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2002 SPRING MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Westin Seattle, Seattle, Washington
March 28–29, 2002

A Spring Meeting of the Association for Symbolic Logic was held March 28–29, 2002, in Seattle, Washington, at the Westin Seattle, in conjunction with the annual meeting of the Pacific Division of the American Philosophical Association. The Program Committee consisted of Aldo Antonelli and Paolo Mancosu (Chair). The program included three invited talks in a Special Session on **Predicativity: problems and prospects**. The following one-hour talks were presented:

Solomon Feferman (Stanford University), *What is predicativity*? *The path to unfolding*.
Geoffrey Hellman (University of Minnesota), *Predicativism as a philosophical position*.
Jeremy Avigad (Carnegie Mellon University), *Methodological predicativity*.

Aldo Antonelli and Richard Zach chaired the two sessions of contributed papers.
Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
PAOLO MANCOSU

Abstracts of invited talks

► JEREMY AVIGAD, *Methodological predicativity*.
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In this talk, I will discuss promising directions for both mathematical and philosophical research on predicativity. With respect to mathematical research, I will discuss issues related to the formalization of combinatorics and measure theory. On the philosophical side, it is an awkward fact that there seems to be no strong case that predicativity is a notion worthy of our attention; I will outline what I take to be the most promising strategy toward showing that it is.

► SOLOMON FEFERMAN, *What is predicativity? The path to unfolding*.
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The identification of what is essential to the concept of predicativity has undergone considerable transformation since it was introduced early in the 20th century by Poincaré. The talk will trace this evolution through, first, its analysis in terms of restricted notions of definability

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of sets of natural numbers via the ramified analytic hierarchy over the constructive ordinals, and then in terms of restricted notions of provability via autonomous transfinite progressions of ramified theories. The limit of the latter was determined in the early 1960s by Schütte and myself to be Γ_0. Though robust in various ways, that answer was not the end of the story. For all of this took the notions of predicative definability and provability to be *absolute* in some sense, given the natural numbers. A further shift came with the reconception of predicativity as a special case of the *relative* notions of *what one ought to accept if one has accepted given concepts and principles*. That has been made precise through the concept introduced in [1] of the *unfolding of a schematic theory* S. In joint work with Thomas Strahm [2], we have shown that the unfolding of a basic schematic system NFA of non-finitistic arithmetic is equivalent in strength to ramified analysis up to Γ_0. In work in progress, we show that the unfolding of the basic schematic system FA of finitist arithmetic is equivalent in strength to PRA. The talk will conclude with problems and prospects for further work on unfolding.

[1] Solomon Feferman, *Gödel's program for new axioms: Why, where, how and what?*, in ***Gödel '96*** (P. Hájek, editor), Lecture Notes in Logic, vol. 6 (1996), pp. 3–22.

[2] Solomon Feferman and Thomas Strahm, *The unfolding of non-finitist arithmetic*, ***Annals of Pure and Applied Logic***, vol. 104 (2000), pp. 75–96.

► GEOFFREY HELLMAN, *Predicativism as a philosophical position*.
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Basic *predicativity requirements* of explicit presentability of objects and predicatively acceptable proof are distinguished from *predicativist theses* of a philosophical character. Familiar among these are expressions of skepticism about the objectivity of full power sets of infinite sets, of uncountable ordinals, and of the classical continuum. Here we consider the epistemological thesis that warranted belief extends only to explicitly presentable objects and "predicatively provable" theorems (i.e., provable in systems of consistency strength $< \Gamma_0$). (The weaker claim that meeting these standards is desirable is not in question.) In particular, we focus on the claim that these standards mark limits to the pure mathematics supportable by indispensability arguments, either of the sort envisioned by Gödel or those of Quine and Putnam concerning scientific applications. Concerning an ongoing debate between Feferman and Friedman bearing on Gödel's program, we call attention to free-variable formulations of theorems such as Kruskal's and the Graph Minor theorem, shown by Friedman to transcend predicative provability. Such theorems escape criticisms of "unmathematicality" and, given their form, ought to be respected by the predicativist. Further challenges are raised concerning mathematical physics and, more generally, the stability and robustness of the predicativist position.

Abstracts of contributed talks

► KATALIN BIMBÓ, *The Church-Rosser property in symmetric combinatory logic*.
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Dual combinators were introduced within structurally free logics LC (cf. [2]). A dual combinator is similar to a combinator except that it takes its arguments from the left. (E.g., the fusion combinator S forms a redex with x, y, z as in the term $((\mathsf{S}x)y)z$, and the dual fusion combinator s forms a redex: $z(y(x\mathsf{s}))$. In *symmetric λ-calculus* (cf. [1]) a λ-abstract $\lambda x.M$ may form a β-redex with N in two ways: $(\lambda x.M)N$ and $N(\lambda x.M)$.

We formally define *symmetric combinatory logic* with combinators that apply to arguments from both sides. E.g., the symmetric fusion combinator $\bar{\mathsf{S}}$ forms a redex in two ways: $z(y(x\bar{\mathsf{S}}))$ and $\bar{\mathsf{S}}xyz$. We prove that if there are at least two (proper) symmetric combinators in the combinatory base, then the generated set of terms (with denumerable many variables included) lacks the Church-Rosser property. This result is similar to the one for dual combinatory logic, however, the proof differs in details. The main reason why a new proof is needed is that in combinatory logic (in dual combinatory logic) if $(M\mathsf{Z})$ $((\mathsf{z}M))$ is a subterm of a term (where Z (z) is a combinator (a dual combinator)), then Z (z) cannot be the head of a redex. In symmetric combinatory logic this conclusion does not follow. Nonetheless, the proof for the symmetric system is considerably simpler than that for the dual one.

[1] F. BARBANERA and S. BERARDI, *A symmetric lambda calculus for classical program extraction*, ***Information and Computation***, vol. 125 (1996), p. 103–117.

[2] J. M. DUNN and R. K. MEYER, *Combinatory logic and structurally free logic*, ***The Logic Journal of IGPL***, vol. 5 (1997), pp. 505–537.

[3] B. K. ROSEN, *Tree-manipulating systems and Church-Rosser theorems*, ***Journal of the Association for Computing Machinery***, vol. 20 (1973), pp. 160–187.

► PHILIP EHRLICH, *Surreal numbers: an alternative construction*.
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J. H. Conway introduced an ordered field *No* of surreal numbers [1] which contains (in a suitable sense that can be made precise [3]) "all numbers great and small." However, in addition to its distinguished structure as an ordered field *No* has a rich algebraico-tree-theoretic structure which depends upon *No*'s structure as a lexicographically ordered binary tree [3]. In the present paper we examine the relative merits of the standard constructions of surreal numbers with the following construction introduced by the author [2, p. 256] which brings *No*'s ordered-tree-theoretic structure to the fore.

CONSTRUCTION OF GAMES. If L and R are any two sets of games, then there is a game (L, R). All games are constructed in this way.

DEFINITIONS. A game x is said to be *simpler than* a game $y = (L_y, R_y)$, written $x <_s y$, if $x \in L_y$ or $x \in R_y$; a chain of games (ordered by $<_s$) is said to be *ancestral* if it is closed under $<_s$; and a partition L, R of an ancestral chain of games is said to be *orderly* if $L_x \subseteq L$ and $R_x \subseteq R$ for each element $x = (L_x, R_x)$ of the chain.

CONSTRUCTION OF NUMBERS. If L, R is an orderly partition of an ancestral chain of surreal numbers, then there is a surreal number (L, R). All surreal numbers are constructed in this way.

DEFINITION OF ORDER. For all surreal numbers x and y, $x < y$ if and only if $x \in L_y$ or $y \in R_x$ or $R_x \cap L_y \neq \emptyset$.

[1] J. H. CONWAY, ***On numbers and games***, Academic Press, 1976.

[2] P. EHRLICH, *All numbers great and small*, in ***Real numbers, generalizations of the reals, and theories of continua***, Kluwer Academic Publishers, 1994, pp. 239–258.

[3] ———, *Number systems with simplicity hierarchies: a generalization of Conway's theory of surreal numbers*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1231–1258.

► ELAINE LANDRY, *Sorts, systems and structure: A category-theoretic analysis of mathematical and logical structure*.
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This paper will consider the historical development and current use of the notion of structure. I will investigate the algebraic history of the mathematical notion of structure (Corry, [1]) and claim that mathematical structuralists (in particular, Shapiro, [3]) implicitly, and problematically, presume a set-theoretic notion of structure. The aim here is to show how a category-theoretic notion of structure allows for a more perspicuous account of both mathematical and logical structure.

If we take a system (mathematical or logical) to be defined as a collection of 'sorts', then a cat-structured system is one which characterizes sorts as 'objects' and 'morphisms' which satisfy the category axioms. The significance of this definition is two-fold: (1) it allows us to avoid set-theoretic underpinnings (i.e., 'objects' need not be elements or sets and 'morphisms' need not be functions), and (2) it allows us to characterize both mathematical and logical structure. Thus, besides offering an alternative to set-theoretic structures, this investigation will be used to inform the "logic as language" versus "logic as calculus" debate.

[1] L. Corry, ***Modern algebra and the rise of mathematical structures***, Springer-Verlag, New York, 1996.

[2] W. Goldfarb, *Frege's conception of logic*, in ***Futures past: Reflections on the history and nature of analytic philosophy*** (J. Floyd and S. Shieh, editors), Harvard University Press, Massachusetts, 2001.

[3] S. Shapiro, ***Philosophy of mathematics: Structure and ontology***, Oxford University Press, Oxford, 1997.

► BENEDIKT LÖWE AND MICHAEL MÖLLERFELD, *Where does the complexity of revision come from?*
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Already from Burgess' [2], it had been known that Revision Theory of Truth leads to rather complicated sets of truth. Since then, several people dealt with the "Complexity of Revision" [1]. Recently, Philip Welch has computed [4] that the revision-theoretically definable reals are exactly the Π^1_2 reals (for a proof sketch, cf. [3]).

In this talk, we shall identify what part of revision theory contributes to this complexity. We shall show that the revision process itself is only responsible for part of the increase in complexity compared to inductive definitions.

[1] Gian Aldo Antonelli, *The complexity of revision*, ***Notre Dame Journal of Formal Logic***, vol. 35 (1994), pp. 67–72.

[2] John P. Burgess, *The truth is never simple*, ***Journal of Symbolic Logic***, vol. 51 (1986), pp. 663–681.

[3] Benedikt Löwe and Philip D. Welch, *Set-theoretic absoluteness and the revision theory of truth*, ***Studia Logica***, vol. 68 (2001), pp. 21–41.

[4] Philip D. Welch, *On revision operators*, preprint.

► RICHARD ZACH, *Hilbert's "Verunglückter Beweis" and the epsilon-theorem.*
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In correspondence from 1929, Paul Bernays mentions a first attempt by Hilbert to prove the eliminability of the ε-symbol from proofs of ε-free formulas. This attempt, the so-called "verunglückte Beweis," suffered from various difficulties which made it unsuitable as the basis for giving consistency proofs for mathematical theories. Hilbert's subsequent approaches led to the development of the ε-substitution method used in Ackermann's dissertation [1]. The "verunglückte Beweis" is nevertheless of some interest, since it is closely connected to the proof given by Bernays of the first ε-theorem in [2]. The basis of a discussion of the "verunglückten Beweis" is a six-page note in Bernays' hand bound with lecture notes to an early course by Hilbert on the foundations of mathematics.

[1] WILHELM ACKERMANN, *Begründung des "tertium non datur" mittels der Hilbertschen Theorie der Widerspruchsfreiheit*, ***Mathematische Annalen***, vol. 93 (1924), pp. 1–36.

[2] DAVID HILBERT and PAUL BERNAYS, ***Grundlagen der Mathematik II***, Springer-Verlag, Berlin, 1939.

Abstracts presented by title

► CYRUS F. NOURANI, *Virtual tree computing, meta-contextual logic, and virtual reality*.
Project Metaai and the Academia; last USA appointment at UCSB; P. O. Box 278, Cardiff By The Sea, California 92007, USA.
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The preliminaries to VR computing logic are presented.

THEOREM 1 ([1]). *Soundness and Completeness-Morph Gentzen Logic is sound and complete.*

DEFINITION 1. A language L is intelligent context free, abbreviated by ICF, if and only if L is intelligent and there is a context free grammar defining L.

DEFINITION 2. A language is String Intelligent if and only if it is an intelligent language and all agent functions in the language are one-to-one functions. The following starts ICF.

THEOREM 2. *String Intelligent languages are ICF.*

PROPOSITION. *Morph Gentzen and Intelligent languages provide a sound and complete logical basis to VR.*

VIRTUAL TREE MODELS. A virtual tree, or virtual proof tree is a proof tree that is constructed with agent languages with free Skolem functions.

DEFINITION 3. A generic diagram for a structure M is a diagram such that there is a proper diagram definition with specific function symbols, for example Σ_1 Skolem functions.

THEOREM 3. *For the virtual proof trees defined for a goal formula from the G-diagram there is an initial model satisfying the goal formulas. It is the canonical model definable from the G-diagram.*

THEOREM 4. *The Hilbert's epsilon technique implies there is a virtual tree model M for the set of formulas such that we can take an existentially quantified formula $w[X]$ and have it instantiated by a Skolem function which can answer the satisfiability question for the model.*

[1] C. F. NOURANI, *MIM Logik*, Summer Logic Colloquium (Prague, August 1998), 1997.

[2] ———, *Intelligent languages—A preliminary syntactic theory*, May 15, 1995, Mathematical Foundations of Computer Science; 1998, 23rd International Symposium, Brno, Czech Republic, August. The satellite workshop on Grammar systems. Silesian University, Faculty of Philosophy and Sciences, Institute of Computer Science.

► ALADDIN M. YAQUB, *Structuralism and mathematical practice*.
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I argue that central aspects of mathematical practice lend strong support to mathematical structuralism, which is the thesis that mathematics is an objective study whose subject matter is abstract structures. The argument is abductive: Compared to its competitors, it is mathematical structuralism that best explains these central aspects of mathematical practice. The aspects under consideration are mathematical explanation and application within mathematics itself. This approach departs from customary philosophical analysis, in which explanation is thought to exist exclusively in the natural and social sciences, and mathematical application is understood to be the application of mathematics in the sciences. Much of the current research in the philosophy of mathematics focuses on the philosophical implications of the applicability of mathematics in science—its external applicability. My goal is to rethink this focus and redirect our attention to the internal applicability of mathematics and to its (usually neglected) explanatory nature. An attractive feature of this approach is its ability to shed some light on what Eugene Wigner famously dubbed "the unreasonable effectiveness of mathematics." This is the striking phenomenon of abstract mathematical models being applicable to diverse, concrete problems in the natural and social sciences. The core idea of the argument, briefly and crudely sketched, is this: If mathematics is the systematic and analytic study of abstract structures, then it should be expected that (1) various parts of mathematics offer explanations and applications in other parts of it, and (2) to the extent that nature and its products are intelligible, many natural systems exhibit (or at least, approximate) structures of which mathematics offers descriptions and analyses.