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2001 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '01

Vienna, Austria, August 6–11, 2001

Logic Colloquium'01, the 2001 European Meeting of the Association for Symbolic Logic, took place at the Vienna University of Technology in Austria, on August 6–11, 2001. The Program Committee consisted of M. Arslanov, M. Baaz, W. Buchholz, L. van den Dries, S. Friedman, D. Isaacson, C. Jockusch, J. Krajíček (Chair), L. Newelski (resigned April 2000), J. Paris, J.-P. Ressayre, B. Veličković, and A. Visser. The Colloquium was very well organized and smoothly run by the Local Organizing Committee whose members included M. Baaz (Chair), A. Beckmann, A. Ciabattoni, C. Fermüller, S. Friedman, M. Goldstern, G. Gottlob, F. Gusel, A. Leitsch , G. Moser, N. Preining, G. Salzer, R.-D. Schindler, H. Veith, and M. Zeman. Generous support from the Kurt Gödel Society, the Austrian Federal Ministry of Education, Science, and Culture, the Vienna Convention Bureau, the European Union, the U.S. National Science Foundation, and the Association for Symbolic Logic is gratefully acknowledged.

The main topics of the conference were Model theory, Set theory, Computability theory, Proof theory and proof search, Philosophy and history of logic, and Computer science and complexity theory. The program included two tutorials, fifteen invited plenary lectures, and seventeen invited talks in six special sessions corresponding to the main topics. There also were 126 contributed talks, and 35 talks presented by title. In total, 248 participants registered for the Colloquium.

The life and work of Kurt Gödel served as the main informal theme of the meeting. K. Gödel lived and worked in Vienna when he proved his famous Incompleteness theorem, published exactly seventy years ago. To commemorate this occasion the organizers arranged for special lectures by G. Kreisel, *A consumer's* (*re*)*view around higher infinities*, and by G. Takeuti, *Incompleteness theorem and its frontier*; the latter was delivered at a splendid banquet in the beautiful City Hall. The organizers also scheduled talks for one afternoon at the institute of the University of Vienna, where Gödel lectured. Last but not least, the atmosphere of numerous Viennese cafes offered the LC'01 participants a glimpse of the world of K. Gödel, R. Carnap, and their many famous contemporaries.

The following tutorials were presented:

Itay Neeman (Berlin), *Determinacy of long games*.
Alexander Leitsch (Vienna), *Resolution theorem proving*.

The following invited plenary lectures were given:

Toshiyasu Arai (Hiroshima), *Wellfoundedness proofs in reflecting universes*.
Steve Awodey (Pittsburgh), *Modal and intensional types in categorical logic*.
James Cummings (Pittsburgh), *Squares, scales and reflection*.
Rod Downey (Wellington), *Calibrating randomness*.
Harvey Friedman (Columbus), *Recent incompleteness*.

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Warren Goldfarb (Cambridge, USA), *Gödel and Carnap: Influence and opposition.*
Olga Kharlampovich (Montreal), *First order properties of free groups.*
Julia F. Knight (Notre Dame), *Quantifiers and complexity.*
Michiel van Lambalgen (Amsterdam), *Moschovakis' notion of meaning as applied to linguistics.*
Chris Miller (Columbus), *Tameness phenomena in expansions of the field of real numbers.*
Ran Raz (Rehovot), *Resolution lower bounds for the weak pigeonhole principle.*
Saharon Shelah (Jerusalem), $(\aleph_n, \aleph_0)$ *may fail* $\aleph_0$*-compactness.*
Jouko Väänänen (Helsinki), *Models, trees and infinitary back-and-forth.*
Frank Wagner (Lyon), *Simplicity theory.*
Stanley S. Wainer (Leeds), *Provable recursiveness and complexity.*

Abstracts of invited and contributed talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
J. Krajíček

Abstracts of invited tutorials

► ALEXANDER LEITSCH, *Resolution theorem proving.*
Institut für Computersprachen, TU of Vienna, Favoriten Strasse 9, 1040 Vienna, Austria.
E-mail: leitsch@logic.at.

The purpose of this tutorial is to present the resolution calculus and to illustrate its impact on theorem proving, deduction and mathematical logic. The main topics are normal form transformation, completeness, refinements and redundancy deletion. In addition to these "traditional" topics of resolution theorem proving we demonstrate the power of resolution as decision procedure and, in particular, as a method to deal with the classical decision problem of first-order logic. Moreover we compare resolution with logical calculi (e.g., the Gentzen calculus LK) and give an analysis of proof complexity. Finally we illustrate the potential of resolution for the development of new proof theoretic algorithms (e.g., concerning cut-elimination).

► ITAY NEEMAN, *Determinacy of long games.*
Department of Mathematics, University of California, Los Angeles, California 90095-1555, USA.
E-mail: ineeman@math.ucla.edu.

I will present the basic ingredients used in determinacy proofs for long games, and illustrate their use in two particular cases: games of length $\omega \cdot \omega$, and games of continuously coded length. These techniques can be used as building blocks for proofs of determinacy in a wide variety of cases. I hope to give an impression of how the different blocks fit together.

Abstracts of invited plenary talks

► TOSHIYASU ARAI, *Wellfoundedness proofs in reflecting universes.*
Graduate School of Science and Technology, Kobe University, Rokko-dui, Nada-ku, Kobe, 657-8501, Japan.
E-mail: arai@kurt.cla.kobe-u.ac.jp.

First order reflecting ordinals were introduced by Richter-Aczel to formulate constructive analogues for indescribable cardinals, and were shown to be related to closure ordinals of first order non-monotonic inductive definitions on ω. For example, Π_3-reflecting ordinals are recursive analogues for weakly compact, i.e., Π^1_1-indescribable cardinals, and the least one is the supremum of closure ordinals of Π^0_2-inductive definitions. Let $\mathrm{KP}\Pi_n$ $(2 < n < \omega)$ denote a set theory whose intended models are L_π with Π_n-reflecting ordinals π. These theories $\mathrm{KP}\Pi_n$ are weaker than the subsystem of second order arithmetic with Σ^1_2-Comprehension Axiom since the least Π_n-reflecting ordinal is smaller than nonprojectible ordinals. Thus the proof-theoretic analyses of $\mathrm{KP}\Pi_n$ form intermediate stages toward nonprojectibles. We have designed a recursive notation system $\mathrm{Od}(\Pi_n)$ of ordinals so that the order type of its countable fragment is the proof-theoretic ordinal of $\mathrm{KP}\Pi_n$. One half of this result was accomplished by cut-elimination. The order was to show that $\mathrm{KP}\Pi_n$ proves the wellfoundedness of $\mathrm{Od}(\Pi_n)$ up to each countable ordinal in it. The first proof was obtained by relativizing iteratedly the maximal distinguished class, a powerful tool for proving wellfoundedness due to Buchholz. The class is Σ_1-definable, and hence a proper class in $\mathrm{KP}\Pi_n$. Behind the iterated relativization, there involved resolving Π_n-recursively Mahlo operations into iterations of Π_2-recursively Mahlo operations. The iteration is supported by Π_n-reflection. Now $\mathrm{KP}\Pi_n$ proves that any Π^0_{n-1}-inductive definition eventually reaches to a closed point. So, instead of distinguished classes we may prove the wellfoundedness by means of inductive definitions. In this talk, we expound the wellfoundedness proofs of recursive notation systems for reflecting ordinals by relevant inductive definitions.

► STEVE AWODEY, *Modal and intensional types in categorical logic*.
Department of Philosophy, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890, USA.
E-mail: awodey@cmu.edu.

Some recent developments in categorical semantics of type theory are presented. Certain modal and intensional type constructs are treated using methods from topos theory. As an application, type-theoretic constructibility is related to classical and intuitionistic first-order validity.

► JAMES CUMMINGS, *Squares, scales and reflection*.
Department of Mathematical Science, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890, USA.
E-mail: jcumming@andrew.cmu.edu.

I will discuss two recent results concerning stationary reflection. In the first result ideas from PCF theory are used to prove a "compactness theorem" for Dzamonja and Shelah's "strong non-reflection" property. In the second result forcing is used to clarify the relationship between squares, stationary reflection and some large cardinal principles introduced by Jensen.

► ROD DOWNEY, *Calibrating randomness*.
Department of Mathematics, Victoria University, Wellington, P. O. Box 600, Wellington, New Zealand.
E-mail: Rod.Downey@mcs.vuw.ac.nz.

This talk will be concerned with some recent work on algorithmic randomness, and in particular Martin-Löf randomness and its relationship to computably enumerable reals. We examine notions of relative randomness based mainly upon relative initial segment complexity. Much of the material is joint with Laforte, Nies, and Hirschfeldt.

[1] R. Downey, *Some computability-theoretical aspects of reals and randomness*, in preparation.

[2] R. Downey, D. Hirschfeldt, and A. Nies, *Density, randomness and computability*, to appear in ***SIAM Journal on Computing***; extended abstract appeared in STACS '01.

[3] R. Downey, D. Hirschfeldt, and G. Laforte, *Randomness and reducibility*, submitted; extended abstracts in MFCS '01.

► HARVEY FRIEDMAN, *Recent incompleteness*.
Mathematics Department, Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210, USA.
E-mail: friedman@math.ohio-state.edu.

We discuss a variety of recent independence results, involving Borel selection, chains in transitive sets and classes, images of sets of integers, and local definability in the Turing degrees. Each of these results provides a new context for incompleteness from set theory with and without large cardinals.

► WARREN GOLDFARB, *Gödel and Carnap: influence and opposition*.
Department of Philosophy, Harvard University, Cambridge, Massachusetts 02138, USA.
E-mail: goldfarb@fas.harvard.edu.

Gödel and the Vienna Circle philosopher Rudolf Carnap had significant interaction in the period 1928–1932; indeed Carnap's 1928 lectures at the University of Vienna played a crucial role in stimulating Gödel's interest in logic and the foundations of mathematics. I survey their interaction, and the consequent nature of their influence on each other. I then discuss their profound philosophical differences, focusing particularly on an issue that has been little discussed in connection with Gödel's Platonism, namely, how the applications of mathematics to the empirical world are to be accounted for.

► OLGA KHARLAMPOVICH, *First order properties of free groups*.
Department of Mathematics and Statistics, McGill University, 805 Sherbrooke St. West, Montreal Quebec, H3A2K6, Canada.
E-mail: olga@math.mcgill.ca.

I will discuss the following results:

Elementary theories of nonabelian free groups of different ranks coincide and decidable. These results prove two of Tarski's conjectures (1945). I will describe the main tools: implicit function theorem for free groups, free regular length functions on fully residually free groups, Makanin-Razborov's method of solving equations and JSJ decomposition for these groups.

This is joint work with A. Miasnikov.

► JULIA F. KNIGHT, *Quantifiers and complexity*.
Department of Mathematics, Notre Dame University, 255 Hurley, Notre Dame, Indiana 46556-5659, USA.
E-mail: knight.1@nd.edu.

I will describe some independent sentences and formulas in arithemtic, and I will indicate how they give information on complexity of models. The "n-diagram" of a structure is the set of Σ_n sentences in the complete diagram. In general, the $(n+1)$-diagram is c. e. in and above the n-diagram, uniformly in n. For non-standard models of arithmetic, the 0-diagram has the same degree as some completion of PA. These two necessary conditions on the sequences of degrees of n-diagrams turn out to be sufficient. For certain other structures A, in particular, for some linear orderings, there are very natural independent formulas, and for any sequence $(C_n)_{n\in\omega}$ such that C_{n+1} is C_n uniformly in n, we can produce B isomorphic to A such that the n-diagram of B is Turing equivalent to C_n.

► MICHIEL VAN LAMBALGEN, *Moschovakis' notion of meaning as applied to linguistics*.
Department of Philosophy, University of Amsterdam, Plantage Muidergracht 24, 1018 TV Amsterdam, Netherlands.
E-mail: michiell@wins.uva.nl.

Yiannis Moschovakis argued in *Sense and denotation as algorithm and value* (1990) that the sense (or intension) of an expression X can be identified with the algorithm which computes the denotation (or extension) of X.

We show that this notion of meaning is also eminently suited to natural language, in particular to capture various forms of intensionality and non-monotonicity found in language. This will be illustrated by a discussion of the process of nominalisation (from the proposition 'John sings' to the event 'John's singing') and of aspect (e.g., the progressive in several languages.

We propose a formalism which embodies Moschovakis' ideas, combining Feferman's theory of truth (1984) and the event calculus in a version due to Shanahan (1997). The computational character of the formalism is brought out by a quantifier elimination theorem, and we shall argue that it agrees quite well with linguistic data.

This is joint work with F. Hamm, Tübingen.

► CHRIS MILLER, *Tameness phenomena in expansions of the field of real numbers*.
Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210, USA.
E-mail: `miller@math.ohio-state.edu`.

What does it mean for a first-order expansion of the field of real numbers to be tame, or well behaved? In recent years, much attention has been paid by model theorists and real-analytic geometers to the o-minimal setting: expansions of the real field in which every definable set has finitely many connected components. But there are expansions of the real field that define sets with infinitely many connected components, yet are tame in some well-defined sense (e.g., the topological closure of every definable set has finitely many connected components, or every definable set has countably many connected components, et cetera). The analysis of such structures often requires a mixture of model-theoretic, analytic-geometric and descriptive set-theoretic techniques. An underlying idea is that first-order definability, in combination with the field structure, can be used as a tool for determining how complicated are given sets of real numbers.

► RAN RAZ, *Resolution lower bounds for the weak pigeonhole principle*.
Department of Computer Science, Weizmann Institute, Rehovot, 76100, Israel.
E-mail: `ranraz@wisdom.weizmann.ac.il`.

We prove that any Resolution proof for the weak pigeonhole principle, with n holes and any number of pigeons, is of length $\Omega(2^{n^{\varepsilon}})$, (for some global constant $\varepsilon > 0$). This implies that (a certain formulation of) the statement $P \neq NP$ doesn't have short resolution proofs.

The *Weak Pigeonhole Principle* (WPHP) states that for $m \geq n + 1$, there is no one-to-one mapping from m pigeons to n holes. WPHP is one of the most fundamental combinatorial principles. In particular, it is used in most probabilistic counting arguments and hence in many combinatorial proofs. Moreover, as observed by Razborov, there are certain connections between WPHP and the problem of $P \neq NP$. Indeed, WPHP (with a relatively large number of pigeons) can be interpreted as an encoding of the following statement: There are no small DNF formulas for SAT. Hence, (in most proof systems), a lower bound for the length of proofs for WPHP implies a lower bound for the length of proofs for (a certain formulation of) the statement $P \neq NP$.

Resolution is one of the most widely studied propositional proof systems. The *Resolution rule* says that if C, D are two clauses and x_i is a variable then any assignment that satisfies both of the clauses $C \vee x_i$ and $D \vee \neg x_i$ also satisfies $C \vee D$. A *Resolution refutation* for a CNF (conjunctive-normal-form) formula F is a sequence of clauses $C_1, C_2, \dots, C_s$, such that: (1) Each clause C_j is either a clause of F or obtained from two previous clauses by the resolution rule. (2) The last clause, C_s, is the empty clause (and hence it has no satisfying assignments). We think of a refutation for an unsatisfiable formula F also as a proof for the tautology $\neg F$.

Resolution proofs for WPHP have attracted a lot of attention in recent years. However, the standard techniques for proving lower bounds for Resolution fail to give lower bounds for WPHP. In particular, for $m \geq n^2$, no non-trivial lower bound was previously known. We prove that any resolution proof for WPHP is of exponential length.

► SAHARON SHELAH, $(\aleph_n, \aleph_0)$ *may fail* $\aleph_0$*-compactness*.
Department of Mathematics, Rutgers University, New Brunswick, New Jersey, USA.
E-mail: shelah@math.huji.ac.il.

Consider a first order theory T with a distinguished unary predicate P. A model M is called a (κ, λ)-model if M has size κ and P is interpreted as a set of size λ.

We consider the following question: If every finite subset of a first order theory T has a $(\aleph_n, \aleph_0)$-model, does the theory T have such a model, too?

We show that the answer "no" is consistent with $2^{\aleph_0} \geq \aleph_n$.

► JOUKO VÄÄNÄNEN, *Models, trees and infinitary back-and-forth*.
Department of Mathematics, University of Helsinki, P. O. Box 4 Yliopistonkatu 5, SF-00014, Helsinki, Finland.
E-mail: jvaanane@cc.helsinki.fi.

I review a recent extension of back-and-forth techniques by means of a transfinite Ehrenfeucht-Fraisse game. This completes the missing blank area in the map of back-and-forth techniques in model theory, namely the case of models of uncountable cofinality whose first order theory is non-classifiable. Examples are provided from linear orders and abelian groups. It turns out to be essential to consider games the "length" of which is not an ordinal but a tree. Thereby new questions about trees emerge from this connection with games and models.

► FRANK WAGNER, *Simplicity theory*.
Institut Girard Desargues, Université Claude Bernard, 43, Boulevard du 11 novembre 1918, 69622 Villeurbanne-cedex, France.
E-mail: wagner@desargues.univ-lyon1.fr.

In his thesis (1996), B. Kim proved the symmetry of forking independence and thereby extended the stability-theoretic machinery to a wider class of theories, called 'simple'. This provided a unified framework for various theories, such as pseudofinite fields or smoothly approximable structures, where stable behaviour had previously been observed in an unstable (but, as it turned out, simple) context. Additional interest arose because of the use of another simple theory, existentially closed fields of characteristic 0 with a generic automorphism, in Hrushovski's proof of the Manin-Mumford conjecture.

A common feature of all known examples is that forking is caused by stable formulas; in this set-up, stability-theoretic proofs adapt reasonably well. The 'stable forking conjecture' claims that this behaviour is universal; if it fails, the model-theoretic analysis becomes more complicated. In particular, in addition to Shelah's imaginaries (whose addition is still first-order), new classes of elements had to be added to a structure: 'hyperimaginaries', and lately 'ultraimaginaries', which were necessary for the generalization of essential structural theorems from stability theory: the existence of canonical bases, and the group configuration theorem.

However, while other basic questions remain open (the binding group theorem, the possible rank one geometries, or rank one groups), it is just as unclear whether these additions are really necessary, or can be eliminated (like hyperimaginaries in supersimple theories).

► STANLEY S. WAINER, *Provable recursiveness and complexity*.
University of Leeds, School of Mathematics, Leeds, LS2 9JK, United Kingdom.
E-mail: pmt6ssw@amsta.leeds.ac.uk.

Fifty years ago, Kreisel characterized the functions computable in (provably recursive in) arithmetic as those definable by recursions over (standard) primitive recursive well-orderings of order-types less than ε_0. His underlying question (then implicit, later explicit): "What more do we know if we have proved a theorem by restricted means than if we merely know that it is true?" became a research programme—reading off the information content of proofs—which has driven much of proof theory and related areas of computer science ever since. In the case of Π^0_2 statements, their truth asserts the mere existence of a recursive function, whereas a proof contains complexity information which can be extracted by normalization or cut-elimination or (originally) epsilon-substitution. This talk will be about the hierarchy-theory of recursive functions which arises in this connection. It will be illustrated by two more-recent results from opposite ends of the subrecursive spectrum.

The first concerns the class of "predicatively accessible" recursive functions, generated according to an autonomy condition of the form: only allow recursions over well-orderings "coded" (in an appropriate sense) at previous levels. Here the methods are those of Girard's Π^1_2 logic, and the hierarchy turns out to characterize provable recursiveness in $\Pi^1_1 - \mathrm{CA}_0$. (Our setting however is quite different to Girard's, being more specific to the subrecursive context.)

The second is a contribution to the search for "natural" theories, without explicit bounds, whose provably recursive functions form feasible, or at least "more realistic", complexity classes than those encountered in classical theories like PA or its inductive fragments. Here the general hierarchy theory itself suggests how to make a simple modification of PA so that, as in earlier work of Leivant, its provably recursive functions are elementary and, in the Σ^0_1-inductive fragment, linear space or polynomial time according as the number-representation is unary or binary.

This is joint work with G. Ostrin.

Abstracts of invited talks in the special session in computability theory

► DENIS HIRSCHFELDT, *A Δ^0_2 set with no infinite low subset in either it or its complement*.
Victoria University, School of Mathematics and Computer Sciences, P. O. Box 600, Wellington, New Zealand.
E-mail: `drh@math.uchicago.edu`.

We construct the set of the title, answering a question of Cholak, Jockusch, and Slaman, and discuss its connections with the study of the proof-theoretic strength and effective content of versions of Ramsey's Theorem. In particular, our result implies that every ω-model of $\mathrm{RCA}_0 + \mathrm{SRT}^2_2$ must contain a nonlow set.

This is joint work with Rod Downey, Steffen Lempp, and Reed Solomon.

► I. SH. KALIMULLIN, *The structure of n-c. e. enumeration degrees and the uniform minimal pair property*.
Kazan State University, Department of Mathematics, 18, Kremlevskaya Str. 420008, Kazan, Tatarstan, Russia.
E-mail: `Iskander.Kalimullin@ksu.ru`.

Let $\mathcal{D}^e_n$ be the upper semilattice of n-c. e. enumeration degrees ($n \geq 2$). It was known [1] that

$$\mathrm{Th}(\mathcal{D}^e_2) \neq \mathrm{Th}(\mathcal{D}^e_n)$$

if $n \geq 3$, and

$$\mathrm{Th}(\mathcal{D}^e_m) \neq \mathrm{Th}(\mathcal{D}^e_n)$$

if $1 < m < 2p \leq n$ for some integer p.

We study the problem of elementarily equivalence between $\mathcal{D}^e_{2p}$ and $\mathcal{D}^e_{2p+1}$ for $p \geq 2$.

Consider the formula Ψ_p:

$$\begin{aligned}(\exists\mathbf{a})\ (\exists\mathbf{c} > \mathbf{0})\ (\exists\mathbf{d})\ (\exists\mathbf{e} > \mathbf{0})\ (\forall\mathbf{b}_1)\dots(\forall\mathbf{b}_p)\ (\forall\mathbf{x})&\\ [[\mathbf{c} \not\leq \mathbf{b}_1\ \&\cdots\&\ \mathbf{c} \not\leq \mathbf{b}_p \Longrightarrow \mathbf{a} \neq \mathbf{b}_1 \cup \cdots \cup \mathbf{b}_p]&\\ \&\ [\mathbf{e} \not\leq \mathbf{b}_1\ \&\cdots\&\ \mathbf{e} \not\leq \mathbf{b}_p \Longrightarrow \mathbf{d} \neq \mathbf{b}_1 \cup \cdots \cup \mathbf{b}_p]&\\ \&\ [(\mathbf{x} \cup \mathbf{a}) \cap (\mathbf{x} \cup \mathbf{d}) = \mathbf{x}]].&\end{aligned}$$

It follows from [1] that $\mathcal{D}^e_{2p} \models \neg\Psi_p$. We show that $\mathcal{D}^e_{2p+1} \models \Psi_p$. To obtain this we must know how to build $(2p+1)$-c. e. e-degrees $\mathbf{a}$ and $\mathbf{d}$ with the property $(\forall\mathbf{x})\ [(\mathbf{x} \cup \mathbf{a}) \cap (\mathbf{x} \cup \mathbf{d}) = \mathbf{x}]$ (it is natural to say that $\mathbf{a}$ and $\mathbf{d}$ form a uniform minimal pair). In fact, this property holds if for some (every) sets $A \in \mathbf{a}$ and $D \in \mathbf{d}$ we have a c. e. set W such that

$$A \times D \subseteq W \quad \text{and} \quad \overline{A} \times \overline{D} \subseteq \overline{W}.$$

The following theorem improves mentioned results from [1].

THEOREM. *If* $1 < m < n$, *then* $\mathrm{Th}(\mathcal{D}^e_m) \neq \mathrm{Th}(\mathcal{D}^e_n)$.

[1] I. SH. KALIMULLIN, *Splitting properties of n-c. e. enumeration degrees*, ***Journal of Symbolic Logic***, to appear.

► FRANK STEPHAN, *Recent results on effective Hausdorff dimension*.
Mathematisches Institut, Universität Heidelberg, Im Neuenheimer Feld 294, 69120 Heidelberg, Germany.
E-mail: fstephan@math.uni-heidelberg.de.

This talk gives an overview on recent results on effective Hausdorff dimension. The classical notion of Hausdorff dimension is an important tool in fractal geometry and dynamical systems. Recently, it has been transferred to various formalizations in complexity and recursion theory. The focus of this talk is on effective versions of Hausdorff dimension defined via martingales which are recursively enumerable from below. Several natural examples for classes of dimension 0, 1 and strictly in between are given. Furthermore, alternative characterizations for the notion of Hausdorff dimension are presented. Finally, relativizations of the concept are considered.

Abstracts of invited talks in the special session in computer science and complexity theory

► DOV GABBAY, *Goal directed logic mechanisms*.
Department of Computer Science, King's College, University of London, Strand, London, WC2R 2LS, United Kingdom.
E-mail: dg@dcs.kcl.ac.uk.

In their book [1], Gabbay and Olivetti presented many classical, intermediate, modal, substructural and other logics in a PROLOG-like, goal-directed way. Such a formulation allows for a very strong CUT property, as proofs have to go through data clauses in a head to body fashion as in PROLOG.

Such proof theory also allows for "metapredicates" (or goal-directed mechanisms) to accompany the main proof algorithms and compute other computational objects such as INTERPOLANTS, FAILURE SETS, RESOURCE USE, ABDUCTION SETS, etc.

The lecture will introduce the goal directed ideas and give examples of interpolation theorems and other mechanisms as well as some frustrating open problems.

[1] D. GABBAY and N. OLIVETTI, ***Goal directed proof theory***, vol. 1, Kluwer Applied Logic Series, Kluwer Academic Publishers, 2000.

► PASCAL KOIRAN, *Circuits versus trees in algebraic complexity*.
Lip, ENS de Lyon, 46, allée d'Italie, 69364 Lyon Cedex 07, France.
E-mail: Pascal.Koiran@ens-lyon.fr.

I will discuss some aspects of the "P = NP?" problem over the real numbers and more general algebraic structures. We shall see that given a structure M, it is important to find out whether NP problems over M can be solved by polynomial depth computation trees, and if so whether these trees can be efficiently simulated by circuits. Point location, a problem of computational geometry, comes into play in the study of these questions for several structures of interest.

► DANIEL LEHMANN, *Choice function semantics for nonmonotonic logics*.
School of Computer Science and Engineering, Hebrew University, Jerusalem, 91904, Israel.
E-mail: lehmann@cs.huji.ac.il.

Tarski gave a general semantics for deductive reasoning: a formula a may be deduced from a set A of formulas if and only if a holds in all models in which each of the elements of A holds. A more liberal semantics has been considered: a formula a may be deduced from a set A of formulas if and only if a holds in all of the preferred models in which all the elements of A hold. Shoham proposed that the notion of preferred models be defined by a partial ordering on the models of the underlying language. A more general semantics is described in this paper, based on a set of natural properties of choice functions. This semantics is here shown to be equivalent to a semantics based on comparing the relative importance of sets of models, by what amounts to a qualitative probability measure. The consequence operations defined by the equivalent semantics are then characterized by a weakening of Tarski's properties in which the monotonicity requirement is replaced by three weaker conditions. Classical propositional connectives are characterized by natural introduction-elimination rules in a nonmonotonic setting. Even in the nonmonotonic setting, one obtains classical propositional logic, thus showing that monotonicity is not required to justify classical propositional connectives.

Abstracts of invited talks in the special session of model theory

► PAOLA D'AQUINO, *Primes in weak fragments of arithmetic*.
Dipartmento di Matematica, Seconda Universita di Napoli, Via Vivaldi 43, Caserta 81100, Italy.
E-mail: Paola.DAquino@unina2.it.

I will consider classical properties of prime numbers in weak fragments of Peano Arithmetic. In particular, I will concentrate on the quadratic reciprocity law. Gauss' second proof uses quadratic forms and group-theoretic arguments. It turns out that many of his arguments work in $I\Delta_0 + \Omega_1$, and I will survey what is known and what is left to check.

► RAHIM MOOSA, *Some model theory of compact complex spaces*.
207-135 Leeward Glenway, Toronto, Ontario, Canada, M3C 2Z6.
E-mail: moosa@math.uiuc.edu.

A compact complex analytic space, X, is viewed as a structure in the model-theoretic sense by naming the analytic subset of all the finite Cartesian powers of X. As such, X admits quantifier elimination and is of finite Morley rank. One of the obstacles to applying stability-theoretic techniques directly to X is that this structure is not sufficiently saturated in the natural language of analytic sets. One is thus often forced to pass to elementary extensions where the underlying complex analytic structure is lost. We introduce the notion of a "saturated compact complex space", spaces that admit a countable language where it is possible to do model theory without having to pass to elementary extensions. Among the results presented will be a characterisation of saturated compact complex analytic spaces in terms of their Douady spaces, and an application of saturation to relative Moishezon reductions.

Abstracts of invited talks in the special session in philosophy and history of logic

► SERGEI ARTEMOV, *Incompleteness theorem and two models for provability*.
Department of Mathematics, Cornell University, 436 Malott Hall, Ithaca, New York 14853, USA.
E-mail: sna2@cornell.edu.

Gödel's Incompleteness theorem and his papers of 1933 and 1938 led to two essentially different models for provability, each having its own areas of applications.

A. Loeb's modal logic L of formal provability (also known as GL, G, PRL, K4.W) where the "non-reflexive" Loeb principle is present. Within this model proofs are represented implicitly by existential quantifiers. The highlights of this model are the propositional formalization of the Second Incompleteness Theorem and Loeb's theorem, De Jongh-Sambin fixed point theorem, and the Solovay completeness theorem. The logic L finds applications in traditional proof theory.

B. Gödel's modal logic for provability (also known as S4) and its counterpart logic of proofs LP with the reflection principle "provable F implies F". Within his model proofs are represented explicitly by combinatory style terms. This model has reached out to such areas as constructive semantics, logics of knowledge, combinatory logic and lambda-calculus, automated deduction and formal verification, and typed programming languages.

The models A and B are compatible because they are both based on the same Gödel provability predicate.

► KARL-GEORG NIEBERGALL, *Natural representations and Gödel's second theorem*.
Institut für Philosophie, Logik und Wissenschaftstheorie, Universität München, Ludwigstr. 31, D-80539 München, Germany.
E-mail: kgn@lrz.uni-muenchen.de.

The philosophical and logical importance of Gödel's second incompleteness theorem rests in part on the relevance of its informal paraphrases—like:

no consistent theory T proves its own consistency.

But we know already from work of Rosser and Feferman that by choosing "unusual" formalizations of "T proves its own consistency", even theories like PA can prove their own consistency (in this "unusual" sense). And there may exist (e.g., non axiomatizable) consistent theories T which prove their own consistency, even when this phrase is formalized in a "natural" way.

In this talk, I will address how the correctness of, e.g., (general) precise renderings of the second incompleteness theorem depends on the representations used in them. In particular, I will present methods for obtaining consistent extensions of, say, PA which prove their own consistency, employing only what seem to be "natural" representations. Finally, I will discuss if these are "really" natural and make some general remarks about what a "natural representation" could be.

► SAHOTRA SARKAR, *Logic and logicism in Carnap's logical syntax of language*.
Faculty of Philosophy, University of Texas at Austin, Waggener Hall 316, Austin, Texas 78712-1180, USA.
E-mail: sarkar@mail.utexas.edu.

The Logical Syntax of Language (LSL, 1936/38) is Carnap's most important logical work. Its philosophical significance has generally been noted though downplayed since Carnap soon went on to endorse semantics, abandoning the pure syntacticism of LSL. This paper explores the purely logical contributions of LSL, most notably, the formulation and proof of the fixed-point lemma as a generalization of Gödel's incompleteness theorem. It also argues that though Carnap manages to rescue and reconcile formalism and logicism to some extent after the negative foundational results of the early 1930s, this is done at the cost of weakening logicism to a form of pure conventionalism.

Abstracts of invited talks in the special session in proof theory and proof search

► ULRICH BERGER, *Modified realizability for classical countable choice*.
Department of Computer Science, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, Wales, U.K..
E-mail: u.berger@swan.ac.uk.

We consider classical arithmetic in all finite types plus the following form of the Axiom of Choice,

$$\text{AC} \qquad \forall x : N\ \exists y : \rho A(x, y) \to \exists f\ \forall x\ A(x, fx)$$

where N is the type of natural numbers, ρ is any finite type, and $A(x, y)$ is an arbitrary formula. Because, classically, AC implies comprehension for sets of natural numbers, this system is at least as strong as full second order arithmetic. In a paper by Berardi, Bezem and Coquand ([1]) a method for extracting witnesses from proofs of Sigma-1 formulas in this system is presented. In the talk we propose a different and simpler method which is closely related to Spector's Dialectica interpretation of AC, but uses Kreisel's modified realizability instead. We also address questions of efficiency of implementations of the method and discuss possible applications in the field of computer aided program synthesis.

[1] Berardi, Bezem, and Coquand, ***Journal of Symbolic Logic***, vol. 63 (1998), no. 2, pp. 600–622.

► ANDREA CANTINI, *Existence and feasibility in applicative theories*.
Dipartimento di Filosofia, Università di Firenze, via Bolognese 52, I-50139 Firenze, Italy.
E-mail: cantini@philos.unifi.it.

Since the mid-seventies, theories with (untyped) operations have been investigated in proof theory and constructive mathematics, especially as the core of Feferman's explicit mathematics (cf. [4], [1], [9]).

In this paper we are concerned with sequent calculi, which embody (forms of extended) combinatory logic with the type W of binary strings, and range in strength from primitive recursive arithmetic to bounded arithmetic (cf. [3]).

In particular, we discuss systems, which comprise in their applicative and logical basis:

(1) weak (= positive and "recursive") comprehension for operations;
(2) uniformity and reflection rules for W;
(3) various forms of W-induction;
(4) modal rules.

While (1) and (2) allow us to relate applicative theories to weak subsystems of analysis (cf. [7] and [5]), (3) and (4) are responsible for calibrating the computational content (i.e., for characterizing the class of functions definable by the closed terms of (provable) type $W \to W$; for related work, cf. [8]). As expected, modal logic is used to recover in the proof theoretic context the so-called intrinsic approach to complexity classes (cf. [2] and [6]).

We show that significant results which hold for the standard arithmetical systems have counterparts within the abstract framework of (untyped) operations.

Cut elimination, asymmetric interpretation and realizability will be the main proof theoretic techniques in use.

[1] M. J. BEESON, ***Foundations of constructive mathematics***, Springer-Verlag, Berlin, 1985.

[2] S. BELLANTONI, *Predicative recursion and computational complexity*, ***Ph. D. thesis***, University of Toronto, 1992.

[3] S. R. BUSS, ***Bounded arithmetic***, Bibliopolis, Napoli, 1986.

[4] S. FEFERMAN, *A language and axioms for explicit mathematics*, in ***Algebra and Logic*** (J. N. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer-Verlag, Berlin, 1975, pp. 87–139.

[5] A. M. FERNANDES and F. FERREIRA, *Groundwork for weak analysis*, ***Journal of Symbolic Logic***, to appear.

[6] D. LEIVANT, *Intrinsic theories and computational complexity*, in ***Logic and computational complexity*** (D. Leivant, editor), Lecture Notes in Computer Science, vol. 960, Springer-Verlag, Berlin, 1995, pp. 177–194.

[7] S. G. SIMPSON, ***Subsystems of second order arithmetic***, Perspectives in Mathematical Logic, Springer-Verlag, Berlin-Heidelberg, 1998.

[8] T. STRAHM, *Theories with self-application and computational complexity*, preprint, Bern, May 17, 2000.

[9] A. S. TROELSTRA and D. VAN DALEN, ***Constructivism in mathematics***, vol. I–II, Studies in Logic, Elsevier-North Holland, Amsterdam, 1988.

► ANDREI VORONKOV, *Proof-theoretic issues in proof-search modal logics*.
Department of Computer Science, University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom.
E-mail: `voronkov@cs.man.ac.uk`.

We discuss a decision procedure for propositional modal logics based on the inverse method. The procedure is based on an "inverted" sequent calculus. To restrict the search space, we prove a number of redundancy criteria for derivations in the calculus based on proof-theoretical analysis of derivations in a tableau calculus.

Two new proof techniques will be demonstrated. One of them is used to prove completeness of proof-search with a strong notion of subsumption. This technique is based on the so-called *traces*. Another technique is based on a *path calculus* which considerably simplifies all proofs as compared to other known presentations of the inverse method.

Abstracts of invited talks in the special session in set theory

► MIRNA DŽAMONJA, *Iterated forcing at the successor of a singular cardinal*.
University of East Anglia, School of Mathematics, Norwich NR 47J, United Kingdom.
E-mail: `M.Dzamonja@uea.ac.uk`.

We shall present a number of results showing that some (weak) versions of iterated forcing at the successor of a singular are possible. The basic idea of these constructions is to start with a Laverised supercompact cardinal and build an iteration of the required forcing along with a filter that will be used for a Prikry forcing at the end of the day. We shall also discuss possibilities of doing this with Radin forcing at the end in place of Prikry forcing, and we shall present some concrete examples of the technique. Results of Gitik and Shelah, Dž. and Shelah, Dž. and Morgan and Dž., Komjath and Morgan will be discussed. Some open questions will be mentioned.

► RALF-DIETER SCHINDLER, *Mutual stationarity in $L[E]$*.
Institut für Formale Logik, Universität Wien, Währingerstrasse 25, 1090 Wien, Austria.
E-mail: rds@logic.univie.ac.at.

Foreman and Magidor ask whether there is always a sequence (S_n) such that each individual S_n is stationary in ω_n and consists of points of a cofinality fixed in advance whereas (S_n) is not mutually stationary. We prove a strong form of this to be true in the core model below $0^{\text{hand-grenade}}$ by exploiting the covering argument. Many interesting questions remain open.

► JINDRICH ZAPLETAL, *Forcing with ideals of closed sets*.
Department of Mathematics, University of Florida, Florida 32611, USA.
E-mail: jinzap@yahoo.com.

Whenever I is a σ ideal generated by an analytic collection of closed sets, then the forcing of I-positive Borel sets ordered by inclusion adds a single real r, which is of an almost minimal degree: If s is a real such that $V[s]$ is contained in $V[r]$ then s is a Cohen real or $V[s] = V[r]$.

Abstracts of contributed talks

► ANDREW ABERDEIN, *Anomalies and the development of non-classical logic*.
Department of Logic and Metaphysics, The University, St. Andrews, Fife, KY16 9AL, Scotland, United Kingdom.
E-mail: andrew.aberdein@dunelm.org.uk.

Imre Lakatos developed an original and profound account of mathematical development through an analysis of how mathematical theories respond to anomalies [1]. I show how this account may be applied to the development of logic by a study of the growth of non-classical systems. I derive a characterization of possible responses to anomalous data, which I term heuristic context: the methodological practices characteristic of a specific stage in the development of a research programme. This measure of how open a programme is to reform and revision generates a range of historically familiar positions, and assembles them into an implicit hierarchy.

The hierarchy begins by distinguishing non-revisionary ways of dealing with recalcitrant natural argumentation, and continues through the adoption of conservatively revisionary systems. The most radical levels provide a novel classification of non-conservative revisions of classical logic. I distinguish between 'restriction,' 'wholesale reform' and 'change of subject matter.' Restriction of logic avoids an anomaly by adopting a new calculus which lacks previously valid inferences and theorems. Wholesale reform of logic builds on this by criticizing and reformulating confused background theories and metalogical concepts, including the nature of consequence, and what it preserves (its inferential goal.) Change of the subject matter of logic is a change of inferential goal precipitated by non-conservative revision of background theories. I argue that in this case the focus of the dispute shifts from the discipline of logic to whatever discipline threw up the conflicting background theories.

[1] Imre Lakatos, ***Proofs and refutations: The logic of mathematical discovery*** (J. Worrall and E. Zahar, editors), Cambridge University Press, Cambridge, 1976.

► ZOFIA ADAMOWICZ, *Incompleteness and non-conservativeness in bounded arithmetic*.
Institute of Mathematics of the Polish Academy of Sciences, Sniadeckich 8, 00-950 Warszawa, Poland.
E-mail: zosiaa@panim.impan.gov.pl.

We prove the second Gödel incompleteness theorem for a version of a bounded arithmetic and the Herbrand version of consistency. We reduce the incompleteness theorem to a fine version of Π_1 non conservativeness in bounded arithmetic.

► MARAT ARSLANOV, *Models of relative computability and the Ershov hierarchy*.
Kazan State University, Department of Mathematics, Kremlevskaja 18, Kazan 420008, Russia.
E-mail: Marat.Arslanov@ksu.ru.

The main focus of my talk will be the n. r. e. hierarchy of sets and the corresponding degree structure. A particular focus will be the definability of the various levels of the n-r. e. hierarchy, both relatively and within wider local structures; more specifically, questions related to the definability of the relations of 'recursively enumerable' and 'recursively enumerable in' in the degree structures of the n-r. e. degrees for $n > 1$.

► ARNON AVRON AND IDDO LEV, *Canonical propositional Gentzen-type systems*.
School of Computer Science, Tel-Aviv University, Ramat Aviv 69978, Israel.
E-mail: aa@post.tau.ac.il.
E-mail: iddolev@post.tau.ac.il.

Canonical propositional Gentzen-type systems are systems which in addition to the standard axioms and structural rules have only pure logical rules which have the subformula property, introduce exactly one occurrence of a connective in their conclusion, and no other occurrence of any connective is mentioned anywhere else in their formulation. We provide a constructive coherence criterion for the non-triviality of such systems, and show that a system of this kind admits cut elimination if and only if it is coherent. We show also that the semantics of such systems is provided by non-deterministic two-valued matrices (2-Nmatrices). 2-Nmatrices form a natural generalization of the classical two-valued matrix, and every coherent canonical system is sound and complete for one of them. Conversely, with any 2-Nmatrix it is possible to associate a coherent canonical Gentzen-type system which has for each connective at most one introduction rule for each side, and is sound and complete for that 2-Nmatrix. We show also that every coherent canonical Gentzen-type system either defines a fragment of the classical two-valued logic, or a logic which has no finite characteristic matrix.

► TOMEK BARTOSZYNSKI, *Strongly meager sets*.
Boise State University, Department of Mathematics, Boise, Idaho 83725, USA.
E-mail: tomek@diamond.boisestate.edu.

A set of reals is strongly meager if it can be translated away from any Lebesgue measure zero sets, and it has strong measure zero if it can be translated away from any set of the first category. In the talk I will discuss several recent results concerning strongly meager sets, with particular focus on the properties that violate the "duality" between these two families of sets.

► ARNOLD BECKMANN, *A non-well-founded primitive recursive tree provably well-founded for co-r. e. sets*.
Institute of Algebra and Computational Mathematics, Vienna University of Technology, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria.
E-mail: Arnold.Beckmann@math.uni-muenster.de.

In [2] we construct by diagonalization a non-well-founded primitive recursive tree, which is well-founded for co-r. e. sets, provable in Σ^0_1-IND. It follows that the supremum of order-types of primitive recursive well-orderings, whose well-foundedness on co-r. e. sets is provable in Σ^0_1-IND, equals the limit of all recursive ordinals ω_1^{CK}.

This work contributes to the investigation of replacing the quantification over all sets in the definition of the proof-theoretic ordinal of a theory by a quantification over certain definable sets such as all arithmetical sets, or certain levels of the arithmetical hierarchy. It is in the line of pathologic well-orderings first treated by Kreisel [3, 4]. We discuss this as a first step towards an investigation of replacing the set parameter in the definition of the dynamic ordinal of fragments of bounded arithmetic (cf. [1]) by certain definable sets in order to obtain non-relativized separation results.

[1] ARNOLD BECKMANN, *Separating fragments of bounded arithmetic*, ***Ph. D. thesis***, Westfälische Wilhelms-Universität, Münster, 1996.

[2] ———, *A non-well-founded primitive recursive tree provably well-founded for co-r. e. sets*, ***Archive for Mathematical Logic***, accepted for publication, 2000.

[3] GEORG KREISEL, *A survey of proof theory*, ***Journal of Symbolic Logic***, vol. 33 (1968), pp. 321–388.

[4] ———, *Wie die Beweistheorie zu ihren Ordinalzahlen kam und kommt*, ***Jahresbericht der Deutschen Mathematiker-Vereinigung***, vol. 78 (1977), pp. 177–223.

► V. BENCI AND MAURO DI NASSO, *A new way of counting infinite sets*.
Dip. Mat. Appl., via Bonanno 25/B, 56126 Pisa, Italy.
E-mail: `benci@dma.unipi.it`.
E-mail: `dinasso@dma.unipi.it`.

Similarly as cardinals and ordinals, we introduce a new set of numbers, called "numerosities", aimed to generalize finite cardinalities. In order to count elements of (possibly infinite) sets, we equip them with "labels". A *labelled set* $\mathbf{A} = \langle A, \ell_A \rangle$ is a pair where A is a set and $\ell_A \colon A \to \mathbb{N}$ is a finite-to-one function ($\ell_A(a)$ is the *label* of a). A *numerosity function* is a function $num \colon \mathcal{L} \to \mathcal{N}$ defined on the class $\mathcal{L}$ of labelled sets onto a linearly ordered set $\langle \mathcal{N}, \leq \rangle$ of *numerosities*, and such that the three axioms below are satisfied. The idea is to count the "numerosity" of $\mathbf{A}$ by counting its finite approximations $A_n \doteq \{ a : \ell_A(a) \leq n \}$. We write $\mathbf{A} \preceq \mathbf{B}$ when the counting process indicates that $\mathbf{A}$ does not contain more elements than $\mathbf{B}$, i.e., when the finite cardinalities $\sharp A_n \leq \sharp B_n$ for all n. We write $\mathbf{B} \subset \mathbf{A}$ if $B \subset A$ (strict inclusion) and $\ell_A(a) = \ell_B(a)$ for all $a \in A$. Denote by $\mathbf{A} \oplus \mathbf{B} = \langle A \uplus B, \ell_A \uplus \ell_B \rangle$ ($A \uplus B$ is the disjoint union) where $(\ell_A \uplus \ell_B)(x)$ equals $\ell_A(x)$ or $\ell_B(x)$ according if $x \in A$ or $x \in B$; and denote by $\mathbf{A} \otimes \mathbf{B} = \langle A \times B, \ell_A \otimes \ell_B \rangle$ where $(\ell_A \otimes \ell_B)(a, b) = \max\{\ell_A(a); \ell_B(b)\}$.

- **Axiom 1**. If $\mathbf{A} \preceq \mathbf{B}$ then $num(\mathbf{A}) \leq num(\mathbf{B})$.
- **Axiom 2**. $\xi < num(\mathbf{A})$ if and only if $\xi = num(\mathbf{B})$ for some $\mathbf{B} \subset \mathbf{A}$.
- **Axiom 3**. If $num(\mathbf{A}) = num(\mathbf{A}')$ and $num(\mathbf{B}) = num(\mathbf{B}')$ then

$$num(\mathbf{A} \oplus \mathbf{B}) = num(\mathbf{A}' \oplus \mathbf{B}')$$

and

$$num(\mathbf{A} \otimes \mathbf{B}) = num(\mathbf{A}' \otimes \mathbf{B}').$$

Sums and products of numerosities are then naturally defined:

$$num(\mathbf{A}) + num(\mathbf{B}) = num(\mathbf{A} \oplus \mathbf{B}) \quad \text{and} \quad num(\mathbf{A}) \cdot num(\mathbf{B}) = num(\mathbf{A} \otimes \mathbf{B}).$$

THEOREM 1. *By assuming a numerosity function, an $\aleph_1$-saturated nonstandard embedding $* \colon V(\mathbb{N}) \to V(\mathcal{N})$ can be defined where the ordered ring of numerosities*

$$\langle \mathcal{N}, +, \cdot, \leq \rangle = \langle \mathbb{N}^*, +, \cdot, \leq \rangle$$

coincides with the hypernatural numbers.

Although defined by means of elementary properties that are naturally satisfied by the intuitive process of counting, ZFC is not enough to get a numerosity function.

THEOREM 2. *A numerosity function exists if and only if there is a selective ultrafilter on $\mathbb{N}$. [The existence of selective ultrafilters is independent of ZFC.]*

► KATALIN BIMBÓ, *The Church-Rosser property in dual combinatory logic*.
ARG, Computer Sciences Laboratory, RSISE, Bld. #115, Australian National University, Canberra, ACT 0200, Australia.
E-mail: bimbo@csl.anu.edu.au.

Dual combinators were introduced in [2] in the context of *structurally free logics*. A *dual combinator* is applicable to its arguments from right to left; e.g., the regular permutator C and its dual c have reduction axioms as follows: $((\mathsf{C}x)y)z \rhd (xz)y$, $z(y(x\mathsf{c})) \rhd y(zx)$. One of the motivations to introduce dual combinators was to extend the Curry-Howard isomorphism to include both implications $\to$ and $\leftarrow$ (when they are distinct).

If we look at combinators and dual combinators (together) as a system of combinatory logic, rather than formulas in structurally free logics, then a natural question to ask is whether the so obtained *dual combinatory logic* has the Church-Rosser property (CR). The full system is known to lack CR (cf. [4]). We prove a general theorem stating that no (sufficiently dual) dual combinatory system has CR. Since our theorem concerns the *lack* of CR (unlike the typical theorems for combinatory logic and λ-calculus that show these systems to have CR), in proving our claim we use a method that is different from the usual proof techniques which demonstrate the presence of CR (cf. [5], [1], and [3]). Lastly, we analyze briefly why the lack of CR in dual combinatory systems causes no harm to the structurally free logic *LC*.

[1] H. B. CURRY and R. FEYS, ***Combinatory logic***, vol. I, North-Holland, Amsterdam, 1958.

[2] J. M. DUNN and R. K. MEYER, *Combinatory logic and structurally free logic*, ***The Logic Journal of IGPL***, vol. 5 (1998), pp. 505–537.

[3] J. R. HINDLEY and J. P. SELDIN, ***Introduction to combinators and λ-calculus***, Cambridge University Press, Cambridge, 1986.

[4] R. K. MEYER, K. BIMBÓ, and J. M. DUNN, *Dual combinators bite the dust* (abstract), ***Bulletin of Symbolic Logic***, vol. 4 (1998), pp. 463–464.

[5] J. B. ROSSER, *A mathematical logic without variables*, ***Annals of Mathematics***, vol. 36 (1935), pp. 127–150.

► DUMITRU BUSNEAG, *MV-algebras*.
University of Craiova, Department of Mathematics, A. I. Cuza Street, 13, 1100, Craiova, Romania.
E-mail: busneag@central.ucv.ro.

An MV-algebra is a structure $A = (A, \oplus, \urcorner, 0, 1)$ satisfying the following equations:

$$\begin{aligned} x \oplus (y \oplus z) &= (x \oplus y) \oplus z, \\ x \oplus y &= y \oplus x, \\ x \oplus 0 &= x, \\ x \oplus 1 &= 1, \\ \urcorner 0 &= 1, \\ \urcorner 1 &= 0, \\ \urcorner(\urcorner x \oplus y) \oplus y &= \urcorner(\urcorner y \oplus x) \oplus x. \end{aligned}$$

MV-algebras were introduced by Chang [2] as the algebraic counterpart of Lukasiewicz logic [3]. They are extensions of boolean algebras just as Lukasiewicz logic is an extension of classical logic: boolean algebras coincide with idempotent MV-algebras.

The scope of my contributed paper is to outline the connections between the MV-algebras and implicative algebras with the origin in mathematical logic [1] (Hilbert algebras, Hertz algebras, Implicational algebras, etc.).

[1] D. BUSNEAG, *Contributions to the study of Hilbert algebras*, ***Ph. D. thesis***, University of Bucharest, 1985.

[2] C. C. CHANG, *Algebraic analysis of many-valued logic*, ***Transactions of the American Mathematical Society***, vol. 88 (1958), pp. 467–490.

[3] A. TARSKI and J. LUKASIEWICZ, ***Investigations into the sentential calculus, logic, semantics, metamathematics***, Oxford University Press, Oxford, 1956, pp. 38–59.

► WOJCIECH BUSZKOWSKI AND BARBARA DZIEMIDOWICZ, *Restricted optimal unification and minimal substitutions*.
Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Matejki 48/49, 60-769 Poznan, Poland.
E-mail: buszko@amu.edu.pl.
Institute of Mathematics and Computer Science, University of Warmia and Mazury, Licznerskiego 4, 10-957 Olsztyn-Kortowo, Poland.
E-mail: basdz@uwm.edu.pl.

Let R denote a finite binary relation on the set of terms of a first-order language. A substitution σ is called a *unifier* of R if $s\sigma = t\sigma$ for all $(s, t) \in R$. The notion of an m. g. u. of R is defined in a standard way. A set X of substitutions is said to be hereditary if $\sigma\eta \in X$ entails $\sigma \in X$. For any substitution σ, we define the relation $\mathrm{Ker}_R\,\sigma$ as follows: $(s, t) \in \mathrm{Ker}_R\,\sigma$ if and only if $(s, t) \in R$ and $s\sigma = t\sigma$. By an *optimal unifier of R restricted to X* (X is a hereditary set of substitutions) we mean a substitution σ such that: (i) $\sigma \in X$, (ii) there is no $\gamma \in X$ such that $\mathrm{Ker}_R\,\sigma \subset \mathrm{Ker}_R\,\gamma$, (iii) σ is an m. g. u. of $\mathrm{Ker}_R\,\sigma$.

Optimal unification is a generalization of the standard term unification, useful in situations in which a formal object admits a nondeterministic term assignment to some units. A working example is categorical grammar: the lexicon usually requires many types assigned to a lexical unit. There have been elaborated learning algorithms for different classes of categorial grammars which essentially use type unification and optimal type unification (Buszkowski and Penn 1990 and Kanazawa 1994, 1996, and 1998). Restricting unification to a hereditary class X corresponds to making the algorithms sensitive to certain negative constraints upon possible outcomes (Marciniec 1994).

In this paper, we find general conditions which guarantee that one can effectively describe all minimal substitutions with respect to a preordering relation $\leq$ on X. We write $\sigma \sim \eta$ if and only if $\sigma \leq \eta$ and $\eta \leq \sigma$, and we write $\sigma < \eta$ if and only if $\sigma \leq \eta$ and $\sigma \not\sim \eta$. A preordering relation $\leq$ is said to be *sound* for R if the following conditions are satisfied: (i) if $\mathrm{Ker}_R\,\sigma \subset \mathrm{Ker}_R\,\eta$ then $\eta < \sigma$, (ii) if $\mathrm{Ker}_R\,\sigma = \mathrm{Ker}_R\,\eta$ then $\sigma \sim \eta$. A substitution $\sigma \in X$ is said to be *minimal* if there is no $\gamma \in X$ such that $\gamma < \sigma$; it is called a *minimal optimal unifier of R restricted to X* if there is no $\gamma < \sigma$ such that γ is an optimal unifier of R restricted to X. Assuming $\sigma \in X$ is decidable, for finite σ, one can effectively find all minimal optimal unifiers of R restricted to X. We prove: if $\leq$ is sound for R, then σ is minimal if and only if there are η and γ such that $\sigma = \eta\gamma$, and η is a minimal optimal unifier of R restricted to X ([1]).

[1] W. BUSZKOWSKI and B. DZIEMIDOWICZ, *Restricted optimal unification*, in ***Formal methods and intelligent techniques in control, decision making, multimedia and robotics*** (Y. Hamamatsu et al., editors), Warsaw, 2000, pp. 1–8.

► LUIS F. CÁCERES, *Ultraproducts and algebraic structures*.
University of Puerto, Mathematics Department, P. O. Box 9018, Mayaguez, Puerto Rico 00681-9018.
E-mail: lcaceres@math.uprm.edu.

The ultraproduct of sets is an alternative way to prove theorems about the ideal structure and the prime ideal structure of a ring. Using this new operation it is possible to obtain easier proofs of some of the classical results and it is possible to obtain new results about the algebraic structure of commutative rings. Therefore we will use ultraproducts to prove old and new properties of ideals of a ring. We also present the propositional theories associated with a ring R and we show how to use the notion of ultraproduct of sets to study these theories.

[1] LUIS CÁCERES, *Ultraproduct of ideals in a Noetherian ring*, in ***Proceedings of the XV SIDIM***, 2001.

[2] ———, *Ultraproduct of sets and ideal theories of commutative rings*, ***Ph. D. thesis***, University of Iowa, 1998.

[3] I. KAPLANSKY, ***Commutative rings***, Polygonal Publishing House, New Jersey, 1974.

[4] GEORGE NELSON, *Compactness, ultralimits, ultraproducts and maximal ideals*, unpublished notes.

► A. A. CHUBARYAN, *On the complexity of proofs in a Frege system*.
Department of Informatics and Applied Mathematics, Yerevan State University, 1 Alek Manoogian str., Yerevan 375049, Armenia.
E-mail: achubaryan@ysu.am.

A formula in DNF is expressed as a set of clauses $\{K_1, K_2, \dots, K_p\}$ where each K_i (conjunction) is a set of literals. A DNF $D = \{K_1, K_2, \dots, K_p\}$ is called full (tautology) if from the axioms $\{K_1, K_2, \dots, K_p\}$ can be proved the empty conjunction (Λ) using the rule of *elimination*

$$\frac{K' \cup \{x\}, K'' \cup \{\bar{x}\}}{K' \cup K''}.$$

The minimal number of the uses of the rule of elimination in the proof of Λ from the DNF D is called the complexity of D and is denoted by $S(D)$.

Let φ be a formula and let $\{x_1, x_2, \dots, x_n\}$ be the set of its distinct variables. We say that the conjunction $K = \{x_{i_1}^{\sigma_{i_1}}, \dots, x_{i_r}^{\sigma_{i_r}}\}$ $(1 \le r \le n)$ is φ-*determinative* if the assignment of values σ_{i_j} to each x_{i_j} $(1 \le j \le r)$ induces the value for φ, without taking into consideration the values of the rest of the variables [1]. A full DNF D is called φ-determinative if every conjunction of D is φ-determinative. Any φ-determinative DNF D, having the minimal complexity is called the *minimal determinative* DNF for φ and is denoted by $D_\varphi^{\min}$. Let $l(\varphi)$ be the size of φ.

THEOREM 1.

(a) *For every tautology φ of size n, $S(D_\varphi^{\min}) \le 2^n$.*
(b) *For a sufficiently large n there are tautologies φ, that*

$$\log_2 l(\varphi) = o(n) \qquad \textit{and} \qquad \log_2 S(D_\varphi^{\min}) = o(2^n).$$

(See formula $\varphi_{n,m}$ of [1] for $m = 2^n - 1$.)

Let F be a Frege system.

THEOREM 2. *Every tautology φ can be proved inside F in less than $c \cdot s \cdot l$ steps, where $s = S(D_\varphi^{\min})$, $l = l(\varphi)$ and c is a constant.*

(The proof follows from the definition of $D_\varphi^{\min}$ and the results of [1].)

THEOREM 3. *For a sufficiently large n and every p $(1 \le p \le (\frac{n}{2}\log_n 2))$ there are tautologies $\varphi_{n,p}$ that $l(\varphi_{n,p}) = o(n)$ and $\varphi_{n,p}$ can be proved inside F in less than $c_1 \cdot n^p$ steps and $c_2 \cdot n^{p+1}$ size, where c_1 and c_2 are constants.*

(The proof follows from Theorem 2 by $s = n^{p-1}$.)

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► CEZARY CIESLINSKI, *Löb's theorem in set theoretical setting*.
Institute of Philosophy, Warsaw University, Krakowskie Przedmiescie 3, 00-047 Warsaw, Poland.
E-mail: cezaryc@mail.uw.edu.pl.

We present a semantic proof of Löb's theorem for theories T containing ZF. Without using the diagonalization lemma, we construct a sentence "HOM_T", which says intuitively that the predicate "homological with respect to T" (i.e., "denoting itself in every model of T") is itself homological with respect to T. In effect, the sentence "HOM_T" states "I follow semantically from T". Next we show that this sentence indeed follows from T and therefore is true. The proof is obtained by trying to reproduce in T the so called Löb's antinomy.

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► RAF CLUCKERS, *Dimension theory for the Presburger structure*.
Department of Mathematics, Katholieke Universiteit Leuven, Celestijnenlaan 200B, B-3001 Leuven, Belgium.
E-mail: raf.cluckers@wis.kuleuven.ac.be.

We consider the integers $\mathbb{Z}$ with the Presburger language $\langle +, \leq, \{\equiv_n\}_n; 0, 1\rangle$, where $\equiv_n$ with $n > 1$ stands for the congruence relation modulo n. For this structure we introduce a dimension theory. This dimension theory behaves well, e.g., the dimension of a Presburger set is invariant under definable bijections and the dimension of the union of two Presburger sets equals the maximum of the dimensions. We are able to give a classification of Presburger sets up to definable bijection by using a cell decomposition theorem and constructing explicit isomorphisms. We obtain that there exists a definable bijection between any two infinite Presburger sets of the same dimension. We also get more general results depending on parameters, when we consider definable families of Presburger sets.

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► M. E. CONIGLIO AND W. A. CARNIELLI, *Belief revision for non-classical logics*.
Center for Logic, Epistemology and the History of Science and IFCH, State University of Campinas, C. P. 6133, 13083-970 Campinas, SP, Brazil.
E-mail: coniglio@cle.unicamp.br.
E-mail: carniell@cle.unicamp.br.

Belief revision systems (BRSs) study the epistemic dynamics of theory change based on extensions of classical logic. Despite their obvious interest, BRSs have not been considered for non-classical logics in full generality (though some particular cases of BRSs for modal logics are treated in [6]).

We show that, by considering abstract logics as two-sorted, first-order structures (with different sorts for formulas and sets of formulas) and *transfers* as certain morphisms for such structures, model theory allows us to express in a powerful and elegant manner several metaproperties of logics (cf. [2] and [3]).

In particular, conservative translations (cf. [4]) are transfers. Also, with slight modifications, the well-known AGM's postulates for BRSs are freed from the constraints of particular languages and from certain restrictions such as *supraclassicality*.

By means of an injective transfer $T : \mathcal{L}_1 \to \mathcal{L}_2$ we show that a BRS in a logic $\mathcal{L}_2$ induces a new BRS in the logic $\mathcal{L}_1$. This method can be applied to a wide variety of logics as long as (conservative) translations are available.

As a concrete example, we show that the relatedness logics R and S (cf. [5]) which are conservatively translated into classical propositional logics can be endowed with a BRS structure fulfilling AGM's postulates. Moreover, different formulations of BRSs within the same logic can be compared via auto-translations. This new method opens an exciting perspective for studying belief revision in a wide context, using the powerful resources of model-theory.

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► MARCELO E. CONIGLIO AND FRANCISCO MIRAGLIA, *Representation of C^*-algebras by sheaves over quantales*.
Department of Philosophy, State University of Campinas, C. P. 6133, CEP 13081-970, Campinas (SP), Brazil.
E-mail: coniglio@cle.unicamp.br.
Institute for Mathematics and Statistics, São Paulo University, C. P. 66281, CEP 05315-970, São Paulo (SP), Brazil.
E-mail: miraglia@ime.usp.br.

The noncommutative version of the Gelfand-Naimark Theorem of Representation of C^*-algebras given by Akemann, Giles and Kummer states the following: let $\mathcal{A}$ be a noncommutative C^*-algebra with identity, and let $\mathcal{B} \supseteq \mathcal{A}$ be the W^*-algebra obtained from the reduced atomic representation of $\mathcal{A}$; an hermitian $a \in \mathcal{B}$ belongs to $\mathcal{A}$ if and only if every spectral projection $F(\lambda)$ of a is open (cf. [1] and [3]). On the other hand, the complete lattice $\tau_{\mathcal{A}}$ of open projections of $\mathcal{B}$ is a right-sided and idempotent quantale with the operation: $e \,\&\, f := e.z(f)$, where

$$z(p) = \bigwedge\{\, z \in \mathcal{B} : z \text{ is a central projection and } p \leq z \,\}$$

for all $p \in \tau_{\mathcal{A}}$ (cf. [2]). In this paper we analyse the category of sheaves over right-sided and idempotent quantales such that the equality map is symmetric; these sheaves were first defined in [4], where they were called *quasi-sheaves*.

THEOREM 1. *The category* $\boldsymbol{Sh}(\mathcal{Q})$ *of (symmetric) sheaves over the right-sided and idempotent quantale* $\mathcal{Q}$ *is a topos, with a real number object* $\widetilde{\mathbb{R}}$ *(obtained by Dedekind cuts).*

We obtain a Theorem of Representation for $\mathcal{A}$: there exists a sheaf in $\mathbf{Sh}(\tau_{\mathcal{A}})$ such that the hermitian elements of $\mathcal{A}$ appear as global sections.

THEOREM 2. *Let* $P_{\mathcal{A}}(e) = \{\, a \in \mathcal{A} : a.e = a = a^* \,\}$ *(for all* $e \in \tau_{\mathcal{A}}$*) and consider the restriction map* $\int \lambda\, dF(\lambda) \mapsto \int \lambda\, d(F(\lambda) \,\&\, f)$ *from* $P_{\mathcal{A}}(e)$ *to* $P_{\mathcal{A}}(e \,\&\, f)$ *(for all* $e, f \in \tau_{\mathcal{A}}$*). Then* $P_{\mathcal{A}}$ *is a presheaf over the quantale* $\tau_{\mathcal{A}}$.

CONJECTURE. *The sheafification of $P_{\mathcal{A}}$ is the sheaf $\widetilde{\mathbb{R}}$ in $\boldsymbol{Sh}(\tau_{\mathcal{A}})$.*

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► HIROSHI DEGUCHI AND MASANAO OZAWA, *Inter-theoretical relations based on indefinite designators in set theory*.

Kyoto University, 1-29-7, Soshigaya, Setagaya-ku, Tokyo, 157-0072 Japan.
E-mail: `deguchi@dis.titiech.ac.jp`.
Graduate School of Information Sciences, Tohoku University, Aoba-ku, Sendai, 980-8579 Japan.
E-mail: `ozawa@math.is.tohoku.ac.jp`.

The conventional approach to inter-theoretical relations admitted the dualism of syntax and semantics; formal theories are written by symbols without meanings and need models to acquire meanings. However, Bourbaki's work showed that even mathematical theories cannot be described under such a clear-cut dualism. For instance, whereas groups are models of the first-order theory of group, topological spaces cannot be considered as models of any purely syntactical theory of topology. A proposal to circumvent this difficulty was to consider every theory as a semantic entity. In fact, mathematicians describe the theory of topology through directly defining what are topological spaces. However, it is difficult to discuss inter-theoretical relations from this approach, since this excludes model theoretical devices based on the dualism. We provide a new framework for discussing inter-theoretical relations that enables us to restore the dualism. For this purpose, we introduce a new linguistic category called indefinite designators in the language of set theory. This concept is necessary for distinguishing notions proper to the theory under consideration from mathematical notions common to every theory such as numbers and functions; the proper notions are expressed syntactically by indefinite designators and can be interpreted as arbitrary sets in a fixed universe of set theory, while the common mathematical notions have the standard interpretations in the same universe of set theory. For a framework to discuss interrelations among various parts of our knowledge, this extension of the set theoretical language can be an inevitable conseqeuence from the conventional view in which set theory is a unified conceptual basis for scientific theories.

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► A. DRAGO AND R. PISANO, *S. Carnot's Réflexions: a theory based on non-classical logic*.

Group of History of Physics, Department of Physical Sciences, Univ. "Federico II", Naples, Via Cinthia, 80125 Napoli, Italy.
E-mail: `adrago@na.infn.it`.
E-mail: `pisanoraffaele@iol.it`.

The discursive part of S. Carnot's book presents 63 double negated sentences (DNSs); being not equivalent to the corresponding affirmative sentences, they involve a non-classical logic [1]. This kind of analysis of a text reveals his fundamental hypotheses underlying his exposition. Such a number of DNSs is an extraordinary result, in comparison to previous analyses performed on original books of different physical theories. They are enough for representing the whole content of Carnot's theory severed in six units of arguing; which may be represented through a flow-chart which characterises the detailed development of Carnot's theory [2]. The theory results to be organized upon a basic problem—put through a DNS—i.e., the existence or not of an upper bound in the conversions of heat in work, and how to determine it. Then, three principles—instead of the commonly two—are evidentiated by further DNSs. Almost in each unit of arguing through DNS, an *ad absurdum* theorem closes the argument; its conclusion is then applied as an operative principle. In total, this kind of analysis shows that Carnot consistently follows a theoretical choice of working within non-classical logic. All that proves that non-classical logic was consistently used in classical scientific theories before the birth of quantum mechanics.

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► PHILIP EHRLICH, *Surreal numbers, Conway names and the simplicity hierarchy*.
Department of Philosophy, Ellis Hall, Room 220D, Ohio University, Athens, Ohio 45701, USA.
E-mail: ehrlich@ohiou.edu.

J. H. Conway's real-closed field of *surreal numbers* [1] contains the reals and the ordinals as well as a great many other numbers including $-\omega$, $\omega/2$, $1/\omega$, $\sqrt{\omega}$ and $\omega-\pi$ to name only a few. Indeed, this particular real-closed field, which Conway calls *No*, is so remarkably inclusive that, subject to the proviso that numbers—construed here as members of ordered "number" fields—be individually definable in terms of sets of von Neumann-Bernays-Gödel set theory with Global Choice, it may be said to contain "All Numbers Great and Small." However, in addition to its distinguished structure as an ordered field *No* has a rich algebraico-tree-theoretic structure, or *simplicity hierarchy*, that emerges from the recursive clauses in terms of which it is defined [2].

Among the striking surreal features is that ordinal exponentiation with base ω extends to an operation $x \mapsto \omega^x : No \to No$ in such a way that every surreal number a can be expressed uniquely as a generalized power series of the form $\sum_{\alpha<\beta} \omega^{y_\alpha} r_\alpha$—called the *Conway name* (or *normal form*) of a [2]—where β is an ordinal, $(y_\alpha)_{\alpha<\beta}$ is a strictly decreasing series of surreals, and $(r_\alpha)_{\alpha<\beta}$ is a series of nonzero real numbers. Indeed, every such expression is the Conway name of some surreal number, the Conway name of an ordinal being just its Cantor normal form [2].

In the present paper the relation between the Conway names of surreal numbers and the simplicity hierarchy is explored.

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► JAFAR S. EIVAZLOO AND MOJTABA MONIRI, *Definably Dedekind complete expansions of ordered fields*.
Department of Mathematics, Tarbiat Modarres University, Tehran, Iran.
E-mail: eivazl_j@net1cs.modares.ac.ir.
E-mail: moniri_m@modares.ac.ir.

We give several equivalent axiomatizations for $\mathcal{L}$-definably Dedekind complete *expansions* of ordered fields. In the language of *OF*, the notion is already known to coincide with *RCF* and with (weak) o-minimality. Our axiomatizations extend *OF* by the schemes expressing d-connectedness; or nontrivial implications between being interval, convex or d-connected for definable subsets; or either of convexity of the image of definable convex subsets under, or IVP for definable continuous functions. In such expansions, we show that one-to-one definable continuous functions on closed bounded intervals are open (second order counterpart characterizing $\mathbb{R}$). Definably Dedekind complete weakly o-minimal expansions are precisely the o-minimal ones, the former has non (weakly) o-minimal examples also. We prove *equivalence* of o-minimality for expansions of ordered fields with definable subsets being finite disjoint unions of d-connected definable subsets.

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► OLIVIER ESSER, *Antifoundation and transitive closure in the system of Zermelo*.
Université libre de Bruxelles, Campus de la plaine, Service de Logique C. P. 211, Boulevard du Triomphe, B-1050 Bruxelles, Belgium.
E-mail: oesser@ulb.ac.be.

The role of the axiom of foundation with respect to the axiom of transitive closure in the system of Zermelo Z has been studied by M. Boffa in [2, 3]. The role of this talk is to study the role of the axiom of antifoundation AFA due to M. Forti, F. Honsell and P. Aczel ([5, 1]). The axiom AFA states that for every graph $(G, \in_G)$, there is a unique function d of domain G satisfying $d(g) = \{\, d(z) \mid z \in_G g \,\}$ (d is called a decoration of the graph G). The relative consistency of Z + "antifoundation" + "transitive closure" is shown by a technique well-known for ZF (see [1]). We introduce further the weak replacement principle WREP: for any set a and F a class-function: if $\{\, F(x) \mid x \in a \,\}$ is a *transitive* class, then it is a set (WREP is an axiom scheme). We study the relations between these three statements via interpretations. Some adaptations for ZF without the axiom of infinity can be given.

This is joint work with R. Hinnion.

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► HORACIO FAAS, *Diagrammatic reasoning and heterogeneous inference*.
Centro de Investigaciones, Facultad de Filosifia y Humanidades, Universidad Nacional de Córdoba, Pabellon Agustin Tosco, Ciudad Universitaria, 5000 Córdoba, Argentina.
E-mail: horacio@goedel.ffyh.unc.edu.ar.

In this paper I give a survey on some recent contributions on diagrammatic reasoning, mainly in logic, computer science, and mathematics. I refer to Shin's work on Venn diagrams, Jamnik's automatic demonstrator for some mathematical theorems, and the applications of Barwise and Etchemendy's Hyperproof. I show how certain rules of this program relate to methods usually applied in the proofs of some classical theorems as Euler's formula for convex polyhedra and its proof by Cauchy. I refer also to Oberlander's use of proofs with this program to elicit the use of abstraction by students using the program, as well as determining the influence of their cognitive styles. I would try some relation between diagrammatic reasoning and mind's activity on this subject on the grounds of recent research in neuroscience.

► DON FAUST, *The concept of negation*: (1) *Aristotle's privatives and contemporary elementary logic and* (2) *logics of 'conflict without contradiction'*.
Department of Mathematics and Computer Science, Northern Michigan University, Marquette, Michigan 49855, USA.
E-mail: dfaust@nmu.edu.

We use Evidence Logic (EL), containing Classical Logic plus machinery for the representation and processing of confirmatory and refutatory evidence. In EL_n $(n > 1)$, $P_c x_1 \dots x_k : e$ asserts confirmation of $Px_1 \dots x_k$ at evidence level e while $P_r x_1 \dots x_k : e$ asserts refutation at level e (for e in the Evidence Space $\{\, i/(n-1) : i = 1, \dots, n-1 \,\}$).

Concerning (1) and using unary predication for simplicity, where Px is a unary predication, non-Px is a privative unary predication. Aristotle argued (e.g., *On interpretation, X and Prior analytics*, Book I, XLVI) for distinction of non-Px and $\neg Px$, maintaining the former implies the latter but not conversely. However, in contemporary elementary logic the term complement non-Px is widely used in such a way that non-Px is logically equivalent to $\neg Px$. After some observations about this peculiar situation, we suggest a faithful implementation of Aristotle's view, an axiomatizable extension T of EL_2 wherein Px is interpreted with $P_c x$ and non-Px with $P_r x$ (and T includes the axiom $P_r x \Longrightarrow \neg P_c x$).

Concerning (2), which is related to aspects of negation particularly relevant to paraconsistency, we construct a family of extensions of EL_n which axiomatically allow conflict of the confirmatory and refutatory while forcing one or the other to be the case. These theories omit axioms reflecting Aristotle's view while including axioms reflecting the converse of his view. As such, these theories, with respect to confirmatory and refutatory predications, allow violation of 'laws of nonconflict' while forcing 'laws of excluded middle'.

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► CHRISTIAN FERMÜLLER, *A natural deduction system for intuitionistic fuzzy logic*.
Technische Universität Wien, E185.2, Favoritenstrasse 9/3.OG, A-1040 Vienna, Austria.
E-mail: chrisf@logic.at.

Intuitionistic fuzzy logic IF was introduced by Takeuti and Titani. It coincides with the first-order Gödel logic based on the real unit interval [0, 1] as set of truth-values. We present a natural deduction system NIF for IF. NIF is defined by suitably translating a first-order extension of Avron's hypersequent calculus for Gödel logic. We discuss soundness, completeness and normal form theorems for NIF; and also hint at some methodological challenges for future work. In particular, we like to view the formulation NIF as a first step towards defining a λ-calculus based semantics for IF logic, which serves to substantiate the claim that Gödel logic relates to functional programming for multi process systems in the same way as intuitionistic logic relates to ordinary functional programming.

This is joint work with M. Baaz and A. Ciabattoni.

► MICHELE FRIEND, *What a proof guarantees for Frege*.
George Washington University; The Old Vicarage, Pytchley, Nr. Kettering, Northamptonshire, England, U. K. NN14 1EP.
E-mail: micheleifriend@yahoo.com.

For Frege, a gapless proof was meant to be truth preserving. That is, if we have a gapless proof, which starts with logical axioms, then the conclusion is also true. This is very close to our modern definition of the validity of an argument, namely: if the premises of the argument are true, then so is the conclusion.

We all know this. What we seem to have forgotten, is that Frege's notion of proof was quite rich. His proofs in logic were meant to preserve both analyticity and universality from the axioms to the conclusion.

In Frege, 'analyticity' is given both a positive and negative definition. The positive definition is that a truth is analytic if and only if it follows from logical axioms and definitions by means of a gapless proof. The negative definition is that to know an analytic truth we need not make appeal to sense experience or to Kantian spatial or temporal intuition. 'Universality' was defined in terms of applying to the universal domain; that is, the domain of all things. This characterisation of universality, together with axiom V, led to contradiction. Both definitions are problematic, but can be salvaged.

There are two questions I wish to address concerning the philosophical richness of Frege's proofs. One is: 'what feature, or set of features, of gapless proofs is meant to guarantee preservation of analyticity and universality?' The other question is: 'could we recognise these features in an arbitrarily chosen formal system of proof?'

Part of the answer lies in Frege's choice of axioms. Part of the answer has to do with Frege's conception of logic as providing ultimate justification. Arguably, the concepts of preservation of universality and analyticity can be identified with some formal systems of proof. Some examples will illustrate this.

► SUMAN GANGULI AND ANIL NERODE, *An effective completeness theorem for a modal logic*.
Department of Mathematics, Cornell University, Ithaca, New York 14850, USA.
E-mail: sg50@cornell.edu.

We initiate the study of computable model theory of modal logic, by establishing an effective completeness theorem for the following modal logic: first-order constant domain K. We give a natural definition of a decidable Kripke model, and then show how to construct such a decidable Kripke model of a given decidable theory.

Our new technique of constructing Kripke models is inspired by the Henkin construction for classical logic. The Henkin construction, however, depends in an essential way on the Deduction Theorem. In its usual form the Deduction Theorem fails for modal logic. In our constructions, the Deduction Theorem is replaced by a result about objects called finite Kripke diagrams. We give an argument that this result can be viewed as an analogue of the Deduction Theorem for modal logic.

► MARTIN GOLDSTERN, *Intervals of clones on infinite sets*.
Department of Algebra, Wiedner Haupstrasse 8-10/118.2, Technische Universität, A-1040 Wien, Austria.

E-mail: Martin.Goldstern@tuwien.ac.at .

A clone on a nonempty set X is a family C of finitary operations on X which contains all projections and is closed under composition.

Let $\mathrm{Cl}(X)$ be the family of all clones on X; $\mathrm{Cl}(X)$ is a complete algebraic lattice.

There are several natural open questions about the structure of $\mathrm{Cl}(X)$; it is expected that set-theoretic or topological (rather than algebraic) methods will be necessary to solve them.

I will discuss some open problems and describe results showing how large this lattice is.

Several of the results were obtained jointly with Saharon Shelah.

► JOANNA GRYGIEL, *Two operations of gluing of lattices*.
Institute of Mathematics and Computer Science, Pedagogical University of Czestochowa, Poland.
E-mail: j.grygiel@wsp.czest.pl.

In [5] A. Wronski introduced an operation of gluing of lattices in order to investigate some intermediate logics. At the same time the similar operation given by Hall-Dilworth was generalized by Ch. Herrmann [3] from the algebraical point of view. We are going to compare these two operations and formulate some theorems concering the decomposition of finite lattices into components of these gluings.

[1] A. Day and C. Herrmann, *Gluings of modular lattices*, ***Order***, vol. 5 (1998), pp. 85–101.

[2] J. Grygiel and P. Wotylak, *The uniqueness of the decomposition of distributive lattices into sums of Boolean lattices*, ***Reports on Mathematical Logic***, vol. 31 (1997), pp. 93–102.

[3] Ch. Herrmann, *S-verklebte Summen von Verbanden*, ***Mathematische Zeitschrift***, vol. 130 (1973), pp. 255–274.

[4] J. Kotas and P. Wotylak, *Finite distributive lattices as sums of Boolean algebras*, ***Reports on Mathematical Logic***, vol. 29 (1995), pp. 35–40.

[5] A. Wronski, *Remarks on intermediate logics with axioms containing only one variable*, ***Reports on Mathematical Logic***, vol. 2 (1974), pp. 63–76.

► JOHANNES HAFNER, *Why (and how) does metamathematics matter to philosophy?*
Group in Logic and the Methodology of Science, Evans Hall, University of California at Berkeley, California 94720-3840, USA.
E-mail: jhafner@math.berkeley.edu.

Apparently there is no straightforward way leading from metamathematical results to philosophical theses. Yet it has been claimed frequently that formal theorems, notably Gödel's incompleteness theorems and the Löwenheim-Skolem theorem, possess philosophical significance or yield philosophical conclusions. How is the gap to be bridged between the two quite different methodologies in play here? Surprisingly, this question has hardly ever been addressed in the philosophical debates surrounding those theorems (rare exceptions being [1, 2]).

In this context Quinean holism may seem a promising position to take since it pronounces formal science, philosophy (and natural science) to be continuous with each other. I explore the prospects of this option and point out that Quine's position renders certain logico-philosophical arguments, e.g., Putnam's model-theoretic argument, *prima facie* unintelligible. This criticism is substantiated by an analysis of Putnam's argument focusing on the (ultimately unsuccessful) attempt to construe it as *a reductio*.

Hence we can conclude that our original methodological problem is not easily dismissed on the basis of Quine's philosophy. However, reflecting on Putnam's use of the Löwenheim-Skolem theorem also leads to a better understanding of the conditions under which metamathematics and philosophy might interact.

[1] P. Benacerraf, *What mathematical truth could not be—I*, in ***Benacerraf and his critics***

(A. Morton and S. P. Stich, editors), Blackwell, Oxford and Cambridge, Massachusetts, 1996, pp. 9–59.

[2] A. George, *Skolem and the Löwenheim-Skolem theorem: A case study of the philosophical significance of mathematical results*, ***History and Philosophy of Logic***, vol. 6 (1985), pp. 75–89.

[3] H. Putnam, *Models and reality*, ***Journal of Symbolic Logic***, vol. 45, pp. 464–482.

► PETR HÁJEK AND ZUZANA HANIKOVÁ, *A contribution on set theory in fuzzy logic*.
Institute of Computer Science, 182 07 Prague, Czech Republic.
E-mail: hajek@cs.cas.cz.
E-mail: zuzana@cs.cas.cz.

Quantified basic fuzzy logic BL$\forall$ is a formal many-valued predicate calculus with a (standard) semantics interpreting conjunction by an arbitrary continuous t-norm and implication by its residuum. BL$\forall\Delta$ has in addition a unary connective Δ. See [2] for a detailed treatment and references.

In BL$\forall\Delta$ we define a ZF-like first-order theory FST ("fuzzy set theory") in the language $\{\in, =\}$. FST as it stands only admits crisp semantics of $=$ (this is largely inspired by [5]). Generally, as shown, e.g., in [1], a careless choice of axioms may prevent a theory from having any non-crisp models; to show that FST allows fuzzy semantics we construct in [3] an analogy of a Boolean-valued universe over a BLΔ-algebra and verify the validity of the axioms of FST in it.

In FST we define a class H of hereditarily crisp sets; then H is an inner model of ZF in FST. Hence FST is equiconsistent with ZF, and moreover, the universe of ZF can be regarded as a subuniverse of a set theoretic universe with fuzzy semantics.

This work draws on a multitude of papers presenting various set theories in many-valued, linear, or intuitionistic logic. Apart from those already quoted, these are, e.g., [6] and [4].

[1] R. J. Grayson, ***Heyting-valued models for intuitionistic set theory***, Lecture Notes in Mathematics, vol. 753, Springer-Verlag, 1979, pp. 402–414.

[2] Petr Hájek, ***Metamathematics of fuzzy logic***, Kluwer Academic Publishers, 1998.

[3] P. Hájek and Z. Haniková, *A set theory within fuzzy logic*, ***Proceedings of the ISMVL 2001 (Warsaw)***, IEEE Computer Society, 2001.

[4] W. C. Powell, *Extending Gödel's negative interpretation to ZF*, ***Journal of Symbolic Logic***, vol. 40 (1975), pp. 221–229.

[5] M. Shirahata, *Phase-valued models for linear set theory*, preprint.

[6] G. Takeuti and S. Titani, *Fuzzy logic and fuzzy set theory*, ***Archive for Mathematical Logic***, vol. 32 (1992), pp. 1–32.

► KAI HAUSER, *What new axioms could not be*.
Technische Universität, Fachbreich 3, Sekr. MA 8-1, 10623 Berlin.
E-mail: hauser@math.tu-berlin.de.

Long before the independence of the continuum hypothesis (CH) from the standard axioms of set theory was established, Gödel suggested looking for new *axioms* which are powerful enough to decide questions like CH and are 'implied' by the concept of set. The large cardinal axioms mentioned by Gödel as well as other strong axioms of infinity studied by set theorists so far describe concepts which are formalizable in set theory. More recently, various attempts have been made to introduce axioms of a different kind that either implicitly or explicitly involve new primitives. One example are 'axioms' of symmetry which were used by Ch. Freiling to give a 'philosophical proof' of the negation of CH based on plausible intuitions about 'random numbers'.

In my talk I will expose the main flaws in this reasoning and examine the larger proposal behind it to discard the set theoretic foundation of mathematics. The analysis draws on philosophical ideas as well as mathematical results that have come to light in the modern development of set theory. I will argue that (in a sense which can be made precise) Gödel's original approach at present holds an advantage over the proposal to axiomatize new primitives.

► ALBERT HOOGEWIJS AND PIETER AUDENAERT, *A pvs-proof for a memory modeling problem is a proof!*
RUG, GALGLAAN 2, B-9000 Gent, Belgium.
E-mail: bh@cage.rug.ac.be.
Kerkstraat 44, B-9080, Lochristi-Zaffelare, Belgium.
E-mail: paudenae@cage.rug.ac.be.

A set $S = \{ s_{pi} \mid 1 \leq p \leq n, 1 \leq i \leq m_p \}$ is considered to be a set of program segments distributed over n processors p. $D_p = \{ (s_{pi}, s_{p(i+1)}) \mid 1 \leq i < m_p \}$ describes the sequential order of the segments for the processor p. Let $D_P = \bigcup D_p$ denote the partial order over all segments. So-called synchronization instructions between segments $s_{p_1 i_1}$ and $s_{p_2 i_2}$ are used in order to avoid conflicts in accessing the shared memory (see [3]). Let D_S denote the set of synchronization relations (denoted as $(s_{p_1 i_1}, s_{p_2 i_2})$), then the indices of an element $(s_{p_1 i_1}, s_{p_2 i_2}) \in D_S$ satisfy the following conditions:

(1) $p_1 \neq p_2$;
(2) $i_1 + 1 < i_2$;
(3) $(s_{p_2(i_2-1)}, s_{pi}) \notin D_S$;
(4) $(s_{pi}, s_{p_1(i_1+1)}) \notin D_S$;
(5) if $i \neq i_2$ then $(s_{p_1 i_1}, s_{pi}) \notin D_S$;
(6) if $i \neq i_1$ then $(s_{pi}, s_{p_2 i_2}) \notin D_S$;
(7) $i_1 + 1 < m_{p_1}$, $i_2 \neq 1$.

Note that these conditions exclude a loop in the partial order relation $D_< = (D_P \cup D_S)^\dagger$ (†: transitive closure). Let $D_>$ be the symmetric relation of $D_<$ and $D_= = \{ (s_{pi}, s_{pi}) \mid 1 \leq p \leq n; 1 \leq i \leq m_p \}$, then $D_\| = S \times S \setminus (D_< \cup D_> \cup D_=)$ represents the parallelism relation between segments.

A more efficient detection of parallel segments relies on so-called vector clocks [2]. We associate a vector $v(p, i) \in \mathbb{N}^n$ to each segment s_{pi} as follows:

(1) $v(p, 1)[p] = 1$ and if $j \neq p$ then $v(p, 1)[j] = 0$;
(2) if $(s_{qk}, s_{p(i+1)}) \in D_S$ then
  (a) $v(p, i+1)[p] = v(p, i)[p] + 1$;
  (b) $v(p, i+1)[q] = \max(v(p, i)[q], v(q, k)[q] + 1)$;
  (c) if $j \neq p$ and $j \neq q$ then $v(p, i+1)[j] = \max(v(p, i)[j], v(q, k)[j])$;
(3) if $(s_{qk}, s_{p)i+1)}) \notin D_S$ then
  (a) $v(p, i+1)[p] = v(p, i)[p] + 1$;
  (b) if $j \neq p$ then $v(p, i+1)[j] = v(p, i)[j]$.

Define $s_{pi} \| s_{qj}$ if and only if $v(p, i)[p] \geq v(q, j)[p]$ and $v(p, i)[q] \leq v(q, j)[q]$. Then parallelism between segments can be defined as $O_\| = \{ (s_{p_1 i_1}, s_{p_2 i_2}) \mid s_{p_1 i_1} \| s_{p_2 i_2} \}$.

We show that $D_\| = O_\|$ using PVS and 91 lemmas (see [1]).

[1] P. AUDENAERT, *Memory modeling in PVS*, in ***Proceedings of the 5th Dutch Proof Tool Day***, Gent, 2000, http://cage.rug.ac.be/prooftool/proceedings/

[2] M. RONSSE and K. DE BOSSCHERE, *RecPlay: a fully integrated practical record/replay system*, ***Transactions on Computer Science***, vol. 17 (1999), pp. 133-152.

[3] N. SLAATS, B. VAN ASSCHE, and A. HOOGEWIJS, *Shared memory synchronization*, in ***Proof in VDM: Case studies*** (J. C. Bicarregui, editor), Springer-Verlag, pp. 123–156.

► LEON HORSTEN, *Canonical notation systems*.
Center for Logic, Department of Philosophy, University of Leuven, Kardinaal Mercierplein 2, B-3000 Leuven, Belgium.
E-mail: Leon.Horsten@hiw.kuleuven.ac.be.

A general theory of *canonicity of notation systems* for sets of objects is outlined. The motivating idea is that the degree of canonicity of a notation system depends on the extent to which it is verifiable, for any two names of the notation system, whether or not they refer to the same object. The paradigmatic example of a highly canonical notation system is the system of standard numerals $\underline{0}, \underline{1}, \underline{2}, \ldots$. An example of a notation system of a lesser degree of canonicity is Kleene's system O of notations for (a subset of the) countable ordinal numbers. A candidate for being a (finite) notation system which is of an even lower degree of canonicity is the system $\{a, b, \underline{1}\}$, where a is defined to be 1 if Goldbach's Conjecture is true and 0 otherwise, and b is defined to be 1 if $a = 0$ and 0 otherwise.

The theory of canonicity of notation systems is formally presented in a second-order framework of *Carnapian* modal logic, where the modal operator is interpreted as 'it is verifiable that'. It will be shown how in this framework, a logical theory of *presentations* of (mathematical) objects can be formulated. Precise definitions of *degrees of canonicity* are then given, the *partial ordering* of this collection of degrees is described, and a notion of *relative canonicity* is formulated.

[1] R. Carnap, ***Meaning and necessity. A study in semantics and modal logic***, University of Chicago Press, 1956 [1947].

► TAKAO INOUÉ, *A modal interpretation for Leśniewski-Ishimoto's propositional ontology*.
Kawagishi-Naka 1-2-24, Okaya, Nagano, 394-0047, Japan.
E-mail: ty5t-inue@asahi-net.or.jp.

Blass [1] proved that Leśniewski-Ishimoto's propositional ontology $\mathbf{L}_1$ (see [4]) is embedded in the modal logic **K** by his proposed translation (see also [2, 3]). In this paper, we shall propose another embedding $NI(\cdot)$ of $\mathbf{L}_1$ in **K** such that the translation $NI(\cdot)$ from $\mathbf{L}_1$ to **K** is defined as follows:

(1) $NI(\phi \vee \psi) = NI(\phi) \vee NI(\psi)$,
(2) $NI(\neg\phi) = \neg NI(\phi)$,
(3) $NI(\varepsilon ab) = \diamond p_a \supset p_a . \wedge \Box(p_a \equiv p_b)$,

where p_a and p_b are propositional variables corresponding to the name variables a and b, respectively. Then we prove:

Embedding theorem. *For any formula* ϕ *of* $\mathbf{L}_1$, $\vdash_{\mathbf{L}_1} \phi \iff \vdash_{\mathbf{K}} NI(\phi)$.

[1] A. Blass, *A faithful modal interpretation of propositional ontology*, ***Mathematica Japonica***, vol. 40 (1994), pp. 217–223.

[2] T. Inoué, *Partial interpretation of Leśniewski's epsilon in modal and intensional logics* (abstract), ***Bulletin of Symbolic Logic***, vol. 1 (1995), pp. 95–96.

[3] ———, *Partial interpretations of Leśniewski's epsilon in von Wright-type deontic logics and provability logics*, ***Bulletin of the Section of Logic***, vol. 24 (1995), pp. 223–233.

[4] A. Ishimoto, *A propositional fragment of Leśniewski's ontology*, ***Studia Logica***, vol. 36 (1977), pp. 285–299.

► ALEXANDRE IVANOV, *Nice enumerations of strongly minimal structures*.
Institute of Mathematics, University of Wrocław, pl. Grunwalldzki 2/4, 50-384, Wrocław, Poland.
E-mail: ivanov@math.uni.wroc.pl.

Let W be an infinite structure and let $<$ be an ordering of W of order type ω. We say that the enumeration of W induced by this ordering is *nice* if any infinite subset $A \subset W$

contains a pair $a_1, a_2 \in A$ such that there exists $g \in G = \mathrm{Aut}(W)$ satisfying $g(a_1) = a_2$ and $(\forall x < a_1)\ (g(x) < a_2)$.

Nice enumerations appear in several model-theoretic topics: quasi-finite axiomatizability (see [1]), finite covers of categorical structures (see [2]).

CONJECTURE. *A modular strongly minimal structure having a nice enumeration is trivial* $(\mathrm{acl}(\bar a \cup \bar b) = \mathrm{acl}(\bar a) \cup \mathrm{acl}(\bar b))$ *or* ω*-categorical.*

THEOREM 1. *If a strongly minimal group G (which can have some extra structure) is modular and has a nice enumeration then G is* ω*-categorical.*

A vector space over a division ring is a typical example where Therem 1 is applicable. It is not known if there is an affine (projective) space over an infinite division ring having a nice enumeration.

THEOREM 2. *Let D be an infinite division ring and W be an affine space of finite dimension over D.*

(1) *If D is the union of an infinite chain of division rings then W does not have a nice enumeration.*
(2) *If D is a finite extension of a field K which is central is D and has an infinite almost finite family of non-trivial valuations, then W does not have a nice enumeration.*

COROLLARY. *A finite dimensional affine space over an infinite field does not have a nice enumeration.*

[1] G. AHLBRANDT and M. ZIEGLER, *Quasi finitely axiomatisable totally categorical theories*, ***Annals of Pure and Applied Logic***, vol. 30 (1986), pp. 63–83.

[2] D. EVANS, A. IVANOV, and H. D. MACPHERSON, *Finite covers*, in ***Model theory of groups and automorphism groups*** (D. Evans, editor), London Mathematical Society, Lecture Note Series, vol. 244, Cambridge University Press, 1997, pp. 1–72.

► PIERRE JORAY, *Nominal negation and predicates' applicability in Lesniewski's ontology*.
Institut de logique, Université de Neuchâtel, Louis-Agassiz 1, CH-2000 Neuchâtel.
E-mail: pierre.joray@unine.ch.

The fact that standard semantics presuposes universal applicability for all predicates had often been considered as an awkward property of predicate logic. Let us consider, for example, the predicate *even*. According to the formula

$$(*) \qquad (Px \vee \neg Px)$$

which is logically true on standard systems, it is clearly true that everything *is even* or *is not even*. But, on the other hand, it is actually not the fact that everything *is even* or *is uneven*, excepted under the consideration we are talking about natural numbers.

This seems to mean that the use of such a predicate is semantically related to what we can call a specific range of applicability. In order to give a formal account to this semantic notion, solutions had been usually proposed in two main directions: intensional logic and many valued logic.

The aim of this paper is to show that it is also possible to give a solution without rejecting any of the usual logical principles: extensionality, excluded middle and bivalence. Thanks to an elementary part of Lesniewski's Ontology, which is an extensional calculus of names, I will emphasize that the above mentionned problem is not a direct consequence of the universal applicability of predicates. I will show that it is due to the impossibility to represent the negation *un-* through the only propositional negation. The way to define in Lesniewski's formal language a family of nominal negations able to deal with the semantic problem of specific ranges of applicability will be discussed.

[1] S. LESNIEWSKI, ***Collected works***, Kluwer Academic Publishers, 1992.

[2] J. SLUPECKI, *S. Lesniewski's calculus of names*, ***Studia Logica***, vol. III (1955), pp. 7–73.

► REINHARD KAHLE AND ANTON SETZER, *Extended predicative universes*.
WSI, Universität Tübingen, Sand 13, D-72076, Tübingen, Germany.
E-mail: kahle@informatik.uni-tuebingen.de.
Department of Computer Science, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, Britain.
E-mail: a.g.setzer@swan.ac.uk.

We give a new formulation of a Mahlo universe in the framework of explicit mathematics. Based on the concept of pre-universes the Mahlo universe is constructed "from below". A certain independence property of the sub-universes gives the approach a predicative nature. Since explicit mathematics provides partial functions, we can give an appropriate induction principle for the Mahlo universe.

The Mahlo case serves only as a starting point to illustrate the underlying concepts. They are also used by the second author to study Π_n reflection and even stronger theories.

The new approach will be discussed in relation to the existing formalizations of Mahlo, e.g., in [4, 5, 1, 2, 3].

[1] GERHARD JÄGER, *Metapredicative and explicit Mahlo: a proof-theoretic perspective*, in ***Logic colloquium 2000***, Lecture Notes in Logic, A. K. Peters, to appear.

[2] GERHARD JÄGER and THOMAS STRAHM, *Upper bounds for metapredicative Mahlo in explicit mathematics and admissible set theory*, ***Journal of Symbolic Logic***, vol. 66 (2001), pp. 935–958.

[3] GERHARD JÄGER and THOMAS STUDER, *Extending the system* T_0 *of explicit mathematics: the limit and Mahlo axioms*, ***Annals of Pure and Applied Logic***, to appear.

[4] M. RATHJEN, *Proof-theoretic analysis of KPM*, ***Archive for Mathematical Logic***, vol. 30 (1991), pp. 377–403.

[5] ANTON SETZER, *Extending Martin-Löf type theory by one Mahlo-universe*, ***Archive for Mathematical Logic***, vol. 39 (2000), pp. 155–181.

► YUZURU KAKUDA AND AKIRA SUZUK, *Boolean-valued design theory*.
Department of Computer and Systems Engineering, Faculty of Engineering, Kobe University.
E-mail: kakuda@kobe-u.ac.jp.
Division of Information Media Science, Graduate School of Science and Technology.
E-mail: sakira@kurt.cla.kobe-u.ac.jp.

In *Abstract Design Theory* [2], we gave a framework of "design theory" based on *Channel Theory* of Barwise-Seligman [1]. By our reasoning the forcing method is a kind of design to get a generic element. For example, consider the forcing method to get a generic real. The corresponding functional scheme $\mathfrak{S}$ consists of

1. The functional theory of $\mathfrak{S}$ is the regular theory such that $\mathrm{typ}(\mathfrak{S}) = \omega$ and $\Gamma \vdash_{\mathfrak{S}} \Delta$ if and only if Γ and Δ have a common element.
2. The attribute space of $\mathfrak{S}$ is the state space canonically obtained by the classification *Gen* such that $\mathrm{tok}(\mathit{Gen})$ is the set of reals, $\mathrm{typ}(\mathit{Gen})$ is the set of *ZF*-sentences and $a \Vdash_{\mathit{Gen}} \varphi$ if and only if $V[a] \vDash \varphi$.
3. The set $N_{\mathfrak{S}}$ of normal tokens of $\mathfrak{S}$ is the set of generic reals.
4. The informorphism $\langle f_{\mathfrak{S}}, g_{\mathfrak{S}} \rangle$ is defined by $f_{\mathfrak{S}}(n) =$ the sentence such that n is a member of the *generic real* $\boldsymbol{a}$, and $g_{\mathfrak{S}}(a) = \langle a, \omega - a \rangle$ for any real a.

This is done in the outer world in which the real world can be considered as a subworld. The functional scheme in the above must be formulated in the $\mathcal{B}$-valued model, where $\mathcal{B}$ is the canonical Cohen algebra. Therefore, a finite specification $\langle X, Y \rangle$ of $\mathcal{S}$ is an aproximate specification of the target reals, that is, the generic reals. Thus, the true reals we want to have cannot be embodied in the real world, but all that we can say is to what boolean degree the real we actually design is fitted to the reals we want to have. This is a common future of actual design. We must be satisfied with a compromise.

In this talk, we develop a boolean-valued design theory in a general setting.

[1] JON BARWISE and JERRY SELIGMAN, ***Information flow***, Cambridge Tracts in Theoretical Computer Science, vol. 44, Cambridge University Press, 1997.

[2] YUZURU KAKUDA and MAKOTO KIKUCHI, *Abstract design theory*, ***Annals of Japan Association of Philosophy of Science***, to appear.

► V. KH. KHAKHANIAN, *Predicates of realizability for intuitionistic set theory*.
Moscow State University of railway communications, Chair of Applied Mathematics - 2, ul Obraztsova 15, Moscow 101475, Russia or Apt. 535, str. Litovskij bulvar 9/7, Moscow 117593, Russia.
E-mail: dvadva@au.ru.

In [1] A. Dragalin suggested a new method of construction of realizability models for HA. We "lift" here this method to the set theory with intuitionistic logic ZFI (for an exact formulation of ZFI see [2]). Predicate T on the set statements of ZFI is called a realizabilit predicate (r-predicate) if (a) ZFI $\vdash \varphi \Longrightarrow T\varphi$; (b) $T\varphi \wedge T(\varphi \to \psi) \Longrightarrow T\psi$ for any φ and ψ. A very simple example of r-predicate is $T\varphi \iff \text{ZFI} \vdash \varphi$. We give an operation of getting a new r-predicate. Let A be a predicate which is defined on the set of atomic ZFI-statements and we have:

(1) $\lambda \Longrightarrow A\varphi \Longrightarrow T\varphi$ for any atomic φ;
(2) $\lambda \Longrightarrow T(\perp)$;
(3) if φ is a true atomic statement then λ;
(4) if φ is a false atomic statement and $A\varphi$ then λ;
(5) the predicate A is extensional on the set atomic statements that is $\forall q\ [A(q \in t) \iff A(q \in r)] \wedge A(t \in p) \Longrightarrow A(r \in p)$;
(6) for any formula $\varphi(x)$ with only one free variable x and for any term t there exists a term q such that $\forall r\ [A(r \in q) \iff A(r \in t) \wedge T\varphi(r)]$.

We define now a new predicate $S(\lambda, A, T)$ on ZFI-statements by induction on φ:

(1) $S\varphi \iff A\varphi$ if φ is an atomic statement;
(2) $S(\varphi \wedge \psi) \iff S\varphi \wedge S\psi$;
(3) $S(\varphi \vee \psi) \iff S\varphi \vee S\psi$;
(4) $S(\varphi \to \psi) \iff (S\varphi \Longrightarrow S\psi) \wedge T(\varphi \to \psi)$;
(5) $(S\forall x\ \varphi(x)) \iff \forall t\ S(\varphi(t) \wedge T(\forall x\ \varphi(x))$;
(6) $S(\exists x\ \varphi(x)) \iff \exists t\ S\varphi(t)$.

We claim: $S\varphi \Longrightarrow T\varphi$; $S(\perp \to \varphi)$; ZFI $\vdash \varphi \Longrightarrow S\varphi$, so that S is an r-predicate. In [3] we used this operation to construct a model to prove the admissibility of Markov's Rule in ZFI.

[1] A. DRAGALIN, *New kinds of realizability and Markov's rule*, ***Soviet Math. Dokl.***, vol. 251 (1980), no. 3, pp. 534–537, in Russian.

[2] J. MYHILL, ***Some properties of intuitionistic Zermelo-Fraenkel set theory***, Lecture Notes in Mathematics, vol. 337, Springer-Verlag, 1973, pp. 206–231.

[3] V. KHAKHANIAN, ***The Markov's rule is admissible in the set theory with intuitionistic logic***, Lecture Notes in Computer Science, vol. 1289, Springer-Verlag, 1997, pp. 163–167.

► PETER KOEPKE AND JOCHEN LÖFFELMANN, *Intermediate models of Prikry generic extensions*.
Mathematisches Institut, Beringstrasse 1, D 53115 Bonn, Germany.
E-mail: koepke@math.uni-bonn.de, jochen@math.uni-bonn.de.

Prikry forcing adjoins a Prikry sequence C to the ground model V; C is an ω-sequence of ordinals cofinal in some measurable cardinal of V. We show that Prikry forcing is minimal in the following sense: every transitive ZFC-model M such that $V \subseteq M \subseteq V[C]$ is of the form $V[C']$ for some $C' \subseteq C$. The intermediate models of the Prikdry extension are thus parametrized by $\mathcal{P}(\omega)/\text{fin}$. This result is contained in the doctoral dissertation of the second author [2]. One may obtain other lattices of intermediate models by modifying Prikry forcing.

[1] K. Prikry, *Changing measurable into accessible cardinals*, ***Dissertationes Mathematicae***, vol. 68 (1970), pp. 5–52.

[2] J. Löffelmann, *Prikry-Forcing und einige Verallgemeinerungen*, ***Doctoral dissertation***, Bonn, 2001.

► ULRICH KOHLENBACH AND PAULO OLIVA, *Effective bounds on strong unicity in L_1-approximation*.

BRICS, Department of Computer Science, University of Aarhus C, Ny Munkegade, DK-8000, Aarhus, Denmark.
E-mail: kohlenb@brics.dk.
E-mail: pbo@brics.dk.

We report the results of another case study [3] in the general project of proof mining—logical analysis of *prima facie* non-effective proofs with the aim of extracting new computationally relevant data (see [2]). More specifically, we analyze a non-effective proof of the uniqueness of the best approximation of continuous functions $f \in C[0,1]$ by polynomials $p \in P_n$ of degree $\leq n$ with respect to the L_1-norm due to Cheney [1]. Cheney's proof is a simplification of Jackson's (1921) original proof in the sense that it avoids measure theory and only uses the Riemann integral. From a logical point of view, though, his proof is highly non-constructive relying both on classical logic and non-computational analytical principles which correspond to the so-called binary ('weak') König's lemma, WKL. From Cheney's proof we extract the first effective (in all parameters f, n, ε) uniform rate of strong uniqueness for L_1-approximation with optimal ε-dependency ([4]). This work is based on [2] where the first author showed how a quite general class of (non-effective) proofs of uniqueness theorems in analysis can be analyzed such that effective moduli of uniqueness can be extracted. Uniform modulus of uniqueness generalizes the concept of strong unicity extensively studied in approximation theory. Using the uniform modulus of uniqueness one can, for instance, compute for any given $f \in C[0,1]$ its best L_1-approximation from P_n ([5]).

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[1] E. W. Cheney, ***Approximation theory***, AMS Chelsea Publishing, 1966.

[2] U. Kohlenbach, *Effective moduli from ineffective uniqueness proofs. An unwinding of de La Vallée Poussin's proof for Chebycheff approximation*, ***Annals of Pure and Applied Logic***, vol. 64 (1993), pp. 27–94.

[3] U. Kohlenbach and P. Oliva, *Effective bounds on strong unicity in L_1-approximation*, ***BRICS Report Series RS-01-14***, BRICS, 2001, 38 pages, submitted.

[4] A. Kroó, *On the continuity of best approximations in the space of integrable functions*, ***Acta Mathematica Academiae Scientiarum Hungaricae***, vol. 32 (1978), pp. 331–348.

[5] P. Oliva, *On the computational complexity of best L_1-approximation*, in preparation.

► SREĆKO KOVAČ, *Gödel and Kant on ontological proof*.

Institute of Philosophy, Vukovarska 54, 10000 Zagreb, Croatia.
E-mail: skovac@filist.fizg.hr.

Gödel's ontological proof is analyzed with respect to Kant's main objections to the ontological proof. His "system of all positive properties" is compared to Kant's "ideal of pure reason". Also, Gödel's concept of necessary existence, $E(x)$, is discussed in relation to Kant's statement that the existence is not a real predicate. The main differences between Gödel's and Kant's concepts in the ontological proof are elaborated. Those differences do not allow for Kan't criticism to be directly applied to Gödel's proof. They are deeply rooted in Gödel's distancing from Kant's conception of "pure" logic and in his adopting of a phenomenological method of intuitive grasping of concepts.

An important feature of at least one version of Gödel's proof is that the positive property, $P(\varphi)$, is taken in "the moral aesthetic sense". Gödel thus puts at the beginning what Kant's philosophy ends with (moral philosophy). At the same time, he corrects what he thinks is inconsistency in Kant's moral postulate of epistemologically not accepted God's existence.

► PIOTR KOWALSKI, *Socle property*.
Department of Mathematics, Wroclaw University, pl. Grunwaldzki 2/4, 50-384 Wroclaw, Poland.
E-mail: pkowa@hera.math.uni.wroc.pl.

Suppose G is a commutative group of finite Morley rank. The socle of G, denoted by $s(G)$, is a subgroup generated by strongly minimal subsets of G containing 1. Hrushovski proved that if G satisfies certain rigidity assumptions, then any irreducible subset of G is contained in a coset of $s(G)$ up to a subset of a smaller Morley rank.

We say that a theory T satisfies the socle property if the above theorem is true for any commutative, definable in T group of finite Morley rank without the rigidity assumptions. Anand Pillay and myself proved that the theory of compact complex spaces satisfies the socle property. This result yields a corollary with a diophantine flavour: any subvariety of a commutative meromorphic group G having a dense intersection with a cyclic subgroup is a coset of a meromorphic subgroup of G. Thomas Scanlon and myself proved that the theory of differentially closed fields of characteristic 0 satisfies the socle property.

► KRZYSZTOF KRUPIŃSKI AND LUDOMIR NEWELSKI, *On bounded type definable equivalence relations*.
Mathematical Institute, University of Wroclaw, pl. Grunwaldzki 2/4, 50-384 Wroclaw, Poland.
E-mail: kkrup@math.uni.wroc.pl.
E-mail: newelski@math.uni.wroc.pl.

Fix a monster model $\mathfrak{C}$ of a complete countable theory T. For bounded 0-type definable equivalence relations E we consider the space $\mathfrak{C}/E$ of E classes. The important examples of such relations are: the relation $\equiv$ (of having the same type over $\emptyset$), the relation $\overset{s}{\equiv}$ (of having the same strong type over $\emptyset$) and the finest bounded type definable equivalence relation $\overset{bd}{\equiv}$. $\mathfrak{C}/E$ is a compact Hausdorff topological space. We consider the question:

($*$) What topological spaces can appear in this way?

We discern three cases.

1. E is coarser then $\equiv$. In this case we get a full answer to ($*$) and show that if T is not small, then any compact metric space can occur.
2. E is finer than $\equiv$, but coarser than $\overset{s}{\equiv}$. In this case $\mathfrak{C}/E$ is 0-dimensional, hence E is an intersection of 0-definable equivalence relations.
3. E is finer than $\overset{s}{\equiv}$, but coarser than $\overset{bd}{\equiv}$. In this case we prove that the connected components of $\mathfrak{C}/E$ correspond to strong types. Using a Haar measure we get an invariant metric on each connected component. This leads us to a new proof of a result of Kim, than in a small theory $\overset{bd}{\equiv}$ equals $\overset{s}{\equiv}$.

Let X be a connected component of the space $\mathfrak{C}/E$ and G be the group of elementary permutations of X. We characterize those E's for which G is a Lie group. There is an example of Poizat where X is a circle S^1. We get that when this happens then G is necessarily a Lie group.

► MICHAŁ KRYNICKI AND KONRAD ZDANOWSKI, *Arithmetics in finite models*.
Department of Mathematics, Cardinal Stefan Wyszyński University, Warsaw, Poland.
Institute of Philosophy, Warsaw University, Poland.
E-mail: krynicki@mimuw.edu.pl.

We consider structures being finite initial segments of standard model of arithmetic. The nth model M_n has as its universe the set $\{0,\dots,n-1\}$. If the result of an operation (e.g., addition or multiplication) is bigger than $n-1$ we put it equal to $n-1$.

We say that $\varphi(k_1,\dots,k_m)$ is *true in all sufficiently large models* ($\models_{\mathrm{sl}} \varphi(k_1,\dots,k_m)$) if there exists n_0 such that for all $n \geq n_0$, $M_n \models \varphi(k_1,\dots,k_m)$ (for details see [1] and [2]). The relation $R \subseteq \mathbb{N}^m$ is *representable in all sufficiently large finite models* (*FM-representable*) if there exists $\varphi(x_1,\dots,x_m)$ such that for arbitrary $(k_1,\dots,k_m) \in \mathbb{N}^m$ we have: $(k_1,\dots,k_m) \in R$ if and only if $\models_{\mathrm{sl}} \varphi(k_1,\dots,k_m)$ and $(k_1,\dots,k_m) \notin R$ if and only if $\models_{\mathrm{sl}} \neg\varphi(k_1,\dots,k_m)$.

In [2] Marcin Mostowski proved that a relation R is *FM*-representable in arithmetical models with addition and multiplication if and only if R is of degree $\leq \mathbf{0}'$ (or, equivalently, Δ^0_2).

We prove that the same holds if we consider the arithmetic with multiplication or exponentiation only. We examine also arithmetic with addition or ordering or successor function only. We show that in these three cases relations representable in sufficiently large finite models are the same as the relations definable in the standard infinite structures.

[1] M. Mostowski, *On representing semantics in finite models*, to appear in ***Proceedings of contributed papers of 19 LMPhSc, Cracow, 1999***.

[2] ———, *On representing concepts in finite models*, ***Mathematical Logic Quarterly***, vol. 47 (2001), pp. 513–523.

► OLIVER KULLMANN, *Resolution as the logic of applying partial assignments*.
Computer Science Department, University of Wales Swansea, Singleton Park, Faraday Building, Swansea SA2 8PP, United Kingdom.
E-mail: O.Kullmann@swansea.ac.uk.

Based on [6], I want to show how resolution for conjunctive normal forms and constraint satisfaction problems can be generalized to *systems of problem instances*, which are given by an (abstract) operation of the *monoid of partial assignments* on the set of problem instances. I will give examples for generalizations from the realm of autarky theory (see [2, 4, 5]) and general lower bounds for resolution (see [1, 3]) to systems of problem instances, and also I want to discuss the relation between the notion of *tree width* from graph theory to the notion of width of resolution proofs.

[1] Eli Ben-Sasson and Avi Wigderson, *Short proofs are narrow—resolution made simple*, ***Technical report TR99-022***, ECCC Electronic Colloquium on Computational Complexity, July 1999.

[2] Allen Van Gelder, *Autarky pruning in propositional model elimination reduces failure redundancy*, ***Journal of Automated Reasoning***, vol. 23 (1999), no. 2, pp. 137–193.

[3] Oliver Kullmann, *Investigating a general hierarchy of polynomially decidable classes of CNFs based on short tree-like resolution proofs*, ***Technical report TR99-041***, Electronic Colloquium on Computational Complexity (ECCC), October 1999.

[4] ———, *Investigations on autark assignments*, ***Discrete Applied Mathematics***, vol. 107 (2000), pp. 99–137.

[5] ———, *Lean clause-sets: Generalizations of minimally unsatisfiable clause-sets*, to appear in ***Discrete Applied Mathematics***, June 2000.

[6] ———, *Upper and lower bounds on the complexity of generalized resolution and constraint satisfaction problems*, submitted to ***Annals of Mathematics and Artificial Intelligence***, September 2000.

► MILOŠ S. KURILIĆ, *Unsupported Boolean algebras and forcing*.
Institute of Mathematics, University of Novi Sad, Trg Dositeja Obradovića 4, 21000 Novi Sad, Yugoslavia.
E-mail: milos@im.ns.ac.yu.

For a cardinal $\kappa \geq \omega$, a complete Boolean algebra B is called κ-supported if the equality

$$\bigwedge_{\alpha<\kappa} \bigvee_{\beta>\alpha} b_\beta = \bigvee_{A\in[\kappa]^\kappa} \bigwedge_{\beta\in A} b_\beta$$

holds for each sequence $\langle b_\beta : \beta < \kappa \rangle$ of elements of B. Otherwise, the algebra B is called κ-unsupported. Every complete Boolean algebra B is κ-unsupported for each singular cardinal κ, and if in addition B is atomic, it is κ-supported for each regular cardinal κ.

There holds $\mathrm{Unsupp}(B) = \{ \kappa \in \mathrm{Reg} : B \text{ is } \kappa\text{-unsupported} \} \subset [\mathfrak{h}_2(B), \pi(B)]$, where $\mathfrak{h}_2(B)$ is the minimal cardinal κ such that B is not $(\kappa, 2)$-distributive and $\pi(B)$ is the algebraic density of B. Under the GCH for each set S of regular cardinals there exists a complete Boolean algebra B satisfying $\mathrm{Unsupp}(B) = S$.

A combinatorial characterization: for a regular cardinal κ, B is κ-supported if and only if for each $C \in [B]^\kappa$ the set of all positive $p \in B$ extending κ elements of C or being compatible with $< \kappa$ elements of C is dense in B. A forcing characterization is: B is κ-supported if and only if in every generic extension κ is a regular cardinal and each new set $X \in [\kappa]^\kappa$ has an old subset of size κ.

For each complete Boolean algebra there holds: B is (κ, κ)-distributive $\Longrightarrow B$ is κ-supported $\Longrightarrow B$ is weakly (κ, κ)-distributive. Also, for a regular cardinal $\kappa \geq \mathrm{cc}(B)$, B is κ-supported if and only if B has κ as a caliber.

Under the GCH each complete atomless Boolean algebra B is $\mathfrak{h}_2(B)$-unsupported and this statement is consistent with MA. Is it true in ZFC?

► MARCIN KYSIAK, *Very meager sets*.
Institute of Mathematics, Polish Academy of Sciences, ul. Śniadeckich 8, 00-950 Warszawa, Poland.
E-mail: mkysiak@impan.gov.pl.

A classical definition (see, e.g., [1]) says that a set $X \subset \mathbb{R}$ is strongly null $(X \in \mathcal{SN})$ if for every sequence of positive real numbers $\langle \varepsilon_n : n \in \omega \rangle$ X can be covered by intervals I_n such that the diameter of I_n is equal to ε_n. It is easy to see that the class $\mathcal{SN}$ is closed under taking subsets and countable unions.

A well-known theorem, due to Galvin, Mycielski and Solovay (see [3]) characterizes strongly null sets as sets X such that for every meager set F we have $X + F \neq \mathbb{R}$.

Using this characterizations we can obtain a dual notion: a set X is called strongly meager $(X \in \mathcal{SM})$ if $X + N \neq \mathbb{R}$ for every null set N. Recently T. Bartoszyński and S. Shelah showed that under the assumption of CH, the class $\mathcal{SM}$ is not closed under finite unions (see [2]).

In our talk, we will present another natural characterization of strongly null sets, similar to the characterization from the theorem of Galvin, Mycielski and Solovay. Using this characterization we will obtain another notion dual to the notion of strongly null set—the notion of *very meager set*. These sets seems to be closely related to strongly meager sets but one can show (in ZFC) that they are closed under taking subsets and countable unions. We will briefly discuss basic properties of very meager sets. In particular, we would like to ask an interesting question, whether every very meager set is a countable union of strongly meager sets.

[1] T. BARTOSZYŃSKI and H. JUDAH, ***Set theory: On the structure of the real line***, A. K. Peters, Ltd., 1985.

[2] T. BARTOSZYŃSKI and S. SHELAH, *Strongly meager sets do not form an ideal*, ***Journal of Mathematical Logic***, vol. 1 (2001), no. 1, pp. 1–39.

[3] F. GALVIN, J. MYCIELSKI, and R. SOLOVAY, *Strong measure zero sets*, ***Notices of the American Mathematical Society*** (1979), p. A-280.

► GYESIK LEE, *Friedman style independence results for* $\mathrm{I}\Sigma_n$ *in terms of binary trees*.
Institut für mathematische Logik und Grundlagenforschung, Einsteinstrasse 62, 48149 Münster, Germany.
E-mail: leegy@math.uni-muenster.de.

Let $(\mathcal{B}, \trianglelefteq)$ be the set of binary trees together with its associated homeomorphic embeddability relation $\trianglelefteq$. Then $(\mathcal{B}, \trianglelefteq)$ is a well partial order. Let f be a unary function, and let $\mathrm{SWP}(f)$ be the Π^0_2-statement that for every natural number K there exists a natural number N such that for every sequence $\beta_0, \dots, \beta_N$ of elements in $\mathcal{B}$ which satisfy the growth rate condition that for $0 \le i \le N$ the number of nodes in β_i does not exceed $K + f(i)$, there exist i, j such that $0 \le i < j \le N$ and $\beta_i \trianglelefteq \beta_j$. According to Kruskal's theorem $\mathrm{SWP}(f)$ is true for all f. H. Friedman, however, has shown that $\mathrm{SWP}(f)$ is in PA unprovable for $f(i) = i$. A. Weiermann has recently shown that $\mathrm{SWP}(f_r)$, where $f_r(i) = r \cdot |i|$, is PA-unprovable for $r > 1/2$ and PA-provable for $r \le 1/2$. We refine this result to $\mathrm{I}\Sigma_n$, the fragment of PA where the induction is restricted to Σ_n formulae. We show that there are also critical numbers r_n for the provability of $\mathrm{SWP}(f_r)$ in $\mathrm{I}\Sigma_n$ and that r_n converge monotone decreasingly to $1/2$ as n grows to infinity. The critical numbers for $\mathrm{I}\Sigma_{2n}$ are $(\log_2(4 \cdot \cos^2(\pi/(n+3))))^{-1}$. In case $\mathrm{I}\Sigma_{2n-1}$ the numbers r_n depend on Chebychev polynomials. The critical number r_1 for $\mathrm{I}\Sigma_1$ is, e.g., $(\log_2 \varphi)^{-1}$, where $\varphi = (1 + \sqrt{5})/2$ is the golden ratio. The proof uses methods from analytical combinatorics; see, e.g., [1, 2].

[1] P. FLAJOLET and R. SEDGEWICK, *The average case of algorithms. Part I: Counting and generating functions*, 1993, preprint, http://algo.inria.fr/flajolet/Publications/publist.html.

[2] ANDREW ODLYZKO, *Asymptotic enumeration methods*, in ***Handbook of combinatorics***, vol. 2 (R. L. Graham, M. Groetschel, and L. Lovasz, editors), Elsevier, 1995, pp. 1063–1229.

[3] STEPHEN G. SIMPSON, *Non-provability of certain combinatorial properties of finite trees*, in ***Harvey Friedman's research on the foundations of mathematics*** (L. A. Harrington et al., editors), 1985, pp. 87–117.

[4] ANDREAS WEIERMANN, *Analytical methods in proof theory: Optimal Friedman style independence results for PA in terms of binary trees and finite sequences*, preprint, Münster, 2000.

► JUI-LIN LEE, *On the uniform-NC^1 completeness of tree*.
Philosophy Department, National Chung-Cheng University, Chia-Yi, Taiwan.
E-mail: jllee@phil.ccu.edu.tw.

In this talk I will prove that the *tree* function is uniform-NC^1 complete. I will use the setting of function algebra N_0 for uniform NC^1: N_0 is the smallest class with basic functions $\mathrm{zero}(x) = 0$, $s_0(x) = 2x$, $s_1(x) = 2x + 1$, $i^n_k(x_1, \dots, x_n) = x_k$, $|x|$ (the length of the binary expression of x), $x \sharp y = 2^{|x|\cdot|y|}$, $\mathrm{Bit}(i, x)$ (the ith bit of the binary expression of x, $\mathrm{tree}(x)$ (for $4^m < |x| \le 4^{m+1}$, apply the binary logic operator AND to bits $x_0, \dots, x_{4^m-1}$ at the first level and then apply OR at level 2, do these alternately until level $2m$), and closed under composition and concatenation recursion on notation (CRN). What we show is exactly the following: For any N_0 function $f(\vec{x})$, there exist a constant c and a polynomial p such that $|f(\vec{x})| \leq p(\mathrm{Max}(|\vec{x}|))$, and for any $y = 2^{2^m}$, $\mathrm{Max}(\vec{x}) \leq y$, and $j \leq p(|y|)$, $\mathrm{Bit}(j, f(\vec{x})) = \mathrm{tree}(2^{4^{cm}} + \sum_{i=0}^{4^{cm}-1} 2^i \cdot G_i)$ where G_i could be 0, 1, $\mathrm{Bit}(l, i_k^n(\vec{x}))$ or $1 - \mathrm{Bit}(l, i_k^n(\vec{x}))$ for some k, l, and the determination of G_i depends on an AC^0 function with inputs i, j, y, which is oblivious to the actual size of $\mathrm{Max}(\vec{x})$.

I will also discuss the N_0 computation for BSVP.

[1] S. Buss, S. Cook, A. Gupta, and V. Ramachandran, *An optimal parallel algorithm for formula evaluation*, ***SIAM Journal on Computing***, vol. 21 (1992), pp. 755–780.

[2] J.-L. Lee, *Count and tree in uniform NC^1*, ***Ph. D. thesis***, Department of Mathematics, University of Illinois at Urbana-Champaign, 1997.

[3] ———, *Counting in uniform TC^0*, Electronic Colloquium on Computational Complexity, TR97-034, 1997.

► BENEDIKT LÖWE, *Regularity properties and determinacy*.
Mathematisches Institut, Rheinische Friedrich-Wilhelms-Universität Bonn, Beringstrasse 6, D-53115 Bonn, Germany.
E-mail: `loewe@math.uni-bonn.de`.

It is a well-known fact from the basics of determinacy theory that $\mathrm{Det}(\Gamma)$ implies that all sets in Γ have a lot of regularity properties (for pointclasses Γ with reasonable closure properties).

In this context, it is very important that we assume determinacy for the whole pointclass and not just for the set under investigation: e.g., you can easily construct a determined set without the Baire property and a determined set whose Banach-Mazur game is not determined.

In this talk we shall look at regularity properties derived from forcing notions $\mathbb{P}$ (called "$\mathbb{P}$-measurability" and "weak $\mathbb{P}$-measurability") and give criteria for a forcing notion $\mathbb{P}$ to satisfy "every determined set is (weakly) $\mathbb{P}$-measurable" or its negation.

► KERKKO LUOSTO, *Equicardinality on linearly ordered structures*.
P. O. Box 4, 00014 University of Helsinki, Finland.
E-mail: `Kerkko.Luosto@helsinki.fi`.

One of the most basic notions that is not definable in first order logic is the equicardinality. The minimal way to resolve this possible defect of FO is to adjoin the *equicardinality of Härtig quantifier I* to the logic FO. The enhanced logic is denoted by $\mathrm{FO}(I)$, the semantics related to I being that for every $\mathfrak{M} \in \mathrm{Str}(\tau)$,

$$\mathfrak{M} \models Ixy(U(x), V(y)) \iff |U^{\mathfrak{M}}| = |U^{\mathfrak{M}}|.$$

Similar enhancement can be made for any given notion of structures. In particular, one can define the *general divisiblity quantifier D* expressing that the size of one unary predicate divides that of another, and for every $n \in \mathbb{N}$ the (ordinary) *divisiblity quantifier D_n* expressing that the size of a unary predicate is divisible by n.

In this work, the expressive power of the Härtig quantifier I on the class $\mathcal{O}$ of ordered finite structures is discussed. We match I against *cardinality quantifiers C_S*, $S \subset \mathbb{N}$, expressing that the size of a unary predicate is in S, and derive a sufficient and necessary condition for

$$\mathrm{FO}(I) \leq \mathrm{FO}(\mathrm{C_S})/\mathcal{O}.$$

From the characterization it will be obvious that such quantities are frequent. This is in sharp contrast to the result of Kolaitis and Väänänen (1995) proving that $\mathrm{FO}(I) \not\leq \mathrm{FO}(\mathcal{Q})/\mathcal{F}$ for any set $\mathcal{Q}$ of cardinality quantifiers where $\mathcal{F}$ is the class of all finite structures.

The motivation of this study comes from the central place of the built-in linear order in descriptive complexity theory and the fact that addition is definable in the logic $\mathrm{FO}(I)$ on $\mathcal{O}$. Furthermore, note the following general fact on built-in relations: For every query q with domain Str($\{\leq\}$), there is a monadic quantifier Q_q such that for any regular $\mathcal{L} \geq \mathrm{FO}(I)/\mathcal{O}$, q is expressible in $\mathcal{L}$ if and only if $\mathcal{L} \geq \mathrm{FO}(Q_q)/\mathcal{O}$. For example, the result of Barrington, Immerman and Straubing can be rephrased as

$$\mathrm{FO}(D) \equiv \mathrm{TC}_0/\mathcal{O}.$$

► BARBARA MAJCHER-IWANOW, *Complexity of conjugacy classes of* Sym(w) *and* Aut(Q, <).
Institute of Mathematics, University of Wrocław, pl. Grundwaldzki 2/4, 50-384 Wrocław, Poland.
E-mail: ivanov@math.uni.wroc.pl.

The group Sym(ω) and its closed subgroups are considered under the (Polish) topology of pointwise convergence. We concentrate on necessary conditions for separation of conjugacy classes by $\mathbf{G}_\delta$-sets. The idea comes from the paper of H. Becker [1], where the notion of elementary embeddability from logic is generalized to the general case of a Polish group action on a Polish space:

> Let G be a Polish group acting continuously on a Polish space X. Let d be a left-invariant metric of G and $x, y \in X$. A Cauchy sequence $g_n \in G$, $n \in \omega$, is an $\imath$-embedding of x into y if $g_n \cdot x$ converges to y.

It is shown in [1] that the definition does not depend on d; we can now say that the sequence g_n is $\imath$-Cauchy. Moreover, the definition is preserved under replacing of x and y by elements of the corresponding orbits and then induces a transitive relation on the set of orbits. Proposition 3.D.5 there asserts that if two orbits of the action are separated by a $\mathbf{G}_\delta$-set then a representative of the orbit outside the set is not embeddable into any element of the orbit inside. We now see that any description when for g, h from the closed subgroup $G \subseteq \mathrm{Sym}(\omega)$ the automorphism g is not embeddable into h under the conjugacy action $\gamma\alpha\gamma^{-1}$ gives a necessary condition for separation of the corresponding conjugacy classes by a $\mathbf{G}_\delta$-set. Our main results give such a description for Sym(ω) and Aut($\mathbb{Q}, <$).

As a consequence the converse to the statement above (3.D.5 from [1]) holds in the case of Sym(ω) and all conjugacy classes in Sym(ω) are Π^0_2. We describe $\mathbf{G}_\delta$-conjugacy classes of Aut($\mathbb{Q}, <$) and prove that there are no non-trivial conjugacy classes belonging to $\mathbf{F}_\sigma$.

[1] H. Becker, *Polish group actions*: *dichotomies and generalized elementary embeddings*, ***Journal of the American Mathematical Society***, vol. 11 (1998), pp. 397–449.

► LARISA MAKSIMOVA, *Projective Beth property in extensions of S4*.
Institute of Mathematics, Siberian Branch of Russian Academy of Sciences, 630090, Novosibirsk, Russia.
E-mail: lmaksi@math.nsc.ru.

Let $\mathbf{x}, \mathbf{q}, \mathbf{q}'$ be disjoint lists of variables not containing y and z, $\Gamma(\mathbf{x}, \mathbf{q}, y)$ a set of formulas. We say that a logic L has the *projective Beth property PBP* if $\Gamma(\mathbf{x}, \mathbf{q}, y), \Gamma(\mathbf{x}, \mathbf{q}', z) \vdash_L y \leftrightarrow z$ implies $\Gamma(\mathbf{x}, \mathbf{q}, y) \vdash_L y \leftrightarrow C(\mathbf{x})$ for some formula $C(\mathbf{x})$. Also we consider two versions of the interpolation property: CIP ($\rightarrow$ interpolation) and IPD ($\vdash$ interpolation).

Interrelations of various versions of interpolation and of the Beth property in modal logics were studied in [1]. In particular, IPD follows from CIP, and IPD implies PBP in the family NE(S4) of normal extensions of S4. The converse does not hold.

Theorem 1. *If* $L \in$ NE(S5), $L' \in$ NE(S4.1) *and both* L *and* L' *have PBP then their intersection has PBP, and it does not possess IPD whenever* L *and* L' *are incomparable.*

We state a useful algebraic criterion (see [2] for definitions):

Theorem 2. *For eacn* $L \in$ NE(S4) *the following are equivalent*:

(i) *L has PBP*,

(ii) *its associated variety $V(L)$ of interior algebras has Strong Epimorphisms Surjectivity SES,*
(iii) *$V(L)$ has Restricted Amalgamation Property and the class of finitely indecomposable algebras in $V(L)$ has SES.*

In [2] the exhaustive list of superintuitionistic logics (SIL) with PBP was found: there exist exactly sixteen logics with PBP. We use their description.

With any $L \in \mathrm{NE(S4)}$, its *superintuitionistic fragment* $\rho(L)$ is associated; L is called a *modal companion of* $\rho(L)$. It was proved [1] that each logic with PBP in NE(S4) is a modal companion of some SIL with PBP.

THEOREM 3.

(i) *There are thirteen SIL which have modal companions with PBP.*
(ii) *There exist three SIL with PBP and without modal companions having PBP.*
(iii) *For each consistent SIL there are infinitely many modal companions without PBP.*

[1] L. MAKSIMOVA, *Projective Beth's properties in modal and superintuitionistic logics*, ***Algebra and Logic***, vol. 38 (1999), pp. 316–333.

[2] ———, *Intuitionistic logic and implicit definability*, ***Annals of Pure and Applied Logic***, vol. 105 (2000), pp. 83–102.

► HEIKE MILDENBERGER AND SAHARON SHELAH, *The splitting number can be smaller than the matrix chaos number*.
Institut für formale Logik, Universität Wien, Währinger Strasse 25, A-1090 Vienna, Austria.
E-mail: heike@logic.univie.ac.at.
Institute of Mathematics, The Hebrew University of Jerusalem, Givat Ram, 91904 Jerusalem, Israel.
E-mail: shelah@.math.huji.ac.il.

Let χ be the minimum cardinal of a subset of 2^{ω} that cannot be made convergent by multiplication with a single Toeplitz matrix. By an application of creature forcing we show that $\mathfrak{s} < \chi$ is consistent. We thus answer a question by Vojtáš. We give two kinds of models for the strict inequality. The first is the combination of an $\aleph_2$-iteration of some proper forcing with adding $\aleph_1$ random reals. The second kind of model is got by adding δ random reals to a model of $\mathrm{MA}_{<\kappa}$ for some $\delta \in [\aleph_1, \kappa)$. It was a conjecture of Blass that $\mathfrak{s} = \aleph_1 < \chi = \kappa$ holds in such a model. For the analysis of the second model we again use the creature forcing from the first model.

► MORTEZA MONIRI, *Intuitionistic weak arithmetics, Kripke models and provably total functions*.
Institute for Studies in Theoretical Physics and Mathematics, P. O. Box 19395-5746, Tehran, Iran.
E-mail: ezmoniri@mail.ipm.ir.

We prove that $i\Pi_1$ is equivalent to the intuitionistic theory axiomatized by $i\Delta_0$ plus the scheme of weak $\neg\neg LNP$ for Π_1-formulas, were universal quantification on the parameters precedes double negation. We also show that a Kripke model which decides bounded formulas forces $i\Pi_1$ if and only if the union of the worlds in any path in it satisfies $I\Pi_1$. Similar results hold for *iop* and $i\forall_1$.

Using the above criterion, we construct ω-framed Kripke models of $i\forall_1$ and $i\Pi_1$, none of whose nodes satisfies $\forall x\, \exists y\ (x = 2y \vee x = 2y+1)$ and $\forall x, y\, \exists z\ \mathrm{Exp}(x, y, z)$ respectively. This will enable us to show that $i\forall_1$ does not prove $\neg\neg\forall x\, \exists y\ (x = 2y \vee x = 2y+1)$ and $i\Pi_1$ does not prove $\neg\neg\forall x, y\, \exists z\ \mathrm{Exp}(x, y, z)$. Therefore, $i\forall_1 \nvdash \neg\neg lop$ and $i\Pi_1 \nvdash \neg\neg i\Sigma_1$. However, it is easy to see that $iop \vdash \forall x\, \neg\neg\exists y\ (x = 2y \vee x = 2y+1)$ and $i\Pi_1 \vdash \forall x, y\, \neg\neg\exists z\ \mathrm{Exp}(x, y, z)$. We also prove that $HA \nvdash l\Sigma_1$ and present some remarks about $i\Pi_2$.

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[2] MORTEZA MONIRI, *Weak arithmetics and Kripke models*, ***Mathematical Logic Quarterly***, accepted.

[3] ———, *Intuitionistic weak arithmetics*, submitted.

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► MARCIN MOSTOWSKI, *Potential infinity and the Church thesis*.
Department of Logic, Institute of Philosophy, Warsaw University, Krakowskie Przedmieście 3, 00-047 Warszawa, Poland.
E-mail: `marcinmo@mail.uw.edu.pl`.

We consider the "mathematical" proof of the Church Thesis. The proof is based on very weak assumptions about *intuitive computability* and *FM-representability theorem* from [1]. It develops and improves the argument from [2]. Our argument essentially depends on the mathematical model of the world we are in. It is justified on the basis of the mathematical model of finite but potentially infinite world similar to that of Mycielski [3].

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[3] J. MYCIELSKI, *Analysis withou actual infinity*, ***Journal of Symbolic Logic***, vol. 46 (1981), pp. 625–633.

► MARCIN MOSTOWSKI AND DOMINIKA MAJSAK, *On the Barwise argument against the Hintikka thesis*.
Department of Logic, Institute of Philosophy, Warsaw University, Karkowskie Przedmieście 3, 00-047 Warszawa, Poland.
E-mail: `marcinmo@mail.uw.edu.pl`.
E-mail: `majdomi@poczta.onet.pl`.

Hintikka has claimed that the logical form of sentences like

($\star$) *Some relative of each villager and some relative of each townsman hate each other*.

should be written down as follows:

$$(\star\star)\qquad \begin{matrix}\forall x\ \exists y\\ \forall z\ \exists w\end{matrix}\Big(\big(V(x)\wedge T(z)\big)\Longrightarrow\big(R(x,y)\wedge R(z,w)\wedge H(y,w)\big)\Big).$$

Hintikka's claim raised an intensive discussion. An attempt to summarize this discussion was given by Barwise in [1]. Barwise considers several arguments against and for the Hintikka thesis; all but one of them support the thesis. The argument against it is based on an empirical test, which mathematically is equivalent to recognizing the truth value of the sentence ($\star$) in finite models.

We prove that: *assuming the Hintikka thesis, the truth value of* ($\star$) *in finite models is an NP-complete problem.*

It was known that: (1) There are statements with Henkin quantifiers expressing NP-complete problems (see [2]). (2) The formula ($\star\star$) is not equivalent to any first order formula (see [1]). (3) The scheme ($\star\star$) is very weak in comparison with the formulae considered by Blass-Gurevich, e.g., in monadic vocabulary it does not go beyond first order logic (see [3]).

We claim that according to the actual state of knowledge we cannot expect that human beings are equipped with a mechanism of recognizing any NP-complete problem. So it follows from our theorem that the result of the Barwise test cannot be treated as a conclusive argument against the Hintikka thesis.

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► ISABEL OITAVEM, *Implicit characterizations: a uniform approach to PTIME, LSPACE and NC*.

Universidade Nova de Lisboa and CMAF—Universidade de Lisboa, Av. Prof. Gama Pinto 2, P-1649-003 Lisboa, Portugal.
E-mail: isarocha@lmc.fc.ul.pt.

Several machine independent approaches to relevant classes of computational complexity have been developed. All of them lead to characterizations of classes of computational complexity where the machine formulation and resources are implicit—*implicit characterizations*.

We work in a free algebra context. That allows us to obtain implicit characterizations of classes as different as *Ptime*, *Lspace* and *NC* just by changing the starting free algebra. By doing so, we give a uniform approach to classes which result from processes as different as deterministic and parallel computations with time, space, or time and space constraints.

[1] S. Bellantoni, *Predicative recursion and computational complexity*, ***Ph. D. thesis***, University of Toronto, 1993.

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[3] S. Bellantoni and I. Oitavem, *Separating NC along the δ axis*, submitted.

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[6] I. Oitavem, *Classes of computational complexity*: *Implicit characterizations*, ***Ph. D. thesis***, Universidade de Lisboa, 2001.

► JACEK PASNICZEK, *A non-standard free logic*.

Department of Logic and Methodology of Science, Maria Curie Sklodowska University, Pl. M. Curie Sklodowskiej 4, 20-031 Lublin, Poland.
E-mail: jpasnicz@bacon.umcs.lublin.pl.

Free logic is defined by Ernano Bencivenga in the following way: "A free logic is a formal system of quantification theory, with identity or without identity, which allows for some singular terms in some circumstances to be thought of as denoting no existing objects, and in which quantifiers are invariably thought of as having existential import".

The aim of this paper is to develop a logic M which fulfills Bencivenga's definition but which is non-standard in the sense that singular terms are treated syntactically and semantically as (generalised) quantifiers. Thus, instead of the classical formula $A(a)$ expressing predication we have in M a quantifier like formula $axA(x)$. Singular terms are semantically correlated with subsets of the power set of the domain of interpretation. The logic M accommodates various kinds of non-existent objects including impossible and incomplete ones such as 'the round square'. According to this logic, the round square possesses the property of being round, being a square, being a geometrical figure but basically, no other properties. And the round square does not exist in the sense of 'existence' defined within M (this sense nicely matches our intuitions concerning existence). When one applies Quine's criterion of ontological commitment to M then it turns out that the logic is committed to the same entities as the classical first-order logic (i.e., is only committed to a non-empty domain of existing individuals). Although non-existent objects are not quantified in M, the quantification over those objects is in a way simulated.

► MATTI PAUNA, *On non-wellfounded sets as limits of approximating wellfounded sets*.
Department of Mathematics, University of Helsinki, P. O. Box 4, Yliopistonkatu 5, SF-00014, University of Helsinki, Finland.
E-mail: pauna@cs.helsinki.fi.

We work in set theory *ZFA* with a class of urelements, denoted by $\mathcal{U}$, and with the Anti Foundation Axiom (*AFA*). We are going to build a domain structure on a subclass of all non-wellfounded sets. We define a partial ordering $\sqsubseteq$ on the class of all non-wellfounded sets. That ordering is defined as a kind of end extension in the tree pictures of sets.

Then the domain D is obtained by taking all inverse limits of $\sqsubseteq$-increasing sequences. This produces a subclass of the universe. We can show that all hereditarily finite sets belong to D. D is a Scott-Ershov Domain as defined in [2].

Thus D consists of those sets that can be obtained as limits of their approximations. We also give another characterization of the elements in D and study the axioms of the theory *ZFA* in D. This work is closely related to Boffa's [1].

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[2] Viggo Stoltenberg-Hansen, Ingrid Linström, and Edward Griffor, ***Mathematical theory of domains***, Cambridge University Press, Cambridge, 1994.

► A. N. PHROLOV, *On the class of recursive sets*.
Department of Mathematics, Kazan State University, 420008, Kazan, Russia.
E-mail: Andrey.Frolov@ksu.ru.

Let A and B be subsets of ω. We will write $A \subset_\infty B$ if $A \subset B$ and $|B - A| = \infty$. A set A is recursive (primitive recursive) if its characteristic function is recursive (primitive recursive).

Theorem 1. *Given recursive sets $A \subset_\infty B$, there is a recursive set C such that $A \subset_\infty C \subset_\infty B$, and there are no primitive recursive sets P and Q such that $A \subset_\infty P \subset_\infty C$ and $C \subset_\infty Q \subset_\infty B$.*

Theorem 2. *If P and Q are primitive recursive sets such that $P \subset_\infty Q$, then there is a recursive no primitive recursive set D such that $P \subset_\infty D \subset_\infty Q$, and for each primitive recursive set A there is a primitive recursive set B such that if $A \subset_\infty D$ then $A \subset_\infty B \subset_\infty D$, and if $D \subset_\infty A$ then $D \subset_\infty B \subset_\infty A$.*

Let $(\mathcal{O}, \leq_\mathcal{O})$ be the Kleene system of notation for constructive ordinals. Further, a hierarchy of α-primitive recursive sets $\{\mathcal{R}_\alpha\}_{\alpha\in\mathcal{O}}$ is defined. It is proved that for $\alpha < \beta$, $\mathcal{R}_\alpha \subset \mathcal{R}_\beta$, and $\{\bigcup_{\alpha\in\mathcal{O}} \mathcal{R}_\alpha\}$ is the class of all recursive sets, and Theorems 1 and 2 are proved for all levels of this hierarchy.

► K. PKHAKADZE, R. ASATIANI, AND M. IVANISHVILI, *Logic of language and paradoxes*.
Tbilisi State University, 380028 Chavchavadze Ave., Tbilisi, Georgia.

According to Wittgenstein, Logic is Language. What does it mean? How do we understand the term "logic of language?" In general, logic is the rules of right thinking, but the right (correct) mental processes, i.e., the right thinking we can understand only as some type of language processes. The task of mathematical logic was and is formalization of mental processes of mathematical type. Frege's mathematical formal language was the main ground of the above-mentioned task. There's a question—what nature has the natural language?—the direct formal-logical description of natural Georgian language shows us the possible to find its isomorphical formal adequate. The words of vocabulary of the Georgian natural language are described by their placeness, by the types of places, and by the type value of this word-operator. On the basis of such described vocabulary the conception of the well-formed expressions of so-called Georgian natural formal language is defined. Classification of the words of vocabulary gives us the possibility to classify formally and mathematically the well-formed expressions.

In any natural language as well as in the Georgian natural language is the class of so called pure mental or logical words. (For example: "da"—"and", "an"—"or", "magram"—"but", "Tu"—"If", "SesaZlebloba"—"possibility", "aucilebloba"—"necessity", ...). In the formal adequate of natural language we have the possibility to study thoroughly (fully) the logical formal—operatoral nature of above-mentioned words. This represents new perspectives in general to understand what is logic. According to all above mentioned we understand the sense of the term 'logic of language.' Here is also the answer of the question about Wittgenstein's famous phrase.

Now very briefly about Paradoxes, especially about the paradox of 'liar.' From the simplified point of view any natural language is the set of typicalized words. Let us assume that W is language, i.e., W is the set of typicalized word. Logical-semantical analysis of the languages shows that from W we can separate the w subset of the basic words of language. We call the elements of $W \setminus w$ set as non-basic or definable or contracted words of the language. (For example: "vtyui"—"I lie" is non-basic word, because of 'I lie' means 'I say something and I know this something is not true.') The formal logical description of the Georgian natural language clarifies that 'I lie' must be regarded as 1-placed operator which operators on L (logic) type words and gives in the result the same types, i.e., the L type word. This means that 'I lie' is $L \to L$ type word and its value is the truth scheme. So to define it as true or false is absurd and forms the paradox.

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[5] ———, ***Ludwig Wittgenstein***, 1951.
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► TOMASZ POLACIK, *Quantifying over propositions in the class of trees*.
University of Silesia, Institute of Mathematics, Bankowa 14, 40007 Katowice, Poland.
E-mail: polacik@us.edu.pl.

We consider second-order intuitionistic propositional logic and its so-called principal Kripke semantics: i.e., we identify the propositions with the upward-closed subsets of the frame and then let the quantifiers range over the set of all propositions. It is known that the set H^2 of all propositional second-order formulae which are valid in every Kripke frame is recursively isomorphic to full classical second-order predicate logic, cf. [1, 3]. Looking for a more plausible interpretation of intuitionistic propositional quantification, we focus on the class $\mathcal{T}$ of all trees and consider the set $\mathsf{H}^2(\mathcal{T})$ of all formulae which are valid in every tree, and the full binary tree T_2 and its logic $\mathsf{H}^2(T_2)$. It follows from the results of M. Rabin, cf. [2], that $\mathsf{H}^2(T_2)$ is decidable. Hence at least one of the inclusions in the chain

$\mathsf{H}^2 \subseteq \mathsf{H}^2(\mathcal{T}) \subseteq \mathsf{H}^2(T_2)$ must be proper. One can show that in fact both of them are. We investigate the logics $\mathsf{H}^2(\mathcal{T})$ and $\mathsf{H}^2(T_2)$, and tackle the following problems: What are the properties of $\mathsf{H}^2(\mathcal{T})$ and $\mathsf{H}^2(T_2)$? Is $\mathsf{H}^2(\mathcal{T})$ (recursively) axiomatizable? What is a natural axiomatization of $\mathsf{H}^2(T_2)$?

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► ANDREJA PRIJATELJ, *On the impact of bounded structural rules*.
Department of Mathematics, University of Ljubljana, Slovenia.
E-mail: Andrea.Prijatelj@fmf.uni-lj.si.

In this talk, we shall discuss some highlights of our BSR Logic tour starting with Lambek Calculus, passing Łukasiewicz Logics and finishing efficiently in Linear Logic. Based on [7], neat and proof theoretically well-behaved BSR Logics have recently been obtained capturing the expressive power of the corresponding Linear Logic bounded modalities. Finally, we shall touch upon complete BSR game semantics on suitable free lattices suggested already in [6]. And this should bring us pretty close to the solution of the Linear Logic completeness problem open since 1987 [1].

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► MARINA PROKHOROVA, *External sets properties in IST*.
Institute of Mathematics and Mechanics UrBr RAS, GSP-384 Ekaterinburg 620219, Russia.
E-mail: pmf@imm.uran.ru.

Kanovei in [1] set up a problem of investigation of "external" analogues of classic theorems in the nonstandard analysis axiomatic IST [2].

There is investigated the external analog of the choice axiom for some class of external equivalence relations, namely for additive relations ρ on $\mathbb{R}$, whose equivalence class ρ_0 of 0 could be defined by Π_1^{st}-formula. It is shown [3] that factor set $\mathbb{R}/\rho$ could be given by some external formula if and only if ρ_0 is proportional to monad of infinitesimals.

There are investigated cuts of $\mathbb{R}$ (such subsets $\mu \subseteq \mathbb{R}$ that $(x \in \mu, y \leq x) \Longrightarrow (y \in \mu)$) in IST. It is shown [3] that:

1. Any external cut is defined by Π_1^{st}- or Σ_1^{st}-formula.
2. Any additive cut is proportional to itself square.
3. Any cut is a shift of some additive cut.

There are obtained the statements of BIST (Bounded Internal Sets Theory), which are equivalent to the hypothesis of measurable cardinal existence. The question is considered: for what nonstandard bounded t all points of X are t-nearstandard [4], where X is a non-rarefied compact. It is shown that any standard set including such t must be measurable. Moreover, for such t all t-standard points of X are standard.

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► SAEED SALEHI, *Unprovability of Herbrand consistency in weak arithmetics*.
Turku Center for Computer Sciences, Lemminkaisenkatu 14A, 4-th floor, FIN-20520, Turku.
E-mail: saeed@cs.utu.fi.

By Herbrand's theorem, a theory T is consistent if and only if every finite set of its Skolem instances is (propositionally) satisfiable.

Herbrand Consistency of a theory T can be formalized as: "for any set of terms, say Λ, there is an evaluation on Λ which satisfies all the available Skolem instances of T".

Provability or unprovability of Herbrand Consistency of weak arithmetics (i.e., proper fragments of $I\Delta_0 + Exp$) in themselves had been an open problem since 1981 (see [2]).

In this paper, we modify the above definition such that its negation gives a *real* Herbrand proof of contradiction, even when *Exp* is not available and show the unprovability of Herbrand Consistency of $I\Delta_0$ (with the modified definition which is implied by the old one) in itself.

This incompleteness theorem has been shown for $I\Delta_0 + \Omega_2$ by Adamowicz [1], also in another unpublished paper, for $I\Delta_0 + \Omega_1$.

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► PETER M. SCHUSTER, *Strong versus uniform continuity: a constructive round*.
Mathematisches Institut, Universität München, Theresienstrasse 39, 80333 München, Germany.
E-mail: pschust@rz.mathematik.uni-muenchen.de.

In the setting of constructive mathematics à la Bishop, uniform continuity is related to strong continuity—that is, the second-order functionality notion naturally associated with apartness spaces. The guiding questions are whether strong continuity in general differs from uniform continuity, and—supposing that it be so—under which circumstances they coincide, and in which instances they are interchangeable.

A mapping between metric spaces is called strongly continuous provided that if the images of two subspaces lie apart from each other, then so do the original subspaces. Every such mapping is pointwise continuous; more specifically, the pointwise continuity of any mapping requires the defining condition of strong continuity only from pairs of subspaces that contain at least one singleton. Moreover, any mapping is uniformly continuous if and only if its two-fold cartesian product is strongly continuous; whence strong continuity follows from uniform continuity, with which it is classically equivalent.

The well-known constructive intermediate value theorems, usually proved for uniformly continuous functions, are valid for strongly continuous functions, too. Any known constructive proof, however, of the implication from strong to uniform continuity requests that either the antecedent is strengthened or the consequent weakened. For instance, every strongly continuous mapping is uniformly sequentially continuous—that is, preserves the relation of sequences approximating each other (a property classically equivalent with, but constructively feebler than, uniform continuity).

Last but not least, each strongly continuous mapping is uniformly continuous provided that its domain is totally bounded. Unlike the classical theorem that continuous functions are uniformly continuous on compact sets, whose constructive validity varies with the model, this result holds within Bishop's constructive mathematics and hence in any of its standard models.

This is joint work with D. Bridges, H. Ishibara, and L. Vîţă, and profited from hints by J.-Y. Girard and F. Richman.

► JOHN H. SEREMBUS, *Last words on the functional completeness of PL*.
Widener University, One University Place, Chester, Pennsylvania 19013, USA.
E-mail: John.Serembus@widener.edu.

This paper completes work begun by the present author in [1] and continued in [2] and [3]. In those papers the author offered a novel representation of truth tables in Propositional Logic (PL) to prove the functional (in)completeness of sets of connectives of PL and explored the significance of that notational representation.

In the present paper, the author summarizes those results and compares them with the traditional literature concerning functional completeness. This comparison reveals some interesting explanations of some of the traditional results. It also suggests a straightforward way to examine the issue of the interdefinability of connectives as well as a decision procedure for determining whether an arbitrary set of connectives of PL of arbitrary degree is complete or not.

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[2] ———, *A study of truth table notation of propositional logic*, presented to the Conference on Notational Engineering, George Washington University, Washington, D. C., May 23–24, 1996.

[3] ———, *A generalized functional completeness proof for propositional logic*, presented to the European Meeting of the Association of Symbolic Logic, Prague, Czech Republic, August 9–15, 1998.

► JAIRO JOSE DA SILVA, *A phenomenological approach to the axioms of set theory*.
Department of Mathematics, Universidade Estadual Paulista (Unesp) at Rio Claro, 13500-230 Rio Claro, SP, Brazil.
E-mail: jairomat@linkway.com.br.

In this paper I argue for the view that the axioms of ZFC, with the exception of extensionality, are analytic truths of the concept of set. That is, they are true only by virtue of what we conceive by a set, and, moreover, can be immediately derived from this conception. Although I assume that the object of set theory is the concept of set, I refrain from taking stance with respect to its ontological status.It is enough that this concept be given to us with a certain sense as the objetive focus of a "phenomenologically reduced" experience. At the core of what we mean by a set there is a relation of ontological dependence between sets and their members, an analysis of which suffices to ground the iterative hierarchy of sets and most of the axioms of ZFC. It is part of my strategy here to identify existence with conceptual possibility: a particular set exists provided it is, as the object of an intentional experience, not ruled out by the meaning of the concept of set. This only works, of course, if existence is understood only as existence in the context of an intentional experience, i.e., in-existence. But this is, I claim, the meaning of existence in classical mathematics (which is expressed by the formula: to exist is to be free from contradiction). Lastly, the non-justifiability of extensionality is due to the fact that the concept of set is consistent with an intensional notion of manifold.

► CHRIS STEINSVOLD AND ROHIT PARIKH, *A modal analysis of some phenomena in child psychology*.
CUNY Graduate Center, 365 Fifth Avenue, New York, New York 10016, USA.
E-mail: ripbc@cunyvm.cuny.edu.

I will give a simple multi-modal analysis of well-known phenomena in the belief systems of very young children. The phenomena are most prevalent in three-year old children, and begin to disappear past this age. I am focusing in particular on variations of the false-belief task experiment. My intention is to give a characterization of a certain 'isomorphism' between a child's relation to others and the child's relation to his/her own past selves, which is exhibited in such experiments.

In typical false-belief task experiments, the 'task' is for children to understand that others can have beliefs that the children know are false. Typically, three-year-olds fail this task. One variation on the false belief task, which will be discussed, is the following: A three year old child is given a crayon box which actually holds candles. The child opens the box to find the candles. When asked what they initially believed was in the box, most three-year-olds reply 'candles'.

There are other interesting phenomena, and some theorems, which may be discussed as well.

► KAZIMIERZ SWIRYDOWICZ, *There exists infinitely many post-complete extensions of intuitionistic linear logic*.
Faculty of Mathematics and Computer Science of Adam Mickiewicz University, Poznań, Poland.
E-mail: swirydow@math.amu.edu.pl.

The language of linear logic consists of propositional variables, propositional constants 1, 0, $\top$, $\bot$ and of the connectives: $\rightarrow$ (implication), $\wedge$ (conjunction), $*$ (weak conjunction) and $\vee$ (disjunction). *Intuitionistic linear logic* (denoted by *ILL*) is defined as the subset of propositional formulae of F which are provable from the set of axiom schemes indicated below, by application of the rule of Modus Ponens (MP; $A,\ A \rightarrow B/B$) and the Rule of Adjunction (AD; $A,\ B/A \wedge B$):

$A1$. $A \rightarrow$A,
$A2$. $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$,
$A3$. $A \rightarrow ((B \rightarrow C) \rightarrow (B \rightarrow (A \rightarrow C))$,
$A4$. $A \rightarrow (B \rightarrow (A * B))$,
$A5$. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A * B) \rightarrow C)$,
$A6$. $(1 \rightarrow 1) \rightarrow 1$,
$A7$. $1 \rightarrow (A \rightarrow A)$,
$A8$. $A \wedge B \rightarrow A$,
$A9$. $A \wedge B \rightarrow B$,
$A10$. $(A \rightarrow B) \wedge (A \rightarrow C) \rightarrow (A \rightarrow B \wedge C)$,
$A11$. $A \rightarrow A \vee B$,
$A12$. $B \rightarrow A \vee B$,
$A13$. $(A \rightarrow B) \wedge (C \rightarrow B) \rightarrow ((A \vee C) \rightarrow B)$,
$A14$. $A \rightarrow \top$,
$A15$. $\top \rightarrow A$.

There exists a matrix semantics for ILL; this logic is algebraizable in the sense of Blok and Pigozzi. Algebras which determine this logic form a variety.

THEOREM. *There exist infinitely many simple algebras for intuitionistic linear logic such that each of these algebras is generated by a constant.*

It solves the "Ackermann problem" for intuitionistic linear logic and proves that there exist infinitely many Post-complete extensions of this logic.

► EUGENIUSZ TOMASZEWSKI, *Generalized characteristic formulas for finite subdirectly irreducible Heyting algebras*.
Department of Logic, Jagiellonian University, Grodzka 52, 31-044 Kraków, Poland.
E-mail: tomaszew@if.uj.edu.pl.

The main object of this paper is to present a new tool for investigating properties of intermediate logics. Although generalized characteristic formulas for finite subdirectly irreducible Heyting algebras are obtained by what might seem to be a small modification of the definition of Jankov formulas (see V. Jankov [1]), they provide a much stronger tool, as they can axiomatize all intermediate logics. This result is an immediate consequence of the fact that there exists an algorithm which, for any intuitionistic formula α, produces generalized formulas $\chi(\mathfrak{A}_1, D_1, \bot), \ldots, \chi(\mathfrak{A}_n, D_n, \bot)$ such that

$$INT + \alpha = INT + \chi(\mathfrak{A}_1, D_1, \bot) + \cdots + \chi(\mathfrak{A}_n, D_n, \bot).$$

The generalized formulas are an algebraic analogue of the so-called canonical formulas introduced by M. Zakharyaschev [2] and the general technique used to prove their completeness with respect to the class of intermediate logics follows some of his ideas. The main difference between these two kinds of formulas lies in the fact that generalized characteristic formulas are related to Heyting algebras while the canonical formulas of Zakharyaschev are related to Kripke frames. This close relationship with algebraic semantics allows us to take advantage of a wide variety of algebraic techniques when dealing with a specific issue.

[1] V. A. JANKOV, *Conjunctively indecomposable formulas in propositional calculi*, ***Izvestiya, Mathematics, Presidium Russian Academy of Sciences, Moscow***, vol. 3 (1969), pp. 17–35.

[2] M. V. ZAKHARYASCHEV, *Syntax and semantics of modal logics containing S4*, ***Algebra and Logic***, vol. 27 (1988), pp. 659–689.

► MARIUSZ URBAŃSKI, *Regular synthetic tableaux*.
Institute of Philosophy, Tadeusz Kotarbiński Pedagogical University, Zielona Góra, Poland.
E-mail: murban@asia.aw.wsp.zgora.pl.

Synthetic tableaux are a certain proof method which is as yet developed for Classical Propositional Calculus and some non-classical propositional logics. Here the concepts of a synthetic inference of a formula and of a synthetic tableau for a formula are introduced as well as the definition of a specific kind of synthetic tableaux—a regular synthetic tableau.

Synthetic tableau method is constructive in the sense that a proof of a formula A is build up as a set of inferences of A from every consistent sets of propositional variables of A or their negations. It is shown, then, that every possible truth-value distribution over the propositional variables of A makes A true. It can be proved, that if A is a satisfiable formula, then there exists a synthetic inference of A. A proof of soundness-completeness theorem is given.

► MLADEN VUKOVIĆ, *Some notes on generalized Veltman models*.
Department of Mathematics, University of Zagreb, Bijenička cesta 30, PP 335, 10002 Zagreb, Croatia.
E-mail: vukovic@math.hr.

Interpretability is a binary relation between extensions of a first-order theory T which are like $T + A$, where A is a formula. We say "$T + A$ is interpretable in $T + B$" if we can "translate" formulas of theory $T+A$ such that all theorems of $T+A$ will be theorems of $T+B$. The interpretability logic *IL* is a modal description of interpretability. It is (non-standard) propositional modal logic with two modal operators: $\Box$ (unary) and $\rhd$ (binary).

We are only interested in *IL* as a system of modal logic. We use in [3] generalized Veltman semantics, developed by D. de Jongh, to determine correspondences between principles of interpretability. In [4] is defined bisimulation of generalized Veltman models. In [1] is proved that each closed formula of interpretability logic is equivalent in the system *ILF* (*IL*+ Feferman's interpretability principle) to a closed formula without $\rhd$. We consider normal forms for the closed fragment of system *IL* and discuss internal questions about generalized semantics.

[1] P. Hájek and V. Švejdar, *A note on the normal form of closed formulas of interpretability logic*, ***Studia Logica***, vol. 50 (1991), pp. 25–28.

[2] A. Visser, *An overview of interpretability logic*, in ***Advances in modal logic '96*** (M. Kracht, M. de Rijke, and H. Wansing, editors), CSLI Publications, Stanford, pp. 307–335.

[3] M. Vuković, *The principles of interpretability*, ***Notre Dame Journal of Formal Logic***, vol. 40 (1999), no. 2, pp. 227–235.

[4] ———, *Characteristic classes and bisimulations of generalized Veltman models*, ***Grazer Mathematische Berichte***, vol. 341 (2000), pp. 9–16.

► PIOTR WASILEWSKI, *Representation theorem for the lattice of equivalence relations and some of its applications*.
Department of Logic, Jagiellonian University, Grodzka 52, 31-044 Kraków, Poland.
E-mail: wasilew@grodzki.phils.uj.edu.pl.

In my paper I present some results of my investigations into so-called indiscernibility relations. They are mainly applied in some issues within Artificial Intelligence and Rough set theory developed by Z. Pawlak (see Z. Pawlak [2] and, e.g., Z. Bonikowski [1]) but their applications in solving some questions concerning the concepts of similarity and supervenience may also prove to be fruitful. I would like, however, to focus especially on the strong connection that takes place between indiscernibility relations and complete subalgebras of the power set algebra of a given set. Using this relationship we are able to give a simple proof of the following theorem:

Representation theorem. *The lattice of equivalence relations on a given set is dually isomorphic to the lattice of complete subalgebras of the power set algebra of the same set.*

In fact our proof enables us to draw additional corollaries concerning indiscernibility relations which in turn give a new approach to the rough set theory. This approach is different but equivalent to that proposed by Z. Pawlak (see Z. Pawlak [3]).

[1] Z. Bonikowski, *A certain conception of the calculus of rough sets*, ***Notre Dame Journal of Formal Logic***, vol. 3 (1992), no. 3, pp. 412–421.

[2] Z. Pawlak, *Rough sets*, ***International Journal of Computing and Information Sciences***, vol. 11 (1982), pp. 341–356.

[3] ———, *Rought sets*: *An algebraic and topological approach*, ***ICS PAS Report, Polish Academy of Sciences*** (1982), no. 482.

► ANDREAS WEIERMANN, *Analytic combinatorics and proof theory*.
Institut für mathematische Logik und Grundlagenforschung der Westfälischen Wilhelms-Universität Münster, Einsteinstrasse 62, D-48149 Münster, Germany.
E-mail: weierma@math.uni-muenster.de.

Using methods from analytic combinatorics we obtain optimal Friedman style independence results for first order Peano arithmetic and its fragments. We get an optimal classification for the slowly well quasi orderedness of the class of finite trees in terms of Otter's tree constant 2.95576... and we obtain a classification for the slowly well orderedness of ε_0 using a result of Yamashita. We further obtain similar classification results for hydra games, Goodstein sequences and we present a refinement of the Paris Harrington result.

[1] A. Weiermann, *An application of graphical enumeration to PA*, to appear in the ***Journal of Symbolic Logic***.

► ROMAN WENCEL, *Cell decomposition for Boolean ordered strongly o-minimal structures*.
Wrocław University, Mathematical Institute, pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland.
E-mail: rwenc@math.uni.wroc.pl.

The notions of o-minimality and strong o-minimality for partially ordered structures may be introduced as in the case of totally ordered structures. We consider o-minimal structures with a very special underlying ordering, i.e., the one derived from a Boolean algebra. Recent research [2, 1, 3, 4] has shown that Boolean ordered o-minimal structures and totally ordered ones enjoy quite a lot of similar properties. The most interesting examples are: the existence and uniqueness of prime model over arbitrary set, the restricted exchange principle and Vaught's conjecture.

In the talk we will discuss the following theorem.

Theorem ([4]). *Let $(M, \leq, \dots)$ be a Boolean ordered o-minimal structure. Then the following conditions are equivalent*:

(a) $(M, \leq, \dots)$ *is strongly o-minimal*,

(b) *For every $n < \omega$ and a definable set $X \subseteq M^n$, X is definable in the language $\{\leq\}$*.

This theorem shows that strongly o-minimal Boolean ordered structures are essentially expansions by constants of pure Boolean algebras with finitely many atoms. The proof involves a decomposition of any definable set into finitely many pairwise disjoint cells.

[1] L. Newelski and R. Wencel, *Definable sets in Boolean ordered o-minimal structures, I*, ***Journal of Symbolic Logic***, to appear.

[2] C. Toffalori, *Lattice ordered o-minimal structures*, ***The Notre Dame Journal of Formal Logic***, vol. 39 (1998), pp. 447–463.

[3] R. Wencel, *Small theories of Boolean ordered o-minimal structures*, ***Journal of Symbolic Logic***, submitted.

[4] ———, *Definable sets in Boolean ordered o-minimal structures, II*, ***Journal of Symbolic Logic***, submitted.

► DAN E. WILLARD, *Generalizations and some exceptions for the incompleteness theorem*.
Computer Science Department, University of Albany, Albany, New York 12222, USA.
E-mail: dew@cs.albany.edu.

This talk will discuss additional generalizations and exceptions to the Second Incompleteness Theorem that we have developed recently [2], but have not published in our June 2001 JSL article [1].

Consider a language that treats Addition and Multiplication as 3-variable predicates, denoted as $A(x, y, z)$ and $M(x, y, z)$, rather than as total functions. All our languages will recognize Predecessor as a total funciton, but many will not recognize Successor as total. In this context, it is natural for our languages to include an infinite number of distinct constant symbols $c_0, c_1, c_2, \dots$, where c_i is encoded using $O(\mathrm{Log}(i))$ bits. We will say an encoding convention is **Type-A** if the symbol c_i represents the number i, **Type-B** if c_i represents the number 2^i, **Type-C** if c_i represents the number 2^{2^i}. We further require that for $i \geq 2$, the *sole formal axiom* for defining c_i under these three conventions be respectively $A(c_{i-1}, 1, c_i)$,

$A(c_{i-1}, c_{i-1}, c_i)$ and $M(c_{i-1}, c_{i-1}, c_i)$.

Our prior exceptions to the Second Incompleteness Theorem in [1] had essentially used the Type-A convenion. We have recently generalized all the exceptions to the Second Incompleteness Theorem from [1] so that the Type-B convention can replace Type-A. At the same time, we have developed a new version of the Second Incompleteness Theorem which states that no consistent axiom system can *simultaneously* employ the Type-C Convention, recognize the non-existence of a Hilbert-style proof of $0 = 1$ from itself and retain an ability to prove all Peano Arithmetic's Π_1 theorems (in this revised notation).

There is also one other result in our new technical report [2]. Let $\text{Pred}^N(x)$ denote an operation that consists of N iterations of the Predecessor function. Say an axiom system α is **Infinitely Far-Reaching** if and only if there exists *a finite subset* of axioms $S \subset \alpha$ such that for *an arbitrary* integer N, this finite set S *is sufficient to prove* $\exists x \;\; \text{Pred}^N(x) = 1$. It turns out some (very awkwardly defined) axiom systems α can verify Peano Arithmetic's Π_1 theorems (in our revised notation) while being simultaneously Infinitely Far-Reaching and able to recognize the non-existence of Hilbert proof of $0 = 1$ from themselves.

[1] D. Willard, *Self-verifying systems, the incompleteness theorem and the tangibility reflection principle*, ***Journal of Symbolic Logic***, vol. 66 (June 2001), pp. 536–596.

[2] ———, *A generalization of the second incompleteness theorem and some exception to it*, recently issued technical report, January 2001.

► REINHARD WINKLER, MARTIN GOLDSTERN, AND JÖRG SCHMELING, *An application of Cantor-Bendixson's theorem to dynamical systems*.

Institut für Diskrete Mathematik, Österreichische Akademie der Wissenschaften, Dr. Ignaz-Seipel-Platz 2, A-1010, Vienna, Austria.
E-mail: `reinhard.winkler@oeaw.ac.at`.
Institut für Algebra und Computermathematik, TU Wien, Wiedner Hauptstrasse 8-10, A-1040, Vienna, Austria.
E-mail: `goldstern@tuwien.ac.at`.
Centre for Mathematical Sciences, LTH, P. O. Box 118, SE-221 00 Lund, Sweden.
E-mail: `joerg@maths.lth.se`.

One of the most important objects in topological and symbolic dynamics is the space X of all infinite sequences $x = (x_n)$ of the symbols 0 and 1, together with the shift operator, mapping (x_n) to (x_{n+1}). Most x, as well from the probabilistic as from the topological point of view, have an orbit which is dense in X. Nevertheless there are sequences x whose orbit closure $A(x)$ is not the whole space X. We are investigating those x for which $A(x)$ is at most countable. Since $A(x)$ is a nonempty compact metric space, Cantor-Bendixson analysis tells us that the topological structure of $A(x)$ is completely given by an ordinal $\alpha < \omega_1$ (the maximal Cantor-Bendixson rank of points in X) and a positive integer n (the number of points of rank α).

Our main result is that each such pair (α, n) occurs for appropriate x, provided that α is not a limit ordinal. We are also discussing the problem of giving a satisfactory description of all x for given (α, n).

► PIOTR WOJTYLAK, *Generalization of proofs by blocks of quantifiers*.

Institute of Mathematics, Silesian University, Bankowa 14, 40-132 Katowice, Poland.
E-mail: `wojtylak@us.edu.pl`.

A proof theory with logical forms of proofs as central object to study, instead of the relation of derivability, is developed by G. Kreisel, see [3]. The problem of generalizing short proofs with deep terms is discussed by M. Baaz and P. Wojtylak in [1]. A very useful method of generalization relies on replacing, in a given short proof, quantifiers by blocks of like quantifiers. Then the more general theorem, received by the translation, can be found by use of semiunification. The method was originally developed in connection with the problem of decidability of k-derivability—in the version of predicate logic in which one introduces a block of quantifiers at one step, instead of a single quantifier, see [2].

[1] M. BAAZ and P. WOJTYLAK, *Generalizing proofs in monadic languages*, to appear.

[2] M. BAAZ and P. ZACH, *Algorithmic structuring of cut-free proofs*, in ***Computer science logic, selected papers from CLS '92***, Lecture Notes in Computer Science, vol. 702, Springer-Verlag, pp. 29–42.

[3] G. KREISEL, *Proof theory: some personal recollections*, appendix to [4], pp. 397–407.

[4] G. TAKEUTI, ***Proof theory***, 2nd ed., 1987.

► ALEXANDER D. YASHIN, *A classification theorem for Novikov complete logics with additional logical constants*.
Udmurt State University, Universitetskaya str., 1, Izhevsk, Russia, 426034.
E-mail: yashin@udm.ru, yashin@uni.udm.ru.

Let the language of the intuitionistic propositional logic *Int* be enriched by additional propositional constants $\bar{\varphi} = \{\varphi_1, \dots, \varphi_n\}$, treated as atomic formulas. Formulas without φ_i are said to be *pure*. Formulas without variables are said to be *constant* (Fm_c will denote the class of them).

By $\bar{\varphi}$*-logic* we mean any set of formulas of the enriched language including *Int* and closed under modus ponens and substitution rules. A $\bar{\varphi}$-logic L is *conservative* (over *Int*) if for any pure formula D $D \in L$ implies $D \in Int$.

A conservative $\bar{\varphi}$-logic is called *Novikov complete* if for any formula $A \notin L$ the $\bar{\varphi}$-logic $L + A$ is not conservative.

In [1, 2, 3] the author proved the following: (a) for $|\bar{\varphi}| \geq 2$ the family of all Novikov complete $\bar{\varphi}$-logics is of the power of the continuum; (b) for single additional constant the intended family is denumerable; (c) for semantic description of Novikov complete $\bar{\varphi}$-logics the *growth technique* has been worked out.

The final result for Novikov completeness problem for constants—the classification theorem—is in the following theorem.

THEOREM. *Every Novikov complete $\bar{\varphi}$-logic L is quite determined by its constant fragment $L_c = L \cap Fm_c$; L is described by the class of all finite tree frames with growths of the constant type L_c.*

[1] A. D. YASHIN, ***The continuality of the family of Novikov complete logics with a new unary connective***, Moscow State University Vestnik, mathematics and mechanics series, vol. 3, 197, pp. 22–25, in Russian.

[2] ———, *On new constant in intuitionistic propositional logic*, ***Fundamentals and Applications of Mathematics***, vol. 5 (1999), no. 3, pp. 903–926, in Russian.

[3] ———, *New intuitionistic logical constants and Novikov completeness*, ***Studia Logica***, vol. 63 (1999), pp. 151–180.

► RICHARD ZACH, *Quantified propositional intuitionistic logic on trees is decidable*.
Department of Philosophy, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4, Canada.
E-mail: rzach@ucalgary.ca.

Quantified propositional intuitionistic logic is obtained from the intuitionistic propositional calculus by adding quantifiers over propositions $\forall p$, $\exists p$. In Kripke semantics, these quantifiers range over subsets of worlds. Philip Kremer has shown that the resulting class of valid formulas is recursively isomorphic to full second order logic if all partial orders are admitted as accessibility relations. It is shown that if the Kripke frames are restricted to trees, the logic is decidable.

► BERISLAV ŽARNIĆ, *A two-dimensional semantics for the language of intentionality?*
University of Split: Teachers College, Teslina 12, 21000 Split, Croatia.
E-mail: berislav@vusst.hr.

The pragmatics of the language using mental vocabulary allows two uses: rationality ascription and irrationality accusation. The semantics of the language enables its different uses. The possibility of an unsuccessful application of inference rules (e.g., failure of substitution of identities) does not indicate a mismatch between its syntax and semantics. Two simple hypotheses can explain the semantical basis of two-fold pragmatics and they can solve the puzzle of syntax-semantics mismatch. First, propositional attitude reports characterize the type to which an agent's state σ belongs. Second, the semantic notions for the language of intentionality are defined by two relations. Horizontal relation is a relation between intentional states. Vertical relation is a correspondence relation between the real world w and an intentional state. In rationality ascription the horizontal relation is being exploited; in irrationality accusation, the vertical one. The rule of substitution of identities is violated when semantically diverse types of sentences get mixed.

Take M to be a family of models σ for the toy language $L = L_P \cup L_B$ (L_P is language of propositional logic, if $\varphi \in L_P$, then $\text{Believes}(a, \varphi) \in L_B$). Let there be a distinguished model $w \in M$. Define

$$[\![\varphi]\!] = \begin{cases} \{w\} & \text{if } \varphi \in L_P, \\ \{\, \sigma \in M \mid \sigma \vDash \varphi \,\} & \text{otherwise.} \end{cases}$$

The horizontal relation of meaning inclusion: $\varphi_1, \dots, \varphi_n \vDash \psi$ if and only if $[\![\varphi_1]\!] \cap \cdots \cap [\![\varphi_n]\!] \subseteq [\![\psi]\!]$. The vertical relation of correspondence: $\varphi_1, \dots, \varphi_n \doteqdot \psi$ if and only if $\varphi_1, \dots, \varphi_n \vDash \psi$ and $w \in ([\![\varphi_1]\!] \cap \cdots \cap [\![\varphi_n]\!])$. On this basis, it can be shown that the failure of substitution is not a violation of standard logical syntax.

► KONRAD ZDANOWSKI, *Undecidable logics with one Henkin quantifier*.
Institute of Philosophy, Warsaw University, Krakowskie Przedmiescie 3, 00-047, Warsaw, Poland.
E-mail: zkonrad@hotmail.com.

We consider the logic with Henkin Quantifiers (for the main concepts see [3]). Marcin Mostowski and Michał Krynicki proved in [2] that the logic with all quantifiers H_n for $n \in \omega$ in pure vocabulary is undecidable. Lately, Marcin Mostowski and Konrad Zdanowski improved this result (see [4]) by showing undecidability for the logic with all quantifiers E_n for $n \in \omega$ where E_n is the quantifier of the form

$$\begin{matrix} \forall x \; \exists y_1 \dots \exists y_k \\ \forall w \; \exists w_1 \dots \exists w_k \end{matrix}.$$

We show that there are single quantifiers H_{n_0} and E_{n_1} such that the logics $L(\mathsf{H}_{n_0})$ and $L(\mathsf{E}_{n_1})$ in pure vocabulary are undecidable in the class of all models. By the result of Yuri Gurevich (see [1]) we obtain that $L(\mathsf{H}_{n_0})$ and $L(\mathsf{E}_{n_1})$ are undecidable also in the class of finite models only.

[1] Yuri Gurevich, *The word problem for certain classes of semigroups*, ***Algebra and Logic***, vol. 5 (1966), pp. 25–35.

[2] Michał Krynicki and Marcin Mostowski, *Decidability problems in language with Henkin quantifiers*, ***Annals of Pure and Applied Logic***, vol. 58 (1992), pp. 149–172.

[3] ———, *Henkin quantifiers*, in ***Quantifiers: Logics, models and computation*** (M. Kryniki, M. Mostowski, and L. W. Szczerba, editors), vol. I, Kluwer Academic Publishers, 1995, pp. 193–262.

[4] Marcin Mostowski and Konrad Zdanowski, *Degrees of logics with Henkin quantifiers in poor vocabularies*, in proofreading.

Abstracts presented by title

► MOJTABA AGHAEI, *Gentzen-style axiomatizations and interpolation theorem for basic predicate logic*.

Isfahan University of Technology, Isfahan, Iran.
E-mail: aghaei@cc.iut.ac.ir.

Basic propositional logic, BPL, was first introduced in [4] by intention of formalizing the concept of provability. A decade later in [3], W. Ruitenburg with a criticism of the constructive interpretations of the logical connectives generalized it to the basic predicate logic BQC. In [1] the interpolation theorem was proved for basic propositional logic. In [2] we introduced a Gentzen style axiomatization for BQC. In this paper, by extending the lanuage by an 'intuitionistic implication' or a 'classic implication', we introduce Gentzen-style axiomatizations for conservative extensions of BQC. Translating these systems into the original language, we prove the interpolation theorems for BQC. In this way, we get another Gentzen style axiomatization for BQC.

[1] M. AGHAEI and M. ARDESHIR, *Gentzen style axiomatizations for some conservative extensions of basic propositional calculus*, ***Studia Logica***, to appear.

[2] ———, *Gentzen style axiomatizations for basic predicate calculus*, submitted.

[3] W. RUITENBURG, *Basic predicate calculus*, ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), pp. 18–46.

[4] A. VISSER, *A propositional logic with explicit fixed points*, ***Studia Logica***, vol. 40 (1981), pp. 151–175.

► M. Z. ARSLANOV AND E. E. ISMAIL, *Measurable cardinals in possibility theory*.

Institute of problems of informatics and control, Almaty 480100, Pushkin str. 125, Kazakhstan.
E-mail: mzarslanov@hotmail.com.

This report considers an existence of possibility distribution function for possibility spaces over power set $P(X)$ of model space X with continuous possibility measure $\Pi\colon P(X) \to [0,1]$. The main result about connection between possibility theory and measurable cardinals is obtained. Remember basic notions of possibility theory [3]. A possibility space is a triplet $(X, \mathcal{B}, \Pi)$, where X is a model space, $\mathcal{B}$ is σ-algebra of the set of its subsets $P(X)$, $\mathcal{B} \subset P(X)$, Π is a possibility measure, $\Pi\colon P(X) \to [0,1]$, satisfying the following properties:

1. $\Pi(\emptyset) = 0$; $\Pi(X) = 1$;
2. $\Pi(A \cup B) = max(\Pi(A), \Pi(B))$.

The most simple and natural way for entering on a set X the structure of a possibility space consists in the definition of possibilities of separate elements by some function $\pi\colon X \to [0,1]$ and following definition of a possibility measure Π by the formula:

$$\Pi(A) = \sup_{x \in A} \pi(x).$$

For such defined possibility measure Π the function π is called the possibility distribution function. It turns out that in possibility theory the lower continuity is weaker than upper continuity.

LEMMA. *If a possibility measure is upper continuous, i.e.,* $B_n \searrow B \Longrightarrow \Pi(B_n) \to \Pi(B)$, *then it is lower continuous, i.e.,* $B_n \nearrow B \Longrightarrow \Pi(B_n) \to \Pi(B)$.

DEFINITION. The spectrum of a possibility space is the range of possibility measure: $S(\Pi) = \{p \in [0,1] \mid \exists A \in P(X) : \Pi(A) = p\}$.

THEOREM 1. *A spectrum of an upper continuous possibility measure is discrete and can have a unique cluster point* 0.

The set X has pdf-property if any upper continuous possibility space $(X, P(X), \Pi)$ has possibility distribution function, i.e., $\Pi(A) = \sup_{x \in A} \Pi(\{x\})$.

THEOREM 2. *An upper continuous possibility space* $(X, P(X), \Pi)$ *has pdf-property if and only if cardinal* $|X|$ *is immeasurable.*

Thus, it is justified the following thesis:

THESIS. *All practical continuous possibility spaces* $(X, P(X), \Pi)$ *have possibility distribution function*.

[1] M. Z. ARSLANOV and E. E. ISMAIL, *Continuity of possibility distributions and measures*, ***Reports of NAS of Kazakhstan***, vol. 6 (2000), pp. 33–37.

[2] C. C. CHANG and H. J. KEISLER, ***Model theory***, North-Holland Publishing Company, Amsterdam, 1973.

[3] D. DUBOIS and H. PRADE, ***Possibility theory***, Plenum Press, New York, 1988.

► ANATOLY BELTIUKOV, *Space-time hierarchies of finite random access machines*.
Mathematics Department, Udmurt University, 1 Universitetskaia Str., Izhevsk 426034, Russia.
E-mail: belt@uni.udm.ru.

Traditional results of the complexity theory are essentially infinite: they consider functions with infinite domains. Therefore they are far from immediate practical use: the relevant arguments and coefficients may be too large. An approach to solve the problem is to reject infinite statements. We consider in this case not infinite computers but classes of finite machines of growing complexity.

Finite random access machine (RAM-N) consists of a finite sequence of registers: $r[0]$, $\dots, r[N-1]$ $(0, \dots, N-1$ are their addresses). Any time any register contains a natural number between 0 and $N-1$. A program is a finite sequence in initial registers (from $r[0]$ to some $r[P-1]$). The next registers contain the input data. The result is to be placed in $r[0]$. The register $r[1]$ always contains the "current instruction address". Any instruction is situated in two consequent registers. The first one contains the instruction code, the next is the operand address. The commands are:

- LOAD operand to $r[0]$,
- STORE $r[0]$ in the operand register,
- ADD operand to $r[0]$,
- SUBTRACT operand from $r[0]$ (negative result is replaced by 0),
- HALT.

Let $\mathrm{BFRAM}(M, K, P, N, T)$ denote the class of all sets of finite sequences of M natural numbers from 0 to $K-1$ that can be recognized by RAM-N with programs of length P within the time T.

THEOREM. *For all natural* M, N, P, *there exist* a, b, c *such that if* $N' > N+a$, $T' > bT+cP$, $a < P$, $P+M < N$, $P < M$, $T < N$, *then* $\mathrm{BFRAM}(M, N, P, N, T)$ *is a proper subset of* $\mathrm{BFRAM}(M, N, P, N', T')$.

This theorem shows that the infinitary approach is not necessary for machine computational complexity results.

► VLADIMIR BELYAEV, *Reducibility* $\leq_m$ *with upper and low limits*.
Odessa National University, Dvoryanskaya 2, Odessa, 65026, Ukraine.
E-mail: vbelyaev@paco.net.

We concern m-reducibility with limited usage of oracle. These limits are introduced in the following way.

Let $\mathfrak{T}$ $(\mathfrak{T}^+)$ be the classes of all total (non-decreasing) functions of $\mathbb{N} \to \mathbb{N}$ kind. Let us introduce $e_f^1(x) = \max\{ f(y) \mid y \le x \}$, $e_f^0(x) = \min\{ f(y) \mid x \le y \}$ for any $f \in \mathfrak{T}$. And for given $X, Y \subseteq \mathbb{N}$, $\mathfrak{R}^i \subseteq \mathfrak{T}^+$, $i = 0, 1$ we write

$$X \le_m \begin{bmatrix} \mathfrak{R}^1 \\ \mathfrak{R}^0 \end{bmatrix} Y$$

if there exists a total recursive function f such that $X \le_m^f Y$ and $\exists h_0 \in \mathfrak{R}^0\ \exists \mathfrak{h}_1 \in \mathfrak{R}^1\ [\mathfrak{h}_0 \trianglelefteq e_f^0\ \&\ e_f^1 \trianglelefteq h^1]$. Here $g \trianglelefteq q$ means that function q majorizes function g almost everywhere.

Criterion is found for classes $\mathfrak{R}^i$, $i = 0, 1$, to define reducibilities. The family of these reducibilities was studying pertaining to structures of their degrees and complete sets. It is shown there are some reducibilities of kind

$$\le_m \begin{bmatrix} \mathfrak{R}^1 \\ \mathfrak{R}^0 \end{bmatrix}$$

with unusual properties. In particular the talk would present some reducibilities such that their structures of recursive set degrees are dense. These results can be interesting in connection with one question by R. Downey.

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► KRYSIA BRODA, DOV GABBAY AND LUÍS C. LAMB, *Labelled natural deduction for first-order conditional logic*.
Department of Computing, Imperial College, London SW7 2BZ, United Kingdom.
E-mail: `kb@doc.ic.ac.uk`.
Department of Computer Science, King's College, Strand, London WC2R 2LS, United Kingdom.
E-mail: `dg@dcs.kcl.ac.uk`.
FACIN/PUCRS, Av. Ipiranga 6681, 90619-900, Porto Alegre, RS, Brazil and PGCC/UFRGS, Cx. Postal 15064, 91501-970, Porto Alegre, RS, Brazil.
E-mail: `lamb@inf.pucrs.br`.

Conditional logics have been the subject of intensive investigation in philosophical logic and, lately, in knowledge representation [8]. However, most of the literature has concentrated on semantic investigations, whereas the proof-theoretical counterpart remained less explored. One of the obstacles in the development of a general proof theory is the different semantic interpretations of conditional sentences [2]. The Labelled Deductive Systems methodology [6] has allowed the development of uniform and general proof systems for a variety of logics. In this work we show how to develop a general proof theoretical approach for the conditional logics of normality [4]. Normality conditionals $\alpha \Longrightarrow \beta$ read "normally, if α then β" have been used in the modelling of default reasoning [5]. Our approach deals with problems associated with general conditional logics, dealing, not only with generic conditional reasoning, but also with situations akin to the lottery paradox. Labelled natural deduction approaches to quantified modal logics have been proposed in Basin et al. [1] and by Russo in [7]. Russo's approach is based on a translation technique into classical first-order semantics, whereas Basin et al. use a classical (modal) relational semantics. Our work builds upon the Compilation approach for Labelled Deductive Systems technique introduced by Broda and Russo [3] in developing an explicit approach to first-order to conditional logic. In our approach the representation of objects existing in particular possible worlds is made explicitly; in addition, this approach can also be seen as an extension of proof theoretical approaches to modal logics allowing conditional reasoning using a natural deduction presentation.

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► ANDREAS BRUNNER AND WALTER CARNIELLI, *Kripke semantics for anti-intuitionistic logics*.
IFCH and CLE-State University of Campinas, 13083 Campinas, SP, Brazil.
E-mail: andreas@cle.unicamp.br.
E-mail: carniell@cle.unicamp.br.

The question about the purported duality between intuitionistic logic and paraconsistent logic is not yet solved, and this note offers new insight into the theme. By dualizing Heyting's calculus I in the usual language L we obtain a system of antiaxioms for the *anti-intuitionistic* logic I^* in the dual-language L_A. Instead of proving new theorems, I^* rejects new conjectures. This anti-intuitionistic logic (cf. [3]) accepts any statement which would be possible in a future world, and only rejects a sentence if it cannot be true in any future world. Thus monotonicity holds for rejecting statements.

We show that this logic is correct and complete with respect to a Kripke semantics, and prove that there is an equivalence between this Kripke semantics and certain algebraic models. Giving a positive Kripke semantics $\Vdash_{I^*}$ in the usual sense, we show that this logic is paraconsistent and prove the following statements for φ, ψ formulas in L_A:

$$\Vdash_{I^*} \psi \vee \neg\psi, \quad \psi, \neg\psi \nVdash_{I^*} \varphi, \quad \psi - \varphi \Vdash_{I^*} \psi \wedge \neg\varphi,$$
$$\neg\neg\varphi \Vdash_{I^*} \varphi \quad \text{and} \quad \varphi \nVdash_{I^*} \neg\neg\varphi.$$

In the same way we construct the dual calculus J^* of the well-known Johansson's minimal logic J, and show that J^* is correct and complete with respect to a Kripke semantics. Although J and J^* are both intuitionistic and paraconsistent, we show that J and J^* are different logics and do not coincide with any known paraconsistent calculus.

Several questions arise (cf. [1]): for example, what is the relation between these anti-intuitionistic logics and the calculi C_ω, $C_{\min}$ and $C_{\lim}$, [2]? Can I^* and J^* be translated into classical logic?

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► VALERIY BULITKO AND VADIM BULITKO, *Self-condensible sets with respect to enumerability*.
Odessa National University, str. Fontanskaya Doroga 67-40, Odessa, 65062, Ukraine.
E-mail: booly@te.net.ua.

We study relative recursive enumerability with restricted usage of oracle. The restrictions being considered are analogous to the ones investigated in [1, 2]. In this paper we deal with the following problem. What sets A allow for an existence of an algorithm $\mathfrak{A}$ such that enumerates x into A using answers to questions from set $\{$ "$y \in A$?" $\mid y < x \}$ only? Unlike Turing reducibility, enumerability with oracles brings some non-trivial examples.

Let $\mathfrak{T}^+$ be $\{ f : \mathbb{N} \to \mathbb{N} \mid f$ is total and non-decreasing $\}$. We write $f \trianglelefteq g$ if $\exists x\ \forall n > x\ [f(n)/leqg(n)]$ and $f \vartriangleleft g$ if $\exists x\ \forall n > x\ [f(n) < g(n)]$.

A set A is called *g-recursively enumerable in B*, $g \in \mathfrak{T}^+$, if $\exists z\ [A = W_z^{\mathcal{C}_B}\ \&\ \forall a\ [a \in A \iff a \in W_z^{\mathcal{C}_B|g(a)}]]$, where $\mathcal{C}_B|x$ is the restriction of characteristic function $\mathcal{C}_B$ of set B onto segment $\{0, \dots, x\}$. We will denote it as $A \Leftarrow_g B$.

When $A \Leftarrow_f B\ \&\ \overline{A} \Leftarrow_f B$ we simply write $A \leq_f B$. This means A is $\leq_T$-reducible to B with bound f (see [1]).

Clearly, if A is not recursive then $A \leq_{\mathcal{I} \dot{-} h} A$ does not take place for any $h \in \mathfrak{T}^+$, $1 \trianglelefteq h$. On the other hand, there are non-recursively enumerable sets A such that $A \Leftarrow_{\mathcal{I} \dot{-} h} A$, $h \in \mathfrak{T}^+$, $1 \trianglelefteq h$.

Set A is said to be *weakly-$\underline{B}$-self-condensible with respect to enumerability* (*w. r. t. e.*) if there exists $h \vartriangleleft \mathcal{I}$ such that $A \Leftarrow_h A$ and $f \not\trianglelefteq h$ for any $f \leq_T B$. Likewise, set A is *strongly-$\underline{B}$-self-condensible* w. r. t. e. if there exists h such that $A \Leftarrow_h A$ and $(\forall f \in \underline{B} \cap \{ g \in \mathfrak{T}^{+,\infty} \mid g \vartriangleleft \mathcal{I} \})\ [h \trianglelefteq f]$.

THEOREM 1. *Let L be a r. e. s. and B be any set such that $L \not\leq_T B$. Then $\underline{L}$ contains a weakly-$\underline{B}$-self-condensible w. r. t. e. set.*

THEOREM 2. *$\underline{\emptyset}'$ contains a strongly-$\emptyset$-self-condensible w. r. t. e. set.*

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► JAAKKO HINTIKKA, *Quantum logic as a fragment of IF logics*.
Department of Philosophy, Boston University, 745 Commonwealth Avenue, Boston, Massachusetts 02215, USA.
E-mail: hintikka@bu.edu.

Independence-friendly (IF) first-order logic arises from ordinary first-order logic when in its game-theoretical semantics players' moves are allowed to be informationally independent of each other. In this paper, the fragment of IF logic is studied where two quantifiers may be mutually dependent of each other. One of the main characteristics of IF logic is that the only negation $\sim$ for which game rules can be given is a strong (dual) negation. Hence IF prepositional logic does not form a lattice, for there does not exist a maximal element. But what is the interpretation of $\sim$?

When a Hilbert space is considered as a state space, this strong negation turns out to correspond to orthocomplementation, and hence can be considered as its generalization. We can similarly generalize other state space concepts to IF languages, for instance the notion of dimension. IF logic can be extended by adding to it a sentence-initial contradictory negation $\neg$. Then the usual Birkhoff-von Neumann "quantum logic" can be interpreted by taking their disjunction to amount to $(\neg \sim A \vee \neg \sim B)$ with conjunction & and negation $\sim$ having their normal meanings. This reintroduces the maximal element. "Quantum logic" can thus be imbedded in a Boolean logic with an additional operation $\sim$ that corresponds to orthocomplementation.

► VLADIMIR KANOVEI, *Borel irreducibility of equality of countable sets of reals*.
MCCME, Bol. Vlasevski 11, Moscow 121002, Russia.
E-mail: `kanovei@math.uni-wuppertal.de`.

The equality of countable sets of reals is an equivalence relation (ER) T_2 defined on $\mathbb{R}^N$ so that $x \mathsf{T}_2 y$ if and only if $\{x_n : n \in \mathbb{N}\} = \{y_n : n \in \mathbb{N}\}$. It plays a key role in studies of ERs induced by actions of the infinite symmetric group [2], in particular, it has recently been established that various ERs are not Borel reducible to T_2. The following theorem is of opposite character.

THEOREM. T_2 *is not Borel reducible to any ER which belongs to any of the following categories*:

(i) (*Hjorth* [1]) *Orbit ERs of continuous actions of CLI groups*;
(ii) *ERs generated by* $\mathbf{F}_\sigma$ *ideals on* $\mathbb{N}$;
(iii) *ERs generated by exhaustive ideals of sequences of LSC submeasures*;
(iv) *ERs obtained from those in* (i)–(iii) *by multiple applications of the operation of Fubini product modulo the ideal of all finite sets.*

A CLI group is a Polish group which admits a compatible (complete) left invariant metric. In (iii), if $\{\varphi_n\}_{n\in\mathbb{N}}$ are LSC (lower semicontinuous) submeasures on $\mathbb{N}$ then the exhaustive ideal is defined by

$$\mathrm{Exh}_{\{\varphi_n\}} = \{X \subseteq \mathbb{N} : \lim_{n\to\infty} \varphi_n(X) = 0\}.$$

The *Fubini product* $\mathsf{E} = \prod_n \mathsf{E}_n/\mathrm{Fin}$ of ERs E_n on sets S_n is an ER on $S = \prod_n S_n$ defined so that $x \mathsf{E} y$ if and only if $x_n \mathsf{E}_n y_n$ for all but finite n.

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► ASYLKHAN KHISAMIEV, *On quasiresolvable models and B-models*.
Sobolev Institute of Mathematics of Siberian Branch of Russian Academy of Sciences, Acad. Koptyug prospect, 4, Novosibirsk, 630090, Russia.
E-mail: `hisamiev@math.nsc.ru`.

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Consider those models that consist of a countable universe and a finite number of predicates and functions. Yu. L. Ershov has introduced in [1] definitions of resolvable set and of quasiresolvable admissible set and has obtained some results: if an admissible set $\mathbb{A}$ is resolvable or quasiresolvable, then there exists a binary Σ-function universal for unary Σ-functions; if $\mathfrak{M}$ is a model of a decidable and model complete theory, then an admissible set $\mathbb{HF}(\mathfrak{M})$ is quasiresolvable. Definitions of internally enumerable model and B-model were introduced by me in [2, 3] and were obtained some results: if a model $\mathfrak{M}$ is an abelian p-group or a boolean algebra, then a model $\mathfrak{M}$ is a B-model; if a model $\mathfrak{M}$ is an internally enumerable, then an admissible set $\mathbb{HF}(\mathfrak{M})$ is resolvable [4].

In the given work are shown:

1. Relations between classes resolvable models, quasiresolvable models, internally enumerable models and B-models.
2. Any countable ordinal is not quasiresolvable. The criterion of an internally enumerable model is obtained in the terms quasiresolvable.
3. Any model of an ω-categorical theory is B-model. The uniformization theorem does not hold in an admissible set $\mathbb{HF}(\mathfrak{M})$ for any B-models.

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► M. V. KOROVINA AND O. V. KUDINOV, *Algorithmic aspects of Σ-definability over the reals*.
BRICS, Department of Computer Science, University of Aarhus, Ny Munkegade 540, Dk-8000 Aarhus C, Denmark.
E-mail: rita@inet.ssc.nsu.ru.
Institute of Mathematics, Koptug pr. 4, Novosibirsk, 630090 Russia.
E-mail: kud@math.nsc.ru.

The concept of Σ-definability is closely related to generalised computability over abstract structures [1, 2, 3, 5], in particular, over the real numbers [3, 4, 6].

Notions of Σ-definable sets or relations on abstract structures generalise those of computable enumerable sets of natural numbers, and play leading role in a specification theory that is used in the higher order computation theory over abstract structures. By working in the reals, with equality and without equality, we investigate properties of subsets of $\mathbf{R}^n$ which are Σ-definable in the set of hereditarily finite sets over the reals. For the definitions we refer to [1, 3, 5].

To algorithmic characterise Σ-definability we consider $\Delta_0 + \text{LFP}$ fixed-point extension of Δ_0 formulas. The operator LFP is an operator which maps computable enumerable sets to computable enumerable sets. It turns us to consider LFP as a natural operator from algorithmic point of view.

One of the main results states that Σ-definable relations on $\mathbf{R}$ are $\Delta_0 + \text{LFP}$-definable.

As direct corollaries we obtain that a relation on $\mathbf{R}$ is Σ-definable if and only if it is the least fixed point of Δ_0-operator. In other words, Σ-definable relations on $\mathbf{R}$ can be produced as the least fixed point of a Δ_0-operator.

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► ALEKSEY G. KRAVTSOV, *Extensions of multidimensional temporal logics*.
Department of Mechanics and Mathematics, Moscow State University, Moscow, 117234, Russia.
E-mail: kravtsov@lpcs.math.msu.ru.

We study some multidimensional modal and temporal logics with functional modalities. These logics can be interpreted as fragments of propositional dynamic logics of deterministic computations. We consider only logics in which all modalities commute. The minimal logics with commuting modalities are the products $\mathbf{SL}^n$ ($\mathbf{SL}$ is the well-known *Tomorrow Logic*) described in [2, Section 14]. The semantics of $\mathbf{SL}^n$ are given by the monoid $\mathbb{N}^n$ with n coordinate shift functions.

We consider arbitrary extensions of $\mathbf{SL}^n$ and especially of $\mathbf{SL.t}^n$ ($\mathbf{SL.t}$ is the *Tomorrow-Yesterday Logic*). We show that all finitely axiomatisable extensions of $\mathbf{SL}^n$ and $\mathbf{SL.t}^n$ are decidable, and all extensions of $\mathbf{SL.t}^n$ have the finite model property. Moreover, we give a complete description of all finitely axiomatisable extensions of $\mathbf{SL.t}^n$: all of them are finite intersections of logics of n-generated abelian groups with coordinate shift functions. By the same method we also give new proofs of Muchnik's theorems on pretabularity of the logics **SL** and **SL.t** [1].

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► OLGA MALAKHOVA, *Understanding "the other" and self-understanding*.
Chair of Philosophy, 11, Studencheskay Street, Izhevsk State Academy of Agriculture, Udmurt Republic, 426069 Russia.
E-mail: Malakhova@akokov.udm.ru.

In modern philosophy conditions of the tendency to self-determining of the personality in the modern world there is the problem of understanding. The Other as self-understanding, where Other is a basis of Self life. This problem is differently decided within the framework philosophy of classical and of unclassical Russian religious tradition. In the correspondence with it there are two main ways of understanding The Other and self-understanding.

Within the framework of both traditions the understanding are interpreted as direction and as Being which can exist. Understanding is a direction to comprehension of something or someone in oder to find a sense of The Other's being. The Self being has possibility to understand The Other and it depends on a degree of their realization.

The realization of possibilities can be presented as an outline of "cognizing subject" possibilities on object (M. Heidegger). The outline has a two-level structure: it is carried out both "outside", and "inside".

In classical philosophy attention is accented on the analysis of realization of possibilities directed "inside", because their "outside" realization contents an eternal conflict. The outline directed "inside" is catching of the objective possibilities and self-understanding as object. It is possible by means of The Other. The situation of meeting with the alien Other creates a boundary situation of fear before the Other, therefore Self recedes from the Other in order to understand itself. The consequence of this situation is self-consciousness as self-understanding. Self possessed self-consciousness as self-understanding has self-being for which the Other is always object. Self body is object or the Other that is become his own world. Therefore understanding without direction to "outside" leads to spiritual loneliness, self-closure and to boundary in self-understanding.

Russian religious philosophers use concepts that describe direction to "inside" as direction to "outside". Self does not consider "We" as passive synthesis "Self" and "the Other". This is a situation of mutual apply, "living" meeting, mutual intrusion of individualities in each other in the form of spiritual interaction. In this situation "the Other" becomes "You", which Self meets him in direction to "outside". "You" speaks about yourself and so Self understand itself.

► GIAN ARTURO MARCO, *Omitting types theorems and the topological Vaught conjecture*.
p. za Statuto n. 17, Turin, Italy.
E-mail: gamarco@tin.it.

Let G be a Polish group acting continuously on a Polish space X; the resulting G-space is Polish and the topological Vaught conjecture (TVC) asserts that for all G and X either the G-space has countably many orbits or else it contains a perfect set of inequivalent elements. This paper is devoted to the study of certain model theoretic properties arising as properties of the action of closed subgroups G of the symmetric permutation group on $\mathbb{N}$.

We introduce the notions of "closure isolation" over a structure and of "orbit completeness": these are descriptive variants of the omitting types property for countable fragments of infinitary languages. Rather surprisingly, these topological versions of the omitting types property are conceptually distinct: while orbit completeness refers to the ideal of meager sets over the orbit, closure isolation refers to the calibrated ideal of orbital closed sets and the former is not reducible to the latter.

The main tools used in this research are the notions of Vaught transform realizations and of fragment topology around the orbit; the main result of the paper is that TVC holds for groups of automorphisms with the closure isolation property. This follows as the corollary of the fact that automorphism actions with the closure isolation property for which there isn't an ergodic nonatomic measure on the space of F-types, have a relative F-saturated model in the closure of the orbit.

In this last part of the paper we briefly discuss some problem connected with the failure of TVC for analytic sets; and the existence of uncountable perfect models for the Scott sentence associated with automorphism groups that are not closed left invariant.

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► VETA MURZINA, *The completeness theorem for modal logic based on linearly ordered f_0-spaces*.
Institute of Mathematics, Russian Academy of Sciences, Siberian Branch, Koptyug Prospect 4, 630090 Novosibirsk, Russia.
E-mail: murzin@iis.nsk.su.

The modal logic based on f-spaces is considered. The conception of f-space was introduced by Ershov [1] with the purpose to develop the theory of computable functionals. It is known that the triple $\langle X, X_0, \leq\rangle$ is defined by f-space if and only if the following conditions are possessed:

(1) $X_0 \subseteq X$;
(2) $\leq$ is a partial order on X;
(3) $x_0, x_0' \in X_0,\ \exists x \in X\ (x_0 \leq x$ and $x_0' \leq x) \Longrightarrow \exists x_0'' \in X_0\ (x_0'' = x_0 \cup x_0')$;
(4) $\forall x \in X\ \exists x_0 \in X_0\ (x_0 \leq x)$;
(5) $x \not\leq y \Longrightarrow \exists x_0 \in X_0\ (x_0 \leq x$ and $x_0 \not\leq y)$.

If the least element exists in X then f-space is called f_0-space.

Frames $\langle X, X_0, R\rangle$ are considered being similar to Kripke's frames. If the frame $\langle X, X_0, \leq\rangle$ is a linearly ordered f_0-space then the frame $\langle X, X_0, R\rangle$, where for any $x, y \in X$ $(x\,R\,y \iff x \leq y$ and $x \neq y)$, is called a linearly ordered f_0-frame.

The tense modalities $\Diamond$ and $\Box$ associated with the relation R are introduced. The set X_0 is represented as a constant β. A logic calculus L' is introduced obtained by adding to the minimal calculus K the following axioms:

(1) $\Diamond\Diamond A \to \Diamond A$;
(2) $\Diamond A_1 \,\&\, \Diamond A_2 \to \Diamond(\beta \,\&\, (\Diamond A_1 \vee A_1) \,\&\, (\Diamond A_2 \vee A_2))$;
(3) $\Box(\Box A_1 \to A_2) \vee \Box(A_2 \,\&\, \Box A_2 \to A_1)$.

The following theorems are proved:

Theorem 1. *In any linearly ordered f-frame the formulas* (1)–(3) *are valid.*

Theorem 2. *Every formula valid in all linearly ordered f_0-frames is a theorem of L'.*

Since any f_0-frame is an f-frame then L' complete with respect to the class of all linearly ordered f_0-frames, and also with respect to the class of all linearly ordered f-frames.

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► D. E. PAL'CHUNOV, *Strongly constructive prime I-algebras*.
Institute of Mathematics, Pr. Koptuga, 4, Novosibirsk-90, 630090 Russia.
E-mail: palch@math.nsc.ru.

We study Boolean algebras with k-distinguished ideals, called as I-algrebras. Operations of ω-mixing and η-mixing of I-algebras were introduced in [2]. If $\mathfrak{A}_n \cong \mathfrak{A}$ for any n then an ω-mixing of $\mathfrak{A}_n$ is called as ω-power of $\mathfrak{A}$, and an η-mixing of $\mathfrak{A}_n$ is called as η-power of $\mathfrak{A}$.

Definitions on constructive algebras one can find in [1]. Using ω-mixing and η-mixing of I-algebras we define ω-power and η-power of a constructive I-algebra $(\mathfrak{A}, \nu)$. Note that ω-mixing, η-mixing, ω-power and η-power are defined not uniquely.

Theorem 1. *If* $(\mathfrak{A}, \nu)$ *is a constructive* I*-algebra,* $(\mathfrak{B}, \mu_1)$ *is an* ω*-power of* $(\mathfrak{A}, \nu)$*, and* $(\mathfrak{C}, \mu_2)$ *is an* η*-power of* $(\mathfrak{A}, \nu)$*, then* $(\mathfrak{B}, \mu_1)$ *and* $(\mathfrak{C}, \mu_2)$ *are constructive* I*-algebras.*

Theorem 2. *If* $(\mathfrak{A}, \nu)$ *is a strongly constructive* I*-algebra,* $(\mathfrak{B}, \mu_1)$ *is an* ω*-power of* $(\mathfrak{A}, \nu)$*, and* $(\mathfrak{C}, \mu_2)$ *is an* η*-power of* $(\mathfrak{A}, \nu)$*, then* $(\mathfrak{B}, \mu_1)$ *and* $(\mathfrak{C}, \mu_2)$ *are strongly constructive* I*-algebras.*

An ω-power $(\mathfrak{B}, \mu)$ of a constructive I-algebra $(\mathfrak{A}, \nu)$ is called as a proper ω-power of $(\mathfrak{A}, \nu)$ if for any η-power $(\mathfrak{C}, \mu_1)$ of $(\mathfrak{A}, \nu)$ we have $\mathfrak{B} \not\equiv \mathfrak{C}$. An η-power $(\mathfrak{C}, \mu)$ of a constructive I-algebra $(\mathfrak{A}, \nu)$ is called as a proper η-power of $(\mathfrak{A}, \nu)$ if for any ω-power $(\mathfrak{C}, \mu_1)$ of $(\mathfrak{A}, \nu)$ we have $\mathfrak{B} \not\equiv \mathfrak{C}$. Denote

$$\mathbf{A}_1 = \{\, (\mathfrak{A}, \nu) \mid (\mathfrak{A}, \nu) \text{ is a strongly constructive two-element } I\text{-algebra} \,\},$$

$$\mathbf{A}_1 = \{\, (\mathfrak{A}, \nu) \mid (\mathfrak{A}, \nu) \text{ is a strongly constructive atomless } I\text{-algebra,}$$
$$\text{where for any } j \leq k \text{ either } I_j = \{0\} \text{ or } 1 \in I_j \,\}.$$

Denote $\mathbf{A} = \mathbf{A}_1 \cup \mathbf{A}_2$. Let $\mathbf{P}$ be a class of constructive I-algebras, generated by $\mathbf{A}$ and closed under proper ω-power, proper η-power, and finite direct products.

Corollary. *If* $(\mathfrak{A}, \nu) \in \mathbf{P}$ *then* $(\mathfrak{A}, \nu)$ *is a strongly constructive* I*-algebra.*

Theorem 3. $\{\, \mathfrak{A} \mid (\mathfrak{A}, \nu) \in \mathbf{P} \,\}$ *is the class of all prime finitely axiomatizable* I*-algebras.*

[1] S. S. Goncharov, ***Countable Boolean algebras and decidability***, Consultants Bureau, New York, London, and Moscow, 1997.

[2] D. E. Pal'chunov, *Countably categorical Boolean algebras with distinguished ideals*, ***Studia Logica***, vol. XLVI (1987), no. 2, pp. 121–135.

► VADIM G. PUZARENKO, *On categoricity in* $\mathbb{HF}$ *language*.
Institute of Mathematics, 4, Koptyug str., Novosibirsk, 630090, Russia.
E-mail: vagrig@math.nsc.ru.

Definition. Let $\mathbb{HF}(\mathfrak{M})$ be hereditarily finite set over arbitrary model $\mathfrak{M}$. We define an iterating collection $\mathcal{TF}$ of definable sets on stages as follows.

Stage 0. $S \in \mathcal{TF}_0$ if and only if $S = \bigvee \Phi[x_0]$, where Φ is a finite set consisting of complete formulas of $\mathrm{Th}(\mathbb{HF}(\mathfrak{M}))$;

Stage $\alpha > 0$. $S \in \mathcal{TF}_\alpha$ if and only if there is a formula $\Psi(x_0, x_1)$ without parameters such that $S = \bigcup_{n\in\omega} \Psi^{\mathbb{HF}(\mathfrak{M})}[n, x_1]$ and for any $n \in \omega$, $\Psi^{\mathbb{HF}(\mathfrak{M})}[n, x_1] \in \mathcal{TF}_{\delta_n}$ for some $\delta_n < \alpha$.

Definition. Let $\mathbb{HF}(\mathfrak{M})$ be hereditarily finite set over the arbitrary model $\mathfrak{M}$. We define the iterating collection $\mathcal{SF}$ of definable sets on stages as follows.

Stage 0. $S \in \mathcal{SF}_0$ if and only if S is finite;

Stage $\alpha > 0$. $S \in \mathcal{SF}_\alpha$ if and only if there is a formula $\Psi(x_0, x_1)$ with parameters such that $S = \bigcup_{n\in\omega} \Psi^{\mathbb{HF}(\mathfrak{M})}[n, x_1]$ and for any $n \in \omega$, $\Psi^{\mathbb{HF}(\mathfrak{M})}[n, x_1] \in \mathcal{SF}_{\delta_n}$ for some $\delta_n < \alpha$.

THEOREM. *Let T be a theory of hereditarily finite set $\mathbb{HF}(\mathfrak{M})$ over some model $\mathfrak{M}$ in some countable language. Then any hereditarily finite sets of this theory are isomorphic if and only if $M^{<\omega} \in \mathcal{TF}_{\omega_1}$.*

THEOREM. *Let T be a theory of hereditarily finite set $\mathbb{HF}(\mathfrak{M})$ over some model $\mathfrak{M}$ in some countable language. Then any hereditarily finite sets of this theory are isomorphic if and only if $M^{<\omega} \in \mathcal{TF}_{\omega_1}$ and $M \in \mathcal{SF}_{\omega_1}$.*

Let $\lambda_0 \leftrightharpoons \omega_0$; $\lambda_{\alpha+1} \leftrightharpoons 2^{\lambda_\alpha}$ and $\lambda_\gamma \leftrightharpoons \bigcup\{\lambda_\alpha \mid \alpha < \gamma\}$, if γ is limit ordinal.

Let $\varkappa_0 \leftrightharpoons \omega_0$; $\varkappa_{\alpha+1} \leftrightharpoons \varkappa_\alpha^+$ where β^+ is a successive ordinal for β; and $\varkappa_\gamma \leftrightharpoons \bigcup\{\varkappa_\alpha \mid \alpha < \gamma\}$, if γ is limit ordinal.

THEOREM.

(1) *For any $\alpha < \omega_1$ there is a model $\mathfrak{M}_\alpha$, $\|\mathfrak{M}_\alpha\| = \lambda_\alpha$ that has no elementary extension $\mathfrak{M}'$, $\|\mathfrak{M}'\| > \lambda_\alpha$, such that $\mathbb{HF}(\mathfrak{M}_\alpha) \preccurlyeq \mathbb{HF}(\mathfrak{M}')$;*

(2) *For any $\alpha < \omega_1$ there is a model $\mathfrak{M}_\alpha$, $\|\mathfrak{M}_\alpha\| = \varkappa_\alpha$ that has no elementary extension $\mathfrak{M}'$, $\|\mathfrak{M}'\| > \varkappa_\alpha$, such that $\mathbb{HF}(\mathfrak{M}_\alpha) \preccurlyeq \mathbb{HF}(\mathfrak{M}')$.*

All the definitions and denotations used in this paper can be found in [1, 2].

[1] YU. L. ERSHOV, ***Definability and computability***, Plenum Publishing Corp., New York, 1996.

[2] C. C. CHANG and H. J. KEISLER, ***Model theory***, North-Holland Publishing Co., Amsterdam, London; Amer. Elseiv. Publ. Comp., Inc., New York, 1973.

► ALEXEJ P. PYNKO, *The completeness of derivable rules of labeled calculi for finite-valued logics*.
Glushkov Institute of Cybernetics, Glushkov Prosp. 40, Kiev 03680, Ukraine.
E-mail: pynko@d105.icyb.kiev.ua.

We follow notions and notations of [1].

LEMMA 1. *Let v be a mapping of a set* **L** *of labels to the set of sets of subsets of* **L**, *and* **C** *a labeled sequent calculus containin, for every $l \in$* **L**, weak : l, cntr : l, *all axioms of the form $\{l\} \cup L$, where $L \in v(l)$, and all cuts of the form $\{l\} \cup f[v(l)]$, where $f : v(l) \to$* **L** *such that, for each $L \in v(l)$, $f(L) \in L$. Assume, for every $\frac{X}{\Gamma} \in$* **C** *and each labeled sequent Δ, the rule*

$$\frac{\{\Theta \cup \Delta : \Theta \in X\}}{\Gamma \cup \Delta}$$

is derivable in **C**. *Then, a labeled sequent rule*

$$\frac{X \cup \{\Gamma\}}{\Delta}$$

is derivable in **C** *if and only if, for each l: $\phi \in \Gamma$ and each $L \in v(l)$, the rule*

$$\frac{X}{\{k : \phi : k \in L\} \cup \Delta}$$

is derivable in **C**.

THEOREM 2. *Let* **M** *be a matrix, S a system of signs over V, and* **C** *the labeled sequent calculus given by* [1, Definition 6.6]. *Then, a labeled sequent rule is derivable in* **C** *if and only if it is sound with respect to* **M** *and S.*

SKETCH OF PROOF. For every $\alpha \in S$, there is some set $v(\alpha)$ of subsets of S such that $V \setminus \alpha = \bigcap\{\bigcup\sigma : \sigma \in v(\alpha)\}$. Then, v and both **C** and the labeled sequent calculus constituted by all rules sound with respect to **M** and S satisfy the conditions of Lemma 1. Thus, [1, Theorem 6.1] and Lemma 1 collectively complete the argument. ⊣

[1] M. Baaz, G. Fermüller, G. Salzer, and R. Zach, *Labeled calculi and finite-valued logics*, ***Studia Logica***, vol. 61 (1998), pp. 7–33.

► S. G. PYRKIN, *On m-equivalence of finitely axiomatizable I-algebras*.
Novosibirsk State University, Lesosechnaya 6, 29. 630058, Novosibirsk, Russia.
E-mail: psg@novosoft.ru.

Boolean algebras with a distinguished ideal (I-algebras) are considered in this paper. We study m-equivalence of non-vanishing finitely axiomatizable I-algebras. It was shown [1] that for complete describing of elementary theories of I-algebras it is sufficient to solve the problem of m-equivalence for finitely axiomatizable I-algebras. The sequence of formulas $V_n,\ n \in \omega$, was constructed via procedures of ω- and η-mixing in [2]. These formulas are exactly describing all elementary theories of finitely axiomatizable non-vanishing I-algebras. We call an I-algebra $\mathfrak{A}$ *ideal* if $1^{\mathfrak{A}} \in I$. In [3, 4] it was described the m-equivalence of considering models in the case if one of I-algebras is constructed from ideal I-algebras. One can find definitions on I-algebras in [2]; on m-equivalence of I-algebras in [2].

Definition. Denote via $\mathfrak{A}^{\omega}$ ($\mathfrak{A}^{\eta}$) the ω-degree (η-degree) of I-algebra $\mathfrak{A}$.

Let $m[\mathfrak{A}, \mathfrak{B}]$ be the maximal m such that I-algebras $\mathfrak{A}$ and $\mathfrak{B}$ are m-equivalent and ∞ if such m does not exist.

Theorem. *Let* $\mathfrak{A}$, $\mathfrak{B}$ *be finitely axiomatizable* I*-algebras and* $\mathfrak{A} \not\equiv \mathfrak{B}$. *Then*

$$m[\mathfrak{A}^{\omega}, \mathfrak{B}^{\omega}] = m[\mathfrak{A}^{\omega}, \mathfrak{B}^{\eta}] = m[\mathfrak{A}^{\eta}, \mathfrak{B}^{\omega}] = m[\mathfrak{A}^{\eta}, \mathfrak{B}^{\eta}].$$

[1] D. E. Pal'chunov, *Finitely axiomatizable Boolean algebras with distinguished ideals*, ***Algebra i Logika***, vol. 26 (1987), no. 4, pp. 435–455 and 525.
[2] ———, *Countably-categorical Boolean algebras with distinguished ideals*, ***Studia Logica***, vol. 46 (1987), no. 2, pp. 121–135.
[3] S. G. Pyrkin, *The m-equivalence of finitely axiomatizable non-vanishing I-algebras*, in ***Logic and Applications***, Novosibirsk, 2000, pp. 86–87.
[4] ———, *On m-equivalence of I-algebras*, in ***XXXVIII international students science conference (ISSC)***, Novosibirsk, 2000, pp. 3–4.

► CHRISTIAN JACQUES RENTERIA AND EDWARD HERMANN HAEUSLER, *Natural deduction for ultrafilter logic—proof theory*.
Pontificia Universidade Catolica do Rio de Janeiro, Brasil.
E-mail: chris@onze.net, chren@inf.puc-rio.br.

We develop a natural deduction system for a first order ultrafilter logic (the logic of 'almost all') [2]. This system is based on a labeling of formulas and marked variables to keep track of what kind of introduction they come from. The system is full normalizable.

[1] Dirk van Dalen, ***Logic and structure***, Springer-Verlag, 1997.
[2] W. A. Carnielli and P. A. S. Veloso, *Ultrafilter logic and generic reasoning*, in ***Computational logic and proof theory (5th Kurt Gödel Colloquium)***, Springer-Verlag, 1997, pp. 34–53.
[3] Robert Goldblatt, ***Lectures on the hyperreals***, Springer-Verlag, 1998.

► IRINA SEMAKINA, *The problem of formalizing of language*.
Docent of Department of Philosophy, Udmurt State University, Izhevsk, Russia.
E-mail: semir@uni.udm.ru.

At the present time modern philosophy directs its attention to the studies of nature of language, which is investigated on different levels of analyses.

It's connected with, at first, a crisis of scientific rationality, the result of which has become awareness of the impossibility to cognize the "objective truth" because it has contradiction. The second was the frustration of attempts to create the perfect scientific language as a "metalanguage", which should be exact and unambiguous. Scientific cognition clashed with the problem of "descriptive language" that couldn't be formalized completely and transformed in "terminological" producing.

The problem of "metaphorical" language, polisemy and sense ambiguous were consequences of that situation. Language refused to be "transparent", exact and definite. Metaphor seemed to be a "reef" of metalanguage, which didn't allow transform language in conceptual logical sphere and formalize fully. The problem of metaphor pointed the limits of scientific cognition, failing of which was connected with "technical" relation to language. Language is not only means or instrument of description.

Formalized language reproduces the concepts that represent the world as space of things where human is a thing among other things. This language is information that is produced by count thinking of man and used for pragmatics goals.

"Technical" relation to language is consequence of scientific human activity that is technical by nature. Human has been conformed in technical counting man and has lost link with foundation of Being. Man is that who possesses gift of speech and language is not important thing for human only that it is used for satisfaction of his necessities, but language is the way by means of human exists and awareness of himself. The reason of formalizing of language is objectivity of human essence and so a man loses possibility to disclose himself as subjectivity in the world that is described by scientific concepts.

Reducing language to informative funciton prevents from solution of questions, such as the problem of sense of life and the problem of freedom. Neither sense of life or freedom are not information and that's why scientific discourse doesn't allow exist to man as human subjectivity.

► VLADIMIR SOTIROV, *Negation hidden in the classical implication*.
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 8, Acad. G. Bonchev Str., 1113 Sofia, Bulgaria.
E-mail: `vlsot@math.bas.bg`.

Maybe Peirce was the first to observe that implication involves many properties of negation if an arbitrary but fixed propositional letter s would be separated and any subformula of the kind $F \Longrightarrow s$ would be interpreted as $\neg F$ (when s does not occur as an antecedent). Later on this idea was successfully explored by Johansson to build his minimal calculus. It was based on the *positive implication* and the axiom of the negation so obtained could be $(A \Longrightarrow \neg B) \Longrightarrow (B \Longrightarrow \neg A)$. A question arises: which kind of negation is hidden in the similar way in the system $\langle \Longrightarrow \rangle$ of the full (*classical*) *implication*. The question was answered by Vakarelov in [3]: the axiom of this negation is the same as that of Johansson's one; denote the system so obtained by $\langle \Longrightarrow, \neg \rangle$. Very surprisingly, Vakarelov showed that $\langle \Longrightarrow, \neg \rangle$ came to be equivalent to the Łukasiewicz 4-valued modal logic Ł [2] where $\neg$ coincides with Łukasiewicz's negation of necessity. However, in [3], the operator '$\Longrightarrow s$' (been read as 'it is doubtful') was going in parallel with the operator '$s \Longrightarrow$' (been read as 'it is verisimilar'), and the appearance of classical negation was used for proving the adequacy of 4-valued semantics. All these peculiarities moved aside the answer of the initial question: which are the feature of the negation hidden in the classical implication? This problem is solved here directly.

Let A be a formula of $\langle \Longrightarrow \rangle$. If T is an arbitrary tautology and s is an arbitrary propositional letter, the *s-negative image* of A in $\langle \Longrightarrow, \neg \rangle$ is given by the transformation ψ_s changing only subformulae containing s (cf. [3]): any subformula of A of the kind $B \Longrightarrow s$ (for $B \neq s$) is replaced with $\neg B$ and any subformula of the kind $s \Longrightarrow B$ (for $B \neq s$) with $\neg T \Longrightarrow B$ ($s \Longrightarrow s$ remains the same).

THEOREM 1. *$\vdash A$ in $\langle \Longrightarrow \rangle$ if and only if $\vdash \psi_s(A)$ in $\langle \Longrightarrow, \neg \rangle$.*

The origin of $\neg$ (the string $\Longrightarrow s$) shows that the theorems of $\langle \Longrightarrow, \neg \rangle$ coincide with that formulae which are tautologies of the classical logic when $\neg$ is treated as both negation and constant 'truth'. Using a result by Kalicki [1], adequate 4-valued matrices for this system are obtained by multiplication of the tables for the classical negation and the 'truth'-constant (see [2]). The resulting matrices consist of ordered pairs (a, b) for a, $b \in \{1, 0\}$ with the designated value $(1, 1)$. The rules of composing Ł are $(a, b) \Longrightarrow (c, d) = (a \Longrightarrow c, b \Longrightarrow d)$, $\neg(a, b) = (1 - a, 1)$.

THEOREM 2. $\vdash A$ *in* $\langle \Longrightarrow, \neg \rangle$ *if and only if A is a* 4*-valued tautology of* Ł.

[1] J. KALICKI, *Note on truth tables*, ***Journal of Symbolic Logic***, vol. 15 (1950), pp. 174–181.

[2] J. ŁUKASIEWICZ, *A system of modal logic*, ***Journal of Computing Systems***, vol. 1 (1953), pp. 111–149.

[3] D. VAKARELOV, *Ein Aussagenkalkül mit Funktoren für "Glaubwürdigkeit" und "Zweifel", Part II*, ***Annuaire de l'Université de Sofia, Fac. de Math.,***, vol. 61 (1966/67), pp. 47–70, Bulgarian with German summary.

► ZH. TALASBAEVA, *On positive numberings in Ershov's hierarchy*.
Department of Mathematics and Mechanics, Kazakh State National University, Masanchi Street 39/47, Almaty, Kazakhstan.
E-mail: t_zhuldyz@hotmail.com.

Positive numberings introduced by A. I. Mal'tsev play an important role in the theory of numberings since they are minimal under reducibility and have simple inner description: numbering α is *positive* if $\{ \langle x, y \rangle \mid \alpha x = \alpha y \} \in \Sigma^0_1$.

Definition of computable numbering of a family of sets in Ershov's hierarchy arises naturally both from general approach to the notion of computability of S. Goncharov and A. Sorbi [2] and from H. Putnam's n-computability notion [3]. A numbering $\alpha \colon \omega \to A$ of a family A of $\Sigma_n - 1$-sets is called Σ^{-1}_n*-computable* if $\{ \langle x, n \rangle \mid x \in \alpha n \} \in \Sigma^{-1}_n$.

THEOREM. *For every $n \geq 1$, if an infinite Σ^{-1}_n-computable family contains the greatest set under inclusion provided n is odd (the least set provided n is even) then it has infinite number of non-equivalent Σ^{-1}_n-computable positive but not decidable numberings.*

COROLLARY. *For every n, the class of all Σ^{-1}_{n+1} sets has infinite number of Σ^{-1}_{n+1}-computable positive non-decidable numberings.*

Conjecture on an existence of computable positive but not decidable numbering in every Σ^{-1}_n, $n \geq 2$, class was formulated in [1, conjecture 15].

It is interesting to find examples of the families of Σ^{-1}_{2n} (Σ^{-1}_{2n+1}) sets which contain the set ω (respectively, empty set) and have no Σ^{-1}_{2n}-computable (correspondingly, Σ^{-1}_{2n+1}-computable) positive numberings.

[1] S. A. BADAEV and S. S. GONCHAROV, *Theory of numberings. Open problems*, ***Contemporary Mathematics***, vol. 257 (2000), pp. 23–48.

[2] S. S. GONCHAROV and A. SORBI, *Generalized computable numerations and nontrivial Rogers semilattices*, ***Algebra and Logic***, vol. 36 (1997), noẇeaktie 4, pp. 359–369.

[3] H. PUTNAM, *Trial and error predicates and the solution to a problem of Mostowski*, ***Journal of Symbolic Logic***, vol. 30 (1965), pp. 49–57.

► LUIS ADRIAN URTUBEY, *Vagueness and contextual information*.
Universidad Nacional de Córdoba, Facultad de Filosofia, Cuidad Universitaria, Cordoba 5000 Argentina.
E-mail: urtubey@ffyh.unc.edu.ar.

It has been pointed out that it is an obvious fact in the case of vague predicates that the extension is underdetermined by meaning: some objects will be neither determined to be within nor outside the predicate extension. Speaker and hearer purposes can then determine whether the predicate is to apply or not to apply to some of these objects, or whether they are to be left as indeterminate. So it would appear that contextual information is an essential feature of vagueness. Moreover vague predicates share this feature with demonstratives singular terms such as 'this' and 'that', firstly studied by Kaplan. In a similar way, this fact would be also of interest to semanticists concerned that speakers' truth-value judgments of vague sentences sometimes violate classical logic. The supervaluational and the multi-valued semantics accounts address the intuition of nearly all sepakers that some vague sentences are neither true nor false. However, judgments of the truth-values of sentences do not always reflect their semantic properties, but they can reflect judgments concerning their informativeness, that also characterize a kind of context. So you would determine that 'tall' is applicable to George if you thought that I thought that it was more likely you would judge 'tall' applicable to men of George's height.

The purpose of this contribution is to evaluate among different alternatives what would be the most adequate construction of these propositional relations an agent is contextually placed—and able–to take into accound. Specifically, I shall consider two kinds of theories of context usually referred to in the literature as objective and subjective theories, respectively, and try to apply each tradition of research to explore the problem of vagueness and contextual knowledge.

▶ WIM VELDMAN, *The Cantor-Bendixson hierarchy revisited by Brouwer*.
Subfaculteit Wiskunde, Katholieke Universiteit Nijmegen, Postbus 9010, 6500 GL Nijmegen, the Netherlands.
E-mail: veldman@sci.kun.nl.

We study countable subsets of Cantor space $\mathcal{C}$ from an intuitionistic point of view. A subset X of $\mathcal{C}$ is a *Cantor-Bendixson-set* or a CB-set if and only if either X is empty or there exists a sequence $X_0, X_1, \ldots$ of CB-sets such that for each i in $\mathbb{N}$ there exists $j > i$ such that X_i coincides with X_j and, for every α in $\mathcal{C}$, α belongs to X if and only if either, for each n, $\alpha(n) = 0$, or there exist n in $\mathbb{N}$, β in X_n, such that n is the least i such that $\alpha(i) = 1$ and, for each j, $\alpha(n + j + 1) = \beta(j)$. For every CB-set X, the closure of X coincides not with X itself but with the double complement of X. For every CB-set X we define the class of the perhapsive extensions of X by the following inductive definition. A subset Y of $\mathcal{C}$ is a *perhapsive extension* of X if and only if either Y coincides with X or there exists a sequence $Y_0, Y_1, \ldots$ of perhapsive extensions of X such that Y is the set of all α in $\mathcal{C}$ with the property that, for some β in $\mathcal{C}$, if a is apart from β, then α belongs to one of the sets $Y_0, Y_1, \ldots$. Every perhapsive extension of X contains X and forms part of the double complement of X. Using stumps, the intuitionistic equivalent of countable ordinals, it is possible to define the *degree of perhapsity* of a given CB-set X. It turns out that the closure of X is itself a perhapsive extension of X and the degree of perhapsity of X is the least stump σ such that σ iterations of the generating operation of the class of perhapsive extensions of X produce the closure of X. This degree proves to be the same as the Cantor-Bendixson degree of X, which is defined much as in classical set theory.

[1] W. Veldman, *The Borel hierarchy and the projective hierarchy in intuitionistic mathematics*, ***Report number 0103***, Department of Mathematics, University of Nijmegen, 2001.

▶ GULNARA YAKHYAEVA, *Applications of the diffuse groups in the linguistics*.
Shevchenko Street, h.157, f.4, Almaty City, 480008 Kazakhstan.
E-mail: gul_nara@mail.ru.

Our group was suggested the new approach at formalisation of fuzzy information. We study the diffuse functions [2]. The result of operation of diffuse function is not one element, but the set of elements. Our group has investigated the diffuse groups [1].

In this paper we suggest to you the some applications of the theory of diffuse groups in linguistics.

Each concept, as a rule, can't be defined by one word of the natural language. We consider the some set of worlds of the natural language. The subset of the set which define the concept is a fuzzy subset of the set. We shall call this set the *semantical field*. The words ω_1 and ω_2 in the common field we shall call *semantic similar words*. We have on the input this fuzzy relation and build the diffuse group in which effectively work our algorithm the finding of the semantical fields.

But what do you do if the collocutor speak in the foreign language?

Let us have $m+1$ sets $\Omega + i$ $(i = 0, \dots, m)$ of worlds of some natural language. If we are given $m+1$ of the fuzzy relations:

- "the word $x_0 \in \Omega_0$ is semantic similar to the word $x_1 \in \Omega_0$"
- "the word $x_0 \in \Omega_0$ is translation of the word $y_i \in \Omega_i$"

we can define the diffuse group in which the algorithms of the concept translation word-book from the one language into the another language works effectively.

[1] V. P. Dobritsa and G. Yakhayayeva, *About group with fuzzy operations*, in ***The calculate systems***, 1999, no. 165, Novosibirsk, pp. 127–138, in Russian.

[2] G. Yakhyayeva, *About the fuzzy functions with one variable*, ***Vestnik KazGY, Almati***, vol. 3 (1999), no. 17, pp. 147–152, in Russian.

► ALEXANDER A. ZENKIN, *Gödel's numbering of multi-modal texts*.
Computing Center of the Russian Academy of Science, Moscow, Russia.
E-mail: alexzen@com2com.ru.

As is known, any Gödel's numbering is defined by the following three conditions: (1) every formal notation (terms, formulas and sequences of formulas) of a given formal system is associated with a single natural number, say, n, (2) the natural number n allows us to reconstruct identically that notation, and (3) any two different formal notations have different associated natural numbers [1, 2].

Any text (containing any symbols, visual and audio data, etc.) is represented in a computer as a binary string, say, S. Consider a new string $S^* = 1S$ which is S with "1" attached to S at its left (since some text can have a corresponding S beginning with "0"). It is obvious that the binary string S^* is a finite natural number, say, n. In such the case, (1) any text will be associated with a single natural number n, (2) such the natural number n written in binary system (with the previously deleted left "1") allows us to reconstruct identically the associated text, and (3) any two texts, differing even one symbol, will have different associated natural numbers.

So, such the computer numbering of any texts is a Gödel one. Denote such the numbering Gz. Now some new, interesting, and quite unexpected problems connecting with the Gz-numbering of different multi-modal texts arise. Some of such the problems that essentially extend main metamathematical, philosophical, and methodological aspects of well-known K. Gödel's achievements will be discussed in my presentation.

[1] K. Gödel, *Über formal unentscheidbare Satze der Principia Mathematica und verwander Systeme I*, ***Monatshefte fur Mathematik und Physik***, vol. 38 (1931), pp. 173–198.

[2] S. Kleene, ***Introduction to metamathematics***, D. Van. Nostrand Company, Inc., New York-Toronto, 1952.