some applications of analytic combinatorics in logic*.
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A famous result in logic is the mathematical incompleteness of Peano arithmetic by Paris and Harrington. Using analytical combinatorics and logical reasoning we classify the largeness conditions in the Paris-Harrington assertion which lead to incompleteness.

We further apply multiplicative number theory to obtain asymptotic bounds for count functions for elementary recursive ordinal notations which are defined via a Schütte style prime number coding. These bounds together with logical methods yield sharp bounds for the resulting Friedman style combinatorial well-foundedness principles.

▶ ROMAN WENCEL, *Boolean ordered o-minimal structures — an overview of my Ph.D. thesis*.
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We consider a notion of o-minimality for Boolean ordered structures. We prove that the notions of quasi o-minimality and o-minimality introduced by C. Toffalori are equivalent in the Boolean case. This is a consequence of weak elimination of imaginaries for Boolean algebras with finitely many atoms.

We develop the theory of definable sets for Boolean ordered strongly o-minimal structures and show that Boolean algebras with finitely many atoms do not admit non-trivial strongly o-minimal expansions. We also investigate the notion of orthogonality of atoms in Boolean subalgebras determined by algebraically closed sets, small theories of Boolean ordered structures (proving that they are $\aleph_0$-categorical) and prime models over sets of parameters. We briefly discuss some possible generalizations of o-minimality for Boolean ordered structures.

[1] L. Newelski and R. Wencel, *Definable sets in Boolean ordered o-minimal structures. I*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1821–1836.

[2] R. Wencel, *Small theories of Boolean ordered o-minimal structures*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 1385–1390.

[3] R. Wencel, *Definable sets in Boolean ordered o-minimal structures. II*, ***The Journal of Symbolic Logic***, vol. 68 (2003), pp. 35–51.

[4] R. Wencel, *Weak elimination of imaginary elements for Boolean algebras*, ***Annals of Pure and Applied Logic***, submitted.

► GUNNAR WILKEN, *Assigning ordinals to elementary patterns of resemblance*.
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In his paper "Elementary Patterns of Resemblance" Timothy Carlson presents a new approach to ordinal notation systems by using the notion of Σ_1-elementary substructure. He introduces a relation $\leq_1$ on the ordinals by recursive definition $\alpha \leq_1 \beta$ iff α seen as structure with ordinal addition considered as partial operation, less than relation and $\leq_1$ is a Σ_1-elementary substructure of β seen as the same kind of structure.

This allows for a characterization of (minimal) ordinals satisfying a finite set of conditions expressed by the constant 0, partial ordinal addition, less than and $<_1$. Carlson conjectures that this notation system provides notations for the ordinals below the proof theoretic ordinal of the system Π^1_1-CA_0 of second order number theory with comprehension restricted to Π^1_1-formulas and parameter free induction.

In classical proof theory a more syntactical notation system using so called Skolem hull operators is employed to perform the ordinal analysis of Π^1_1-CA_0.

In this talk we investigate the correspondence between these two ways of characterising ordinal numbers. On the one hand this indicates an assignment of well known ordinal terms to elementary patterns of resemblance. On the other hand this gives a semantical interpretation of proof theoretically significant ordinals given in syntactical terms by a finite ordered pattern expressed in terms of Σ_1-elementary substructure.

► DAN WILLARD, *A level-2 generalization and a level-1 exception for the Semantic Tableaux version of the Second Incompleteness Theorem*.
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The notion of a Π_n^- sentence was defined by us in [1]. It is roughly the same as conventional Arithmetic's Π_n sentences, except that Π_n^- sentences are written in a language that treats Addition and Multiplication as a 3-way relations, rather than as total functions. Let us say an axiom system α possesses a Level-k comprehension of its Semantic Tableaux consistency iff it can recognize that there exists assuredly no Π_k^- sentence ϕ where simultaneous Semantic Tableaux proofs of both ϕ and its negation derive from α's axioms. Let us say an axiom system α possesses a Level-k^* comprehension of its Semantic Tableaux consistency iff it can recognize that there exists assuredly no sentence ϕ, which is a Boolean combination of several Π_k^- sentence, such that α supports simultaneous Semantic Tableaux proofs of both ϕ and its negation. In this talk, we discuss a generalization of the Second Incompleteness Theorem for Level-2 Tableaux, and an exception to it for Level-1.

Our generalization of the Second Incompleteness Theorem is an extension of results from [2] and states there exists a Π_1^- sentence W, provable from the $I\Sigma_0$ fragment of Peano Arithmetic, such that no axiom system α can simultaneously prove Addition is a total function, prove the sentence W and prove a theorem indicating the Level-2^* consistency of its Semantic Tableaux proofs.

Our new boundary-case exceptions to the Second Incompleteness Theorem indicate that it is possible to construct axiom systems that simultaneously prove Addition is a total function, their Level-1 style Semantic Tableaux consistency and all Peano Arithmetic's Π_1^- theorems.

[1] Dan E. Willard, *Self-verifying systems, the incompleteness theorem and the tangibility reflection principle*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 536–596.

[2] Dan E. Willard, *How to extend the semantic tableaux & CutFree versions of the second incompleteness theorem almost to Robinson's Arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–496.

► ALAN R. WOODS, *Bounded arithmetic with prime counting*.
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$\mathrm{I}\Delta_0$ is the axiom system similar to Peano Arithmetic, but with induction hypotheses restricted to being Δ_0 formulas, *i.e.*, arithmetic formulas with only bounded quantifiers. An old question of Wilkie asks what, if anything, needs to be added to $\mathrm{I}\Delta_0$ in order to be able to prove the existence of arbitrarily large prime numbers. From work of Wilkie, Paris and myself, it is known that it suffices to add the axiom "$\forall n(n^{\log n})$ exists" or a particular instance of $\mathsf{PHP}(\Delta_0)$, the axiom schema asserting the pigeonhole principle for Δ_0 definable functions. From this latter result, it follows by general considerations, that adding the cardinality function

$$|\{n \leq x : \theta(n)\}|$$

counting the elements $n \leq x$ satisfying a particular, rather complicated, Δ_0 formula $\vartheta(n)$ (with parameters) is enough. If $\vartheta(n)$ is taken to be the predicate "n is prime" then the associated counting function is $\pi(x)$, the number of primes less than or equal to x. Twenty-one years ago, I conjectured that adding $\pi(x)$ should suffice.

Define $\xi(x, y, e)$ to be the number of primes p less than or equal to x for which the integer part $[y/p^e]$ is odd. Let $\mathrm{I}\Delta_0(\xi)$ be $\mathrm{I}\Delta_0$ augmented by ξ (allowed in induction hypotheses) and its obvious recursive definition.

THEOREM. $\mathrm{I}\Delta_0(\xi) \vdash$ "There exist arbitrarily large prime numbers".

This theorem is a step in the direction of the conjecture, because $\pi(x)$ can be defined, and its recursive definition proved, in $\mathrm{I}\Delta_0(\xi)$.

► ANNA ZALEWSKA, *Metainduction versus redundancy in some Gentzen-like systems*.
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The main obstacle in the mechanization of reasoning in logics of programs are, first of all, ω-rules.

In this talk, two Gentzen-like systems for algorithmic logic will be presented and compared. Each of them overcomes ω-rules but each of them tries to do it in the different way. The first one replaces ω-rules by metainduction. It allows to prove only that the proof exists instead of carrying out full proof for a given sequent. The second system introduces the notion of redundant nodes, i.e., looping nodes satisfying some conditions. Both systems are sound, complete and decidable for propositional algorithmic logic (***PAL***).

The systems can be extended to the first-order algorithmic logic (***FAL***) the more so as any valid formula of ***PAL*** becomes a valid formula of ***FAL*** following a substitution of formulas for propositional variables and programs for programs variables. Of course, the extended systems are not complete but one can compare the classes of provable formulas. Let CM (CR) be the class of the first-order formulas which are provable in the first (second) system. It can be proved that CR is included in CM. This fact is very important from the automated reasoning point of view the more so as the idea of metainduction can be applied to another logics of programs.

► BERISLAV ŽARNIĆ, *Prima facie consequence in update semantics for change expressions*.
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In the paper a variant of the notion of prima facie consequence for Lemmon style imperative logic [1] is developed within the framework of eliminative update semantics [3]. The formal apparatus utilizes the notion of an information empty context which updated with a sequence of sentences T generates the minimal context $0[T]$ in which T is accepted. Within the framework of eliminative update semantics a defeasible conclusion is not a persistent sentence and, therefore, it must be a non-eliminative sentence. Informational incompleteness provides a natural habitat for non-eliminative sentences such as indicative "It might be the case that ..." or imperative "It might be good to produce (sustain) ...". In the set of contexts $C(T)$ in which a sequence of sentences T is accepted there is a minimal context $0[T]$ such that for any other context c from $C(T)$ there is a sentence accepted in c and not accepted in $0[T]$. Since $0[T]$ has the smallest informational content in $C(T)$, it follows that prima facie consequence relation can be explicated using three notions: the notion of non-eliminative sentence, the notion of incomplete premises, and the notion of consequence relation defined over the minimal context. The examination of conflicting results in imperative logic shows that in the typical case the controversial arguments are the ones in which premises are incomplete and where, consequently, the purported conclusion should be a non-eliminative sentence.

[1] E. J. LEMMON, *Deontic logic and the logic of imperatives*, ***Logique et Analyse***, vol. 8 (1965), pp. 39–71.

[2] K. SEGERBERG, *To do and not to do*, (B. J. Copeland, editor), ***Logic and reality: Essays on the legacy of Arthur Prior***, Clarendon Press, 1996, pp. 301–313.

[3] F. VELTMAN, *Defaults in update semantics*, ***Journal of Philosophical Logic***, vol. 25 (1996), pp. 221–261.

► ERNST ZIMMERMANN, *Peirce's Rule in Natural Deduction*.
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In the very contrast to Natural Deduction for Intuitionistic Logic Natural Deduction for Classical Logic suffers from some well known limitations. Although normalisation can be proved, the standard proofs are highly language dependent. Dropping falsum and negation leads immediately to non classical logic. Such facts raise some doubts about the proof theoretical significance of the classical negation rule, the reductio ad absurdum. Instead of this rule Peirce' Rule may be chosen, a purely implicative elimination rule, proposed for Classical Natural Deduction by Curry in 1963. In this talk it is shown that with Peirce's Rule two different theorems of weak normalisation for classical predicate logic are provable, one for the language and respective rules implication, conjunction, all quantification, falsum, the other for this language extended with disjunction and existence quantification. These proofs remain proofs for classical logic in any fragment of logic containing implication. The talk presents the paper [3].

[1] H. B. CURRY, ***Foundations of mathematical logic***, 1963.

[2] D. PRAWITZ, ***Natural deduction. A proof-theoretical study***. 1965.

[3] E. ZIMMERMANN, *Peirce's rule in natural deduction*, ***Theoretical Computer Science***, vol. 275 (2002), nos. 1–2, pp. 561–574.

Abstracts presented by title

► ADIB BEN JEBARA, *All things are numbers (continuation)*.
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This poster is the continuation of the abstract "all things are numbers":

`http://www.logic.at/LC2001/files/ben_jebara.adib.txt`

Previously, we used a restricted axiom of choice, $\mathsf{CC}(2 \text{ through } m)$, countable choice for a family of sets of 2 through m elements, and we saw that we can move from a world with non restricted Evil to a world with restricted Evil because the numbers of attributes of Evil are infinite products of integers and "most" of them do not exist in the right mathematical universes. Now, we are going to see other consequences of restricting Evil. A hint for a proof that souls are immortal: The number of attributes of souls are integers. Souls are the counterparts in the philosophical universe of integers in the mathematical universes. And we have an infinity of universes in which integers are the same. Thus, souls are immortal.

The equation with infinite products $z \cdot z \cdot z \cdot \ldots = x \cdot x \cdot x \cdot \ldots + y \cdot y \cdot y \cdot \ldots$ with $z > y$ has no solution in the universe where only the restricted axiom $\mathtt{CC}(2 \text{ through } x)$ is true.

It is because otherwise the infinite products $x \cdot x \cdot x \cdot \ldots$ and $y \cdot y \cdot y \cdot \ldots$ exist but not $z \cdot z \cdot z \cdot \ldots$ and we cannot have a side of the equation existing and the other not.

The counterpart is that the numbers of attributes of Evil cannot be combined in the right philosophical universes. Once the principles of Good applied, history stops because principles of Good are not connected to time. The analogy between mathematical concepts and philosophical ones and surrounding the relation number of attributes can be reversed to better understand the mathematical universes from the understanding of the philosophical universes. As time, the counterpart of history, cease also to exist, we could be at once in all the infinity of mathematical universes where only $\mathtt{CC}(2 \text{ through } m)$ is true for every integer m.

Now, if Evil is restricted, there is a problem about fate. It could be argued that people are doing evil because of their fate. To that we reply that fate has multiple readings. And because of the multiple readings, freedom of choice is allowed.

► ALEXANDER DENISOV, *Limit points and homogeneous decidable extensions.*
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Let $\mathcal{M}_{\mathrm{T}}^{(d_2,d_3)}$ be a structural semiordering of the theory T. Let the diagram $\mathcal{D}_1$ is obtained from the first diagram of [1] by replacing $\mathcal{O}'$ bu $\mathcal{O}^1/\ \mathcal{D}_2$ is $\mathcal{D}_1$ without $\mathcal{O}$ by the needle. Next result precedes to the statement of [1].

Theorem. There exists $\mathcal{O}$-element $\mathcal{M}$ in $\mathcal{M}_{\mathrm{T}}^{(d_2,d_3)}$ such that $\mathcal{D}_2$ can be complemented, but $\mathcal{D}_1$ can not be complemented by indicated way.

Proof. Fix an effective sequence $\hat{A}_0, \hat{A}_1, \hat{A}_2, \ldots$ of infinite subsets of $\mathbb{N} = \{0, 1, 2, \ldots\}$, $\hat{A}_i \cap \hat{A}_j = \emptyset$ $(i \neq j)$, such that $\bigcup\{\hat{A}_{2n} \mid n < \omega\}$ and $\bigcup\{A_{2n+1} \mid n < \omega\}$ are a recursively inseparable recursively enumerable sets [2, §10.4]. Denote $\hat{A} = \bigcup\{A_n \mid n < \omega\}$. The numbers of $\hat{A}$ will correspond to all elements of some part of necessary $\mathcal{M}$. Add them with the elements of the first part through the binary predicate $\mathcal{Q}$. The structural diagram of $\mathcal{M}$ will be composed from two similiar phragments $\mathcal{A}$ and $\hat{\mathcal{A}}$. For $\hat{\mathcal{A}}$, we draw to

the points for $\hat{C}_0, \hat{C}_2, \hat{C}_4, \ldots$ both $\ldots, \hat{C}_5, \hat{C}_3, \hat{C}_1$ and represent $\mathcal{Q}$-connections of $\hat{C}_0$ with the elements corresponding to $\hat{A}_0$, $\hat{C}_2$ with the elements corresponding to $\hat{A}_0 \cup \hat{A}_2$, $\hat{C}_4$ with the elements corresponding to $\hat{A}_0 \cup \hat{A}_2 \cup \hat{A}_4$, $\hat{C}_5$ with the elements corresponding to $A \setminus (\hat{A}_2 \cup \hat{A}_3)$, $\hat{C}_2$ with the elements corresponding to $\hat{A} \setminus \hat{A}_1$, $\hat{C}_1$ with the elements corresponding to $\hat{A}$ through the horizontal straight brackets. The structural diagram of $\mathcal{A}$ will be obtained if we wash off all ˆ and provide at least one limit point for an element, say C_ω, $\mathcal{Q}$-connected with whole massive $\bigcup\{A_{2n} \mid n < \omega\}$.

$\mathcal{M}$ with its natural numeration is evidently decidable, i.e., $\mathcal{O}$-element. Besides, every homogeneous extension of $\mathcal{M}$ existing by the classical theorem must contain certain element of $\hat{C}_\omega$-type (on the cartesian plane appears again a limit point). So the model "$\mathcal{A} + \mathcal{A}$" satisfies to $\mathcal{D}_2$. But if some homogeneous extensionof $\mathcal{M}$ is decidable then $\hat{C}_w$ will give the recursive set separating $\bigcup\{\hat{A}_{2n} \mid n < \omega\}$ and $\bigcup\{\hat{A}_{2n+1} \mid n < \omega\}$. Hence $\mathcal{D}_1$ can not be complemented as necessary.

[1] A. S. DESINOV, *The most left branch in computable tree* (Unrefereed abstract), ***The Bulletin of Symbolic Logic***, vol. 6 (2000), no. 1, pp. 111–112.

[2] H. ROGERS JR, ***Theory of recursive functions and effective computability***, McGraw-Hill Book Company, New York, 1967.

▶ JAFAR S. EIVAZLOO AND MOJTABA MONIRI, *Proof theoretic arguments for ordered fields which are not real closed.*
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In addition to various other axiomatizations for real closed fields (like ordered fields with the IVP principle for polynomials), they are also precisely those ordered fields which have no (parametrically) definable gaps. Fix a positive integer d which is either 2 or odd. We investigate proof theoretic arguments for some of the previously model-theoretically established consequences for ordered fields F which are not d-real closed. This includes the one on existence of a monic unary polynomial P of degree d over F such that the set of u's in F with the property that P is negative on all $v < u$ does not have a least upper bound in F. Also of interest will be a proof theoretic approach to any nontrivial downward closed subset of F traversed by a definable function on F which is divergent at infinity is a gap and further to, any gap of F traversed by a definable divergent Cauchy function is regular (that is of zero distance to its complement).

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[4] C. MILLER, *Expansions of dense linear orders with the intermediate value property*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1783–1790.

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▶ ARTEM GOLUBEV, *Structural properties of computable sets under polynomial-time reducibilities.*
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In my talk I study structural properties of degrees of computable sets, classifying these sets in respect to polynomial-time reducibilities. Among obtained results splitting, cupping and capping of several classes of computable sets. The question on the embeddability of particular finite lattices are also considered.

▶ RAMI GROSSBERG, *Stability in abstract elementary classes.*
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THEOREM. Let K be a tame abstract elementary class satisfying the amalgamation property without maximal models. There exists a cardinal μ_0 such that for every $\mu > \mu_0$ and every member M of K of cardinality at least μ^+ if K is Galois-stable in μ then there exists a subset J of M of cardinality μ^+ which is Galois-indiscernible. Moreover J can be chosen to be a Morley sequence.

I will explain the concepts involved in the statement, will say something about the proof and will explain the significance of this to classification theory for Abstract Elementary Classes.

This is a joint result with Monica VanDieren.

▶ BARBARA MAJCHER-IWANOW, *Inert subgroups of uncountable universal locally finite groups.*
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Let κ be an infinite cardinal. We say that a subgroup H of G is κ-inert if for every $g \in G$, $|H : H \cap H^g| < \kappa$. If G has an infinite subgroup H such that H is $|H|$-inert and $\bigcap\{K < H : |H : K| < |H|\}$ is trivial (for example H is residually finite) then G is non-trivially topologized.

A locally finite group G is *universal locally finite* if every finite group is embedded into G and any isomorphism between finite subgroups of G can be realized by an inner automorphism of G. There is a unique countable universal locally finite group and there are 2^κ universal locally finite groups of cardinality $\kappa > \omega$ ([1] and [2]).

We give two constructions of uncountable universal locally finite groups with inert residually finite subgroups:

1. There is a universal locally finite group of cardinality ω_1 with a countable ω-inert residually finite subgroup.
2. For every uncountable cardinal κ there exists a universal locally finite group G of cardinality κ with the following properties:
   (a) there exists a κ-inert residually finite subgroup $H < G$ of cardinality κ;
   (b) if κ is regular then for any $\delta < \kappa$ there is no δ-inert subgroup of cardinality δ.

It is worth noting that the sentence that H is an ω-inert residually finite subgroup of G can be expressed by a single sentence of $L_{\omega_1,\omega}$ in the language with a unary predicate or H.

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[2] A. Macintyre and S. Shelah, *Uncountable universal locally finite groups*, ***Journal of Algebra***, vol. 43 (1976) pp. 168–175.

▶ CYRUS F. NOURANI, *Functorial Hasse models.*
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Start with the infinitary language $L_{\omega_1,\omega}$. Let $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$ be the category of models definable with generalized diagram $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$ and their morphisms. An embedding from a model definable in $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$ to the limit created from the functor from the Infinitary language category $\boldsymbol{L}_{\omega_1}$ is defined.

Define a functor $F : \mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle \to \boldsymbol{L}^{\mathrm{Op}}_{\omega_1,\boldsymbol{B}}$ by universal embedding from the diagram functions. A computing view to the functorial models are presented by defining Hasse diagrams on the $\boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ fragments. The limit model is defined by computing Hasse diagram limits and natural transformations on the limit cones.

Lemma 1. *The model defined from the* G*-diagram is initial in* $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$.

Define a functor $F : \boldsymbol{L}^{\mathrm{Op}}_{\omega_1,\boldsymbol{B}} \to \mathbf{Set}$ to be the generic functor. Define a functor $V : \mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle \to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ by universal embedding from the diagram functions.

THEOREM 1. *V creates a limit for F.*

THEOREM 2. *The power set of an inductive theory defined on $\boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ generates a generic model for the theory, provided the inductive theory has a defined G-diagram for its initial model. Next, the functor $V: \mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle \to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ is on the category of models definable in $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$. The models generated are similar to generic set foundations models.*

LEMMA 2. *The model defined from the G-diagram, $d\langle F, M\rangle$ is initial in $\mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle$.*

Starting with the completeness proof for the fragment category $\to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ to view the limiting cones in $\boldsymbol{L}^{\mathrm{Op}}_{\omega_1,\boldsymbol{B}}$. For a functor $F: J \to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}$, $j \to k$ corresponds to a preorder arrow on $\boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ example.

THEOREM 3. *The functor $V: \mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle \to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}$ defines a generic model for*

$$\mathrm{V}: \mathrm{D}\langle \mathrm{A}, \mathrm{G}\rangle \to \boldsymbol{L}_{\omega_1,\boldsymbol{B}}.$$

▶ JACEK PASNICZEK, *On bracketing names and quantifiers in first-order logic.*
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In this paper I will explain the motives categorially distinguishing names and quantifiers in classical logic and then set forth arguments for categorical unification of these two kinds of expressions. Conscpicously, for every two categories of expressions or categories of entities we can arbitrarily create a new category embracing the two. But such a move is sensible if we have sufficient philosophical, linguistic, or logical reasons for doing that (type-free logics are examples of a unification of individuals, properties, relations, and propositions). I will show that putting names and quantifiers in the same category is not only reasonable but it deepens our understanding of syntactic and semantic features of names and quantifiers and finally suggests a surprisingly simple first-order logic. Although this new logic is extensional in the most straightforward meaning of the word some basic intensionality phenomena may be rendered in it as well (e.g., these concerning the meanings of singular terms). The categorical unification of names and quantifiers makes the language of the new logic capable of comprising a broad class of noun-phrases and thus makes this language similar to natural language. However, the logic in question still closely resembles classical logic and this makes it useful in potential applications.

▶ AHTI-VEIKKO PIETARINEN, *Language-games and ludics: Some Wittgensteinian themes in logic and computation.*
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Wittgenstein's notion of a language-game can be put into a wider service by looking at its contribution to some contemporary notions in logic and the semantics of computation. In this talk, I will compare two paradigms: the role of language-games in the game-theoretic approach to logical semantics, and the contribution of language-games to the recent advancements in computer science. The paradigm of computation known as *ludics* [1], for instance, has a striking proximity to Wittgenstein's language-games. The interrelations that are emerging can be scrutinised by drawing parallels from logics that virtually necessitate some game-theoretic conceptualisations. These conceptualisations in turn demonstrate that the meaning of logical constants can in many instances be understood as Wittgensteinian language-games of 'showing or telling what one sees' [2].

The following contacts can be highlighted: (i) Some language-games are ones of winning and losing, and even employ the concept of a winning strategy. Wittgenstein connected these to the notions of true and false in arithmetic. (ii) The relation between truth-conditional and verificationistic semantics in logic is comparable with the relation between denotational and operational semantics in computation only if the notion of strategic meaning is accommodated. (iii) Neither language-games nor ludics warrant an exploration of something beyond the system of rules. Yet in both, the concept of a game is fundamental, and the concept of a rule is derived from it. (iv) Game-theoretically and ludically, negation means a role-switch, and hence its meaning lies in the geometry of the actions it is capable of giving rise to, not in the rules that attempt to define it. (v) The notion of consistency is more delicate and general than just a formal contradiction. (vi) Over and above the idea that at least some language-games are those of verification and falsification, the activities can be explained in terms of what Wittgenstein termed to be one of the most fundamental language-game: showing or telling what one sees in the context of an assertion.

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[2] L. WITTGENSTEIN, ***Wittgenstein's Nachlass, The Bergen electronic edition***, Diplomatic transcription. The Wittgenstein Trustees, The University of Bergen, Oxford University Press, 2000.

▶ VLADIMIR SOTIROV, *The majority voting parliament is either oligarchic or inconsistent*.
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One of the interpretations of Kenneth Arrow's celebrated Impossibility Theorem is in the theory of voting. Consider a set of alternatives together with a set of voters each of them arranging the alternatives according to his or her own it preference ordering. The central problem is to formulate a general rule (the social choice function) which would produce a preference relation arranging the alternatives so to reflect "the common will". The main result is that it is impossible to find a choice function satisfying en bloc certain conditions appearing to be natural and avoiding the existence of a dictator, that is a person whose preference of a certain alternative dictates this alternative to be the social choice. Among the numerous proofs of Arow's theorem, the "logical" proofs use the fact that any ultrafilter on a finite set coincides with the family of all subsets containing a fixed element.

Arrow showed that in the case of two alternatives, a social welfare function satisfying all "good" conditions (and avoiding the existence of a dictator) does exist and an example of such a function is the majority voting. It is shown in this paper that the majority choice between two alternatives is vulnerable as well, because the dictatorship can be recognized in the behavior of a certain group (an oligarchy).

The parliament is a non-empty set of deputies. Majority is any group containing more than half of the deputies. Sentences to be voted are called bills; they will be treated as propositional formulae and hence their set is closed under Boolean connectives. If A is a bill, denote by $[A]$ the set of all deputies voting pro A. A group of deputies a imposes a bill A when $a = [A]$ and a is a majority. In such a case, A will be called a law. In some cases however, a decision can be imputed to the parliament without having been voted and that is when it is a consequence of certain laws voted before. Namely, the parliament will be said to produce A either when A is a law or when it is a logical consequence of laws already passed.

The deputies are supposed to be honest (no of them votes pro some bill together with its negation; competent (they never abstain); consequent (having voted pro A and pro $A \Rightarrow B$ a deputy will vote pro B as well; logical (they vote pro any propositional tautology).

The parliament is inconsistent when there is a contradiction among its decisions. In the opposite case it is consistent. A majority a is stable $\iff$ each time when B is a law, a is a subset of $[B]$. The parliament is said to be oligarchic if a stable majority exists.

THEOREM. *Any consistent parliament is oligarchic.*

Define a decisive set to be a group of deputies involving a majority imposing some law. Then one of the possible proofs uses the well-known fact that any filter on a finite set consists of all its subsets involving a certain fixed subset.

▶ URSZULA WYBRANIEC-SKARDOWSKA, *On logical explication of syntactic and semantic sense*.
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The word 'sense' has many senses (meanings). In syntax and semantics of language we use this word, respectively, when we speak about 1) *operative* and 2) *eidetic* senses of its expressions (as some conventional sign-*types*). An expression-*type* E has an *operative sense* if we know how to use it in accordance with the syntactic rules of the language. Then E is a it well-formed expression and has a **syntactic sense**. E has an *eidetic* sense if we know what E means and what E denotes. Then E has to have a *meaning* and a *denotation*, so a **semantic sense**. In a logical conception of language these syntactic and semantic notions must be differentiated and explicated. This is possible on the basis of the author's formal-logical theory of language syntax (1991) with its expansion of semantic (1998, 2001) and pragmatic components. The theory is formalized on two levels: *tokens* and *types*. On the *token*-level are defined the notions of simple and complex *well-formed expression tokens* and the operations of *using* and *interpreting tokens*. By means of these concepts it is possible to formalize on the *type*-level the notions: *well-formed expression-type, possessing same manner of use types* and *possessing same manner of interpreting types, meaning* and *denotation* of expression-*types* (cf. Ajdukiewicz 1934; Wittgenstein 1953), and also the notion *interpretation* of expression-*types*. In this paper the author proves some relations among these notions.

[1] K. AJDUKIEWICZ, *Sprache und Sinn*, ***Erkenntnis***, vol. IV (1934), pp. 100–138.

[2] L. WITTGENSTEIN, ***Philosophical investigations***, Blackwell, Oxford, 1953.

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[4] U. WYBRANIEC-SKARDOWSKA, *Logical and philosophical ideas in certain approaches to language*, ***Synthese***, vol. 116 (1998), pp. 231–277.

[5] U. WYBRANIEC-SKARDOWSKA, *On denotation of quantifiers*, ***Logical ideas of Roman Suszko***, (M. Omyla, editor) Warsaw 2001, pp. 89–119.

▶ TATIANA YAVORSKAYA, *Axiomatic description of operations on proofs and labels*.
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Logic of proofs $\mathcal{LP}$ was introduced by S. Artemov in [1]. It is formulated in the propositional language extended by atoms of the form $[t]F$ which represent the proof predicate *"t is a proof of a formula F"*. Proofs are denoted by proof terms constructed by three elementary operations on proofs (namely, application, (positive) proof checking and nondeterministic choice). In [1] $\mathcal{LP}$ is proved to be complete with respect to the intended semantics.

We extend the language of the logic of proofs by the additional labelling predicate $x : F$ with the informal reading *"x is a label for a formula F"*. A typical example of a labelling predicate is the relation *"x is a code of a finite set of formulas containing F"*. Labelling predicate allows to specify operations on proofs depending on the formula argument. For example, we can specify an operation for the proof of disjunction

$$[t]A \wedge x\colon B \to [\mathsf{disj}(t,x)](A \vee B)$$

and negative proof checker

$$\neg[t]F \wedge x\colon F \to [\mathsf{check}^-(t,x)](\neg[t]F).$$

Different choices of operations on proofs and labels result in different logics. We have proved that for a wide class of operations the corresponding logics are complete with respect to the intended arithmetical semantics. Also, we study some natural examples of such logics.

The work is partially supported by the Russian Foundation for Basic Research.

[1] S. ARTEMOV, *Explicit provability and constructive semantics*, ***The Bulletin for Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

[2] K. GÖDEL, *Vortrag bei Zilsel (1938)*, (S. Feferman, editor) ***Kurt Gödel Collected Works***, vol. III, Oxford University Press, 1995.

▶ ROSTISLAV YAVORSKY, *Arithmetically complete fragment of the logic QGL*.
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The logic ***QGL*** is formulated in the first order language extended by the modal provability operator $\Box$. It is axiomatized over the first order calculus in the extended language by the following three schemes of axioms:

1. $\Box(A \to B) \to (\Box A \to \Box B)$
2. $\Box A \to \Box\Box A$
3. $\Box(\Box A \to A) \to \Box A$

The rules of inference are modus ponens, generalization and modal necessitation: $A \vdash \Box A$.

An arithmetical interpretation f_T of the language of ***QGL*** maps every atomic formula to an arithmetical formula with the same set of free variables. It commutes with boolean connectives, quantifiers and the operation of renaming of free variables. The modal operator $\Box$ is interpreted by the Gödel provability formula $\mathrm{Pr}_T()$ for an arithmetical theory T.

It is known that the set of arithmetically valid modal first order formulas is not axiomatizable, so ***QGL*** is not arithmetically complete. Let ***QGLb*** denote the following restricted fragment of the logic ***QGL***: we stipulate that for any subformula of the kind $\Box F$ the formula F has no free variables.

THEOREM. *The following are equivalent*:

(*i*) ***QGLb*** $\vdash A$;

(*ii*) *for any extension T of Peano Arithmetic by a true arithmetical sentence and for any arithmetical interpretation f_T one has $T \vdash f_T(A)$.*

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[1] G. BOOLOS, ***The logic of provability***, Cambridge University Press, 1993.

[2] R. E. YAVORSKY, *On arithmetical completeness of first-order logics of provability*, ***Advances in Modal Logic***, vol. 3 (2002), CSLI Publications (to appear).

▶ KONRAD ZDANOWSKI, *A criterion for equivalence of PFP(**Q**) with FO(**Q**)*.
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We give the model theoretic condition on a class of models $\mathcal{K}$ for equivalence on $\mathcal{K}$ of logic with partial fixed point operator extended by any set of generalized quantifiers ***Q*** with first order logic extended by quantifiers in ***Q***. In particular, when $\mathcal{K}$ is a class of models in monadic vocabulary or countable models of "almost sure" theory of graphs **PFP**(***Q***) $\equiv_{\mathcal{K}}$ ***FO***(***Q***), for any set of generalized quantifiers.

It follows then, that if an extension of partial fixed point logic (inflationary fixed point logic) by a class of generalized quantifiers ***Q*** captures **PSPACE** (**PTIME**), then on models in monadic vocabulary ***FO***(***Q***) captures **PSPACE** (**PTIME**).

We compare our criterion with game theoretic condition for equivalence of $L^{\omega}_{\infty\omega}$ with ***FO*** given in [1, Prop. 2.3.20].

[1] HEINZ-DIETER EBBINGHAUS and JÖRG FLUM, ***Finite Model Theory***, Springer, 1995.

▶ XUNWEI ZHOU, *Implicit inductive composition—the third way of logical theorem proving*.
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Mutually-inversistic logic is constructed by the author. In mutually-inversistic logic, logical theorem is proved by the third way: implicit inductive composition (the first two ways are logical axiomatic system and natural deduction system, both are erroneous).

In mutually-inversistic logic, logical operators are divided in to compounders and connectives, $\neg$, $\wedge$, $\vee$, etc. are compounders. $\overset{-1}{\leq}$ (mutually inverse implication, universality), $\overset{-1}{/\!\!\wedge}$ (mutually inverse conjunction, existenciality), etc. are connectives which are associated with inductive compostion and decomposition. The truth table of inductive composition of $\overset{-1}{\leq}$ is shown in Table 1, that of decomposition of $\overset{-1}{\leq}$ in Tables 2 and 3.

Table 1

A	B	$A \overset{-1}{\leq} B$
F	F	T
F	T	F/T
T	F	F
T	T	T

Table 2

$A \overset{-1}{\leq} B$	A	B
F	F	F/T
F	T	F/T
T	F	F/T
T	T	T

Table 3

$A \overset{-1}{\leq} B$	B	A
F	F	F/T
F	T	F/T
T	F	F
T	T	F/T

F/T in Table 1 means "needn't determine". F/T in Tables 2 and 3 means "unable to determine". The aim of Table 1 is to establish $A \overset{-1}{\leq} B$, the aims of Table 2 (modus ponens) and Table 3 (modus tollens) are to use the established $A \overset{-1}{\leq} B$ to make hypothetical inference.

$\{P \overset{-1}{\leq} Q\} \overset{-1}{\leq} \{Q \overset{-1}{/\!\!\wedge} P\}$ (corresponding to subalternation of immediate inference in Aristotelian logic) is proved by implicit inductive composition described in Table 1 like this: Suppose $A \overset{-1}{\leq} B$ is true, if $\{Q \overset{-1}{/\!\!\wedge} P\}$ is necessarily true, then $\{P \overset{-1}{\leq} Q\} \overset{-1}{\leq} \{Q \overset{-1}{/\!\!\wedge} P\}$ is proved.

From the viewpoint of mutually-inversistic logic, logical axiomatic system is wrong in that it intends to make third order decomposition which doesn't exist; natural deduction system is wrong in that it employs both $\rightarrow I$ (corresponding to implicit inductive composition) and $\rightarrow E$ (corresponding to decomposition), resulting in circular argument.

[1] RALPH M. EATON, ***General logic***, Charles Scribner's Sons, New York, 1931.

[2] A. G. HAMILTON, ***Logic for mathematicians***, Cambridge University Press, Cambridge, 1978.