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2001–2002 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Diego Marriott Hotel and Marina, San Diego, California, January 8–9, 2002

A Winter Meeting of the Association for Symbolic Logic was held on January 8–9, 2002, at the Marriott Hotel in the San Diego Convention Center in conjunction with the annual Joint Mathematics Meetings. The members of the Program Committee were G. Hjorth (Chair), C. Miller, J. Remmel, and V. Vianu. The program included nine invited 50-minute talks and ten contributed talks. The ASL hosted a welcoming reception on January 8th in the Cardiff Room of the Marriott. In addition, on January 6–7, prior to the ASL Meeting, there was a joint AMS-ASL Special Session, **Set Theory and Classification Problems**, organized by Simon R. Thomas, and an AMS Special Session, **Computability Theory with Applications**, organized by Douglas Cenzer and Jeffrey Remmel.

The 50-minute invited addresses were:

Michael Benedikt (Bell Laboratories Research), *Model theory, finite automata, and string relations*.

John D. Clemens (California Institute of Technology), who gave the talk on behalf of Daniel Jackson (MIT) as co-author of their paper, *Classifying Borel automorphisms*.

Rod Downey (Victoria University), *Algorithmic randomness, computability, and reals*.

Sergey S. Goncharov (Novosibirsk State University), *Invariants in the theory of computable models*.

Martin Grohe (University of Edinburgh), *Learning in finite models*.

Leonard Libkin (University of Toronto), *Expressing graph and topological connectivity over the reals*.

Philipp Rothmaler (Ohio State University at Lima), *Elementary epimorphisms*.

Moshe Y. Vardi (Rice University), 0-1 *Laws for fragments of existential second order logic: a survey*.

Martin Zeman (University of California, Irvine), *Some applications of inner models*.

Leonard Libkin was unable to attend the conference due to the birth of a child, and Michael Benedikt kindly stood in to give not only his own talk but also that of Libkin.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
GREG HJORTH

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Abstracts of invited talks

► MICHAEL BENEDIKT, *Model theory, finite automata, and string relations*.
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I'll consider several models based on the set of words over an alphabet, and the set of trees. First, I'll present results about the basic model-theoretic properties of the resulting structures: quantifier-elimination, definability, independence, and connections to automata theory. I'll then move to definability in expansions of these structures with a free predicate symbol, and (if time permits), talk about relationship of the resulting languages to applications in querying trees and strings. The talk also includes work with Leonid Libkin, Thomas Schwentick, and Luc Segoufin.

► JOHN D. CLEMENS, *Classifying Borel automorphisms*.
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I will discuss the difficulty of classifying Borel automorphisms of a Polish space up to isomorphism, where two automorphisms are said to be isomorphic if they are conjugate via a third Borel automorphism. I will show, in particular, that any Borel equivalence relation is reducible to this isomorphism relation, and that the relation is Σ^1_2-complete, so it is quite complicated.

► ROD DOWNEY, *Algorithmic randomness, computability, and reals*.
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There are various notions of algorithmic randomness, such as Schnorr randomness, Martin-Löf randomness, Chaitin randomness, etc. These are usually based on effective topology ("random means avoid all effectively null properties"), martingales ("random is hard to predict") or compressibility ("random strings are hard to compute"). We will look at the background in this area, and discuss recent results by the author together with Hirschfeldt, Laforte, Stephan, and Nies. In particular, we will look at recent work calibrating randomness, and a surprising solution to Post's problem.

► SERGEY S. GONCHAROV, *Invariants in the theory of computable models*.
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The subject of my talk deals with some methods and their connections with results in the theory of computable models. We will discuss the existence problems for theories with different properties based on methods that used certain types of invariants. We will consider some old and new results about countably categorical as well as uncountably categorical theories. Another direction in the theory of computable models is connected with describing complexities of problems relative to some hierarchies and definitions. We study the complexity of invariant properties and their dependence on different computable presentations and definability. This approach started from the work of Nerode and Ash. This research is supported by NSF and RFBR grants.

► MARTIN GROHE, *Learning in finite models*.
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The general problem of concept learning is to identify an unknown set from a given class of sets. The given class, referred to as the concept class, represents the possible classifications of the elements of the universe, and the unknown set, also called the target concept, is the actual classification. We consider the setup of predicate logic learning in finite structures, where the universe is the set of elements (or, more generally, tuples of elements) of a finite relational structure, and concepts are classes of sets definable by predicate logic formulas.

The identification of the target concept can be exact, as for example in the model of learning with equivalence queries, or approximate, as in the model of probably approximately correct (PAC) learning. It is an interesting fact that several notions of learning complexity turn out to be closely related to certain combinatorial parameters of the concept class. In particular, the sample size for PAC learning is of the same order of magnitude as the Vapnik-Chervonenkis dimension of the concept class, and the query complexity of learning with equivalence queries is closely related to the strong consistency dimension of the concept class.

While it is easy to see that these combinatorial parameters are unbounded for predicate logic learning on arbitrary finite structures, we will show that they can be bounded on certain restricted classes of structures such as trees or planar graphs, for the former even if concepts are defined by formulas of monadic second-order logic. These bounds imply positive learnability results for the PAC and equivalence query learnability of definable sets over these structures.

This is joint work with Gyorgy Turan.

► LEONID LIBKIN, *Expressing graph and topological connectivity over the reals*.
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We consider expressing properties of finite and definable sets embedded in an infinite structure. This setting attracted database researchers about ten years ago, as it represents a model of query languages for geographical data, as well as for data over an interpreted domain. Motivated by geographical applications, we consider several structures over the reals, and study the expressive power of first-order logic over the language that involves an extra predicate, interpreted as a finite or a definable set. We show that the finite case is crucial for characterizing the expressive power, and survey a number of results that relate expressibility over a finite set to the model-theoretic properties of the underlying structure. From these results we derive inexpressibility of graph and topological connectivity. As those are desirable properties from the point of view of query languages, we consider extensions of first-order logic that can express them. In finite model theory, such extensions are usually obtained by adding a fixed point construction. In our case, these extensions are obtained by adding quantification over continuous curves, and we show that the resulting logics have a number of nice properties.

► PHILIPP ROTHMALER, *Elementary epimorphisms*.
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I will discuss a notion dual to that of elementary monomorphism (the standard morphism used in model theory).

► MOSHE Y. VARDI, 0-1 *laws for fragments of existential second-order logic*.
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The probability of a property on the collection of all finite relational structures is the limit, as n goes to infinity, of the fraction of structures with n elements satisfying the property, provided the limit exists. It is known that the 0-1 law holds for every property expressible in first-order logic, i.e., the probability of every such property exists and is either 0 or 1. Moreover, the associated decision problem for the probabilities is solvable.

In this survey, we consider fragments of existential second-order logic in which we restrict the patterns of first-order quantifiers. We focus on fragments in which the first-order part belongs to a prefix class. We show that the classifications of prefix classes of first-order logic with equality according to the solvability of the finite satisfiability problem and according to the 0-1 law for the corresponding existential second-order fragments are identical, but the classifications are different without equality.

The talk represents joint work with Phokion Kolaitis, UC Santa Cruz.

► MARTIN ZEMAN, *Some applications of inner models*.
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I will discuss some applications of inner model theory and infinite combinatorics.

Abstracts of contributed talks

► OTÁVIO BUENO, *On the referential indeterminacy of logical and mathematical concepts*.
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Hartry Field has recently examined the question whether our logical and mathematical concept are referentially indeterminate (see Field [1] and [2]). In his view, (1) certain logical notions, such as second-order quantification, are indeterminate, but (2) important mathematical notions, such as the notion of finiteness, are not (they are determinate). In this paper, I assess Field's analysis, and argue that claims (1) and (2) turn out to be inconsistent. After all, given that the notion of finiteness can only be adequately characterized in pure second-order logic, if Field is right in claiming that second-order quantification is indeterminate (see (1)), it follows that finiteness is also indeterminate (contrary to (2)). After arguing that Field is committed to these claims, I provide a diagnosis of why this inconsistency emerged, and I suggest an alternative, consistent picture of the relationship between logical and mathematical indeterminacy.

[1] H. Field, *Are our logical and mathematical concepts highly indeterminate*?, in ***Midwest studies in philosophy***, vol. 19 (P. A. French, T. E. Uehling, Jr., and H. K. Wettstein, editors), 1994, pp. 391–429.

[2] ———, *Do we have a determinate conception of finiteness and natural number*?, in ***The philosophy of mathematics today*** (M. Schirn, editor), Clarendon Press, Oxford, 1998, pp. 99–129.

► JOHN A. CHISHOLM, *Steps toward a "linkless" proof of Lachlan's non-bounding theorem—Preliminary report*.
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Lachlan's non-bounding theorem states that there exists a noncomputable c. e. degree with no c. e. minimal pair below it. We show how adding additional constraints for opening gaps alleviate complexities in $0'''$-arguments of traditional proofs.

► TIM COOKSON, *Independence from cardinal arithmetic*.

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Many statements in the language of set theory which are provably independent of ZFC are decided by ZFC + (assumptions about cardinal arithmetic). We say an extension $M \subseteq N$, $M, N \models ZFC$ is *mild* if and only if $\text{Card}^M = \text{Card}^N$, and $\forall \kappa, \lambda \in \text{Card}^M\ [(\kappa^\lambda)^M = (\kappa^\lambda)^N]$. Similarly we say a forcing notion P is mild for M if and only if for all G, P generic for M, $M[G]$ is a mild extension of M. If for any model M of ZFC we can find two mild forcings $\mathbf{P}$ and $\mathbf{Q}$ such that $\mathbf{P}$ forces our sentence to be true and $\mathbf{Q}$ forces it to be false, then we say this sentence is independent from cardinal arithmetic. I will present a particular sentence that is independent from cardinal arithmetic.

I will also present some reals in the random $\times$ random real extension that I have constructed in an attempt to classify all reals in this extension, up to equivalence over the ground model. In particular I will discuss the construction of a Cohen $\times$ random $\times$ random generic real.

► OMAR DE LA CRUZ, *Non-well-orderable sets*.

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We compare non-well-orderable sets by studying the structure obtained from each set (assumed to contain only atoms) by repeated application of the power set operation.

► RUMEN DIMITROV, VALENTINA HARIZANOV, AND ANDREI MOROZOV, *Dependence relations in computably rigid computable vector spaces*.

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Metakides and Nerode showed that the study of computably enumerable (c. e.) vector spaces can be reduced to the study of c. e. subspaces of V_∞, an $\aleph_0$-dimensional vector space over a computable field F, consisting of all finitely nonzero ω-sequences of elements of F under pointwise operations. We assume that F is infinite and rigid. The k-dependence degree ($k \geq 1$) of a space V is the Turing degree of the set

$$\{\, (x_1, \dots, x_k) \in V_\infty^k : x_1, \dots, x_k \text{ are dependent over } V \,\}.$$

It is easy to see that for any c. e. subspace, its k-dependence degrees form a uniformly increasing sequence of c. e. degrees. Shore [3] established that any such sequence of dependence degrees is possible.

Morozov [2] constructed a computable vector space V such that the space V_∞ mod V has trivial computable automorphism group. That is, the only computable automorphisms of V_∞ mod V are given by scalar multiplications. Morozov asked whether it is possible for such a space to have a k-dependence algorithm and not a $(k+1)$-dependence algorithm. We answer this question positively by establishing the following result.

THEOREM. *Suppose that $\delta_2 \leq \cdots \leq \delta_m$ is an arbitrary increasing sequence of c. e. Turing degrees. Then there is a computable space V such that V_∞* mod *V has trivial computable automorphism group, and for every $k \in \{2, \dots, m\}$, the k-dependence degree of V is δ_k.*

[1] R. D. DIMITROV, V. S. HARIZANOV, and A. S. MOROZOV, *Dependence relations in computably rigid computable vector spaces*, in preparation.

[2] A. S. MOROZOV, *Rigid constructive modules*, ***Algebra and Logic***, vol. 28 (1989), no. 5, pp. 570–583 (Russian); (1990), pp. 379–387 (English translation).

[3] R. A. SHORE, *Controlling the dependence degree of a recursively enumerable vector space*, ***Journal of Symbolic Logic***, vol. 43 (1978), pp. 13–22.

► JEFFRY L. HIRST, *Reverse mathematics of transfinite triangular numbers*.
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In [1], the authors prove that if γ is an ordinal and f is a non-decreasing ordinal valued function with domain ω^γ, then

$$\sum_{\alpha<\omega^\gamma} f(\alpha) = \sup\{ f(\alpha) \cdot \omega^\gamma \mid \alpha < \omega^\gamma \}.$$

This lemma generalizes an exercise in Sierpiński [2] which asserts that for each natural number n, $\sum_{\alpha<\omega^n} \alpha = \omega^{2n-1}$. Working in the framework of reverse mathematics, we can show that RCA_0 can prove each case of Sierpiński's exercise and that the lemma is equivalent to ATR_0, proving that the lemma is much stronger than Sierpiński's exercise.

[1] JAMES P. JONES, HILBERT LEVITZ, and WARREN D. NICHOLS, *On series of ordinals and combinatorics*, ***Mathematical Logic Quarterly***, vol. 43 (1997), pp. 121–133.

[2] WACŁAW SIERPIŃSKI, ***Cardinal and ordinal numbers***, Polska Akademia Nauk, Monografie Matematyczne, Państwowe Wydawnictwo Naukowe, Warszawa, 1958.

► TIMOTHY H. MCNICHOLL, *Homomorphisms of strong reducibility structures*.
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For all n, let $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ be the set of all truth-table degrees that contain an n-c. e. set. Let $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}} = \mathcal{D}^{\mathrm{tt}}_{1\text{-c. e.}}$. We show that for all n, any proper final segment of $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ can be embedded into any proper final segment of $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ in such a way that zeros and ones are preserved. We also show that for all n and any $d \in \mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ there is an embedding of $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ into $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ that preserves zeros and ones and that hits d. We answer a question of Downey by showing that for all n, $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ is co-embeddable with the c. e. wtt-degrees in a way that preserves zeros and ones.

We also discuss current work on constructing a non-trivial automorphism of the c. e. weak truth-table degrees.

► ROBERT S. MILNIKEL, *Explicit non-provability and nonmonotonic S4*.
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In modal nonmonotonic logics, lack of proof of a formula φ is sufficient to conclude $\neg\Box\varphi$. We will make use of explicit modal operators from Artemov's Logic of Proofs [1] to establish non-provability of formulas in S4. Our eventual goal is to incorporate the these into sequent proofs for S4 skeptical consequence.

[1] Sergei N. Artemov, *Explicit provability and constructive semantics*, ***Bulletin of Symbolic Logic***, vol. 7 (2001), pp. 1–36.

► WIM RUITENBURG, *Classical model theory and Kripke models*.
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Part of this paper is derived from joint work with Mohammad Ardeshir. We combine the main completeness theorems for T-normal Kripke structures, and for Kripke structures satisfying bounded-extension constraints, into a single general completeness theorem. We include applications to the Kripke model theory of intuitionistic arithmetic and algebra.

[1] Mohammad Ardeshir, Wim Ruitenburg, and Saaed Salehi, *Intuitionistic axiomatizations for bounded extension Kripke models*, 1999, preprint (submitted).

[2] Samuel R. Buss, *Intuitionistic validity in T-normal Kripke structures*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 159–173.

► DAN E. WILLARD, *Further generalizations and exceptions for the second incompleteness theorem*.
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All our results concern axiom systems α that treat Addition and Multiplication as 3-variable predicates, denoted as $A(x, y, z)$ and $M(x, y, z)$, rather than as total functions. There were two notable theorems from our full-length papers [1, 2], not mentioned in [3]'s conference talk.

The first concerns a phenomena, called the "Continuously Expansion" Property. Define an axiom system α to be *Continuously Expanding* if and only if there exists an increasingly infinitely-long sequence of constants $k_1 < k_2 < k_3 < \dots$ such that the set of α's axioms whose Gödel number is less than k_i can prove the existence of an integer greater than k_{i+1}. We observed in [2] that our older results, published in Section 3 of [1], generalize when their "Group-zero" subschemes are made to correspond to certain particular Continuously Expanding systems. These unusual exceptions to the Second Incompleteness Theorems, can *simultaneously* recognize their Hilbert consistency, prove all Peano Arithmetic's Π_1 theorems (when they are rewritten in a language using the $A(x, y, z)$ and $M(x, y, z)$ symbols), and additionally satisfy a "Tangibility Reflection" property, analogous to that published about "ISREF(A)" in [1].

A second topic, not mentioned in [3], was "*How its theorems would change* (?)", if we considered axiom systems that proved all the Π_1 theorems of "$\mathrm{I}\Sigma_0$" rather than of "Peano Arithmetic"? The brief answer to this question is that all its results would remain valid, verbatim, if we replaced the words "Peano Arithmetic" with "$\mathrm{I}\Sigma_0$". More precisely, the Second Incompleteness Theorem clearly becomes more interesting when it applies to weaker formalisms, but the boundary-case exceptions to it become *substantially weaker* in this same case. Thus while all [3]'s results remain valid when one changes the words "Peano Arithmetic" to "$\mathrm{I}\Sigma_0$", this change in terminology improves only half its results.

[1] D. Willard, *Self-verifying systems, the incompleteness theorem and the tangibility reflection principle*, ***Journal of Symbolic Logic***, vol. 66 (June 2001), pp. 536–596.

[2] ———, *A generalization of the second incompleteness theorem and some exceptions to it*, recently issued report (January 2001) that has been submitted for journal publication.

[3] ———, *Generalizations and some exceptions for the incompleteness theorem*, a talk given at the 2001 ASL European Summer Meeting (Logic Colloquium '01, Vienna) in August 2001, whose abstract will soon be published by the ***Bulletin of Symbolic Logic***.