THE BULLETIN OF SYMBOLIC LOGIC
Volume 8, Number 1, March 2002

REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The journal of symbolic logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Herbert Enderton (Coordinating Editor), Geoffrey Hellman, Thomas Jech, Wolfram Pohlers, and Philip Scowcroft. Authors and publishers are requested to send, for review, copies of books to *Herbert Enderton, Association for Symbolic Logic, Mathematical Sciences Bldg. 7332, UCLA, Los Angeles, California 90095-1566, USA*.

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

STEWART SHAPIRO. ***Thinking about mathematics. The philosophy of mathematics.*** Oxford University Press, Oxford and New York 2000, xiii + 308 pp.

Stewart Shapiro has written a very good, very useful textbook on the philosophy of mathematics. The book is also sorely needed, for while there are a few textbook-style overviews of the philosophy of mathematics in the literature, they are all badly out of date. What is good about Shapiro's book is that (i) a large chunk of the book (about a third) is dedicated to bringing the reader up to date on contemporary work in the philosophy of mathematics, and (ii) it takes the old logicism-formalism-intuitionism debate for what it was, namely, as a historical episode, and not as somehow capturing the three main views that one might endorse about mathematics. To most contemporary writers, all three of those views are mistaken (though, of course, there are still some contemporary advocates, and Shapiro covers them). It seems to me that there was something of a divide in the middle of 20th-century philosophy of mathematics: there were very few significant works published on the metaphysics and epistimology of mathematics between 1951, the year that Curry (certainly an old-schooler) published his book on formalism, and 1965, the year that Benacerraf published the first of the two famous papers that, in my view, ushered in the contemporary era in the philosophy of mathematics. The latter era is now a significant intellectual episode in its own right, and it is about time we had a book that provided a survey of this episode. Shapiro's book does this and does it well.

The book is divided into four parts. Part I provides some background for the rest of the book. Shapiro begins by discussing the question of whether the philosophy of mathematics

1079-8986/02/0801-0004/$2.60

ought to *dictate* how mathematics should be practiced or *account* for how it is practiced; he points out that the latter view is standard today. Then he discusses the notions of necessity and a priority, and describes the three main ontological views about mathematics (realism, idealism, and nominalism). Next, Shapiro discusses what he calls realism-in-truth-value, "the view that mathematical statements have objective truth-values, independent of the minds, languages, conventions, and so on of mathematicians" (p. 29). (Shapiro claims that some ontological anti-realists, e.g. Field and the intuitionists, are also anti-realists-in-truth-value, but that others, e.g. Chihara and Hellman, are realists-in-truth-value. One might question this characterization of Field; Shapiro points out that for Field, mathematical statements have only vacuous truth values; but Field thinks that, for example, '3 is prime' is objectively false, i.e., false independently of us, because of the nature of the world. It seems to me that there is a sense in which Field's view is more realist than Chihara's or Hellman's, semantically speaking, because unlike them, he is a *face-valuist* about the truth conditions of mathematical statements.) After this, Shapiro discusses the problem of the applicability of mathematics, and then he ends Part I by running through some of the particular mathematical propositions that philosophers have been especially interested in, for example the Löwenheim–Skolem theorem, the continuum hypothesis, and the incompleteness theorem.

Part II begins the historical survey of the philosophy of mathematics. In particular, it covers Plato, Aristotle, Kant, and Mill. The section on Kant is particularly good. Anyone who has ever tried to summarize Kant's philosophy of mathematics knows how difficult it can be to pin this view down. Shapiro does an excellent job of pulling the various strands together. He also does a good job making Mill's view seem more plausible than it is sometimes made to seem. For example, Shapiro points out that Mill has responses to several of the standard Fregean arguments against his view.

Part III is about logicism, formalism, and intuitionism. Within the logicist camp, Shapiro covers Frege, Russell, Carnap, and the contemporary neo-logicists, Wright and Hale. In connection with formalism, he begins with the views that Frege attacked, namely, those of Heine and Thomae, which, as Shapiro points out, both contain elements of term formalism and game formalism; Shapiro then discusses the early Hilbert's deductivism, the later Hilbert's finitism (and the Hilbert program), and Curry's metamathematical version of formalism. (In addition to these, I would have liked to see a discussion here of the early Putnam, who is a more contemporary advocate of deductivism.) As for intuitionism, Shapiro covers Brouwer, Heyting, and Dummett. Overall, Shapiro's treatment of all these philosophers is excellent.

(One thing I would have liked included in Part III is a discussion of the strangeness of the very fact that these three views were once thought of as opponents and, indeed, the only "live options" in the philosophy of mathematics. In fact, the three views here are all compatible with one another—they are not even answers to the same question—and as subsequent work has shown, they do not come close to exhausting all of the viable philosophies of mathematics. Indeed, it seems to me that formalism is the only one of them that even counts as a full-blown philosophy of mathematics, for it is the only one that provides a semantics and ontology for mathematics. Logicism and intuitionism were historically aligned with certain semantic and ontological views, but this is not central to those views—the one is a thesis about the logical relationship between mathematics and logic, and the other is a thesis about the cogency of certain kinds of mathematical proofs—and what is more, it seems to me that both of them can, with just as much plausibility, be aligned with radically different semantic and ontological views.)

Finally, Part IV covers contemporary views. It focuses, as it should, on views about the semantics and ontology of mathematics, in particular, on the question of whether or not mathematical realism is true. On the realist side of the debate, Shapiro discusses the views of Gödel, Quine, and the early Maddy. On the anti-realist side, he covers the views of Field, Chihara, Burgess and Rosen, Azzouni, and myself. He then ends with a chapter on structuralism, covering the views of Benacerraf, Hellman, Resnik, and himself. I am not so sure that structuralism really deserves to be singled out in this way; if I were organizing a book like this, I would include Resnik–Shapiro *ante rem* structuralism with the other contemporary realist views, and I would include Hellman's modal structuralism with the other contemporary anti-realist views. But of course, this is Shapiro's book, not mine, and it is understandable that he thinks that structuralism deserves a bit more space.

For the most part, Shapiro's discussions of the various contemporary views are accurate and very good. If I have any complaint at all here, it is that I think he might have devoted a bit more space, in the chapter on realism, to bringing out the different varieties of post-Gödel–Quine ontological realism that have emerged in recent years, and perhaps more importantly, to the different ways in which realists have tried to answer the Benacerrafian epistemological challenge. Shapiro says quite a bit on this topic in various places in the book—for example, in the chapter on structuralism, he discusses the ontology and epistemology of (his and Resnik's) *ante rem* structuralism, and in the chapter on anti-realism, he describes my own work on the ontology and epistemology of realism—but I think it would have been nice for the reader to see the various contemporary realist views brought together.

In sum, Shapiro has written an excellent book. It will be most useful, I think, as a secondary source for (graduate and undergraduate) students in philosophy of mathematics courses; and for graduate students and professional philosophers who do not know much about the philosophy of mathematics but want to be introduced to the area (anyone with a half-way decent philosophical background should be able to follow the book); and for professional philosophers of mathematics, who can use the text as a quick (but, of course, incomplete) source book on the views of the various figures that Shapiro covers.

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W. HUGH WOODIN. ***The axiom of determinacy, forcing axioms, and the nonstationary ideal.*** De Gruyter series in logic and its applications, no. 1. Walter de Gruyter, Berlin and New York 1999, vi + 934 pp.

In the 1980's, the author of the book under review showed that in the presence of large cardinals (a proper class of Woodin cardinals suffices) the theory of the inner model $L(\mathbb{R})$, the smallest model of ZF containing the reals and the ordinals, cannot be changed by set forcing. Combined with results of Martin and Steel (JSL LVII 1133), the techniques used in this proof soon yielded that the existence of infinitely many Woodin cardinals below a measurable cardinal implies that the axiom of determinacy (AD) holds in $L(\mathbb{R})$. In the other direction, AD gives a rich structure theory for $L(\mathbb{R})$, including the fact, again due to the author, that there are inner models of ZFC in which certain countable ordinals are Woodin cardinals. Applying the existence of such models, this book introduces a method for forcing over models of determinacy, and uses this method to lift the influence of determinacy to models of the axiom of choice.

The main results of the book are best stated in terms of a strong logic introduced in the last chapter. A sentence is said to be valid in Ω-logic if it holds in all models of ZFC with a certain second-order property (A-closure for a certain set of reals A). For example, a sentence that holds in all well-founded models of ZFC is valid in Ω-logic, and any sentence that can be forced to hold is Ω-consistent; indeed, the question whether the statement "φ is valid in Ω-logic" is equivalent to "φ cannot be forced to be false" is the key open question in this area (the positive answer is known as the Ω-conjecture). Under the assumption of a proper class of Woodin cardinals, every sentence for $H(\omega_1)$ is either valid or false in Ω-logic. Further, there are sentences corresponding to the forcing construction introduced in this book that give a similarly complete axiomatization of the theory of $H(\omega_2)$ in Ω-logic. This phenomenon is inconsistent with the continuum hypothesis, however: if the theory of $H(\omega_2)$ can be finitely axiomatized over ZFC in Ω-logic then there is a definable surjection from the reals onto ω_2. While Ω-logic provides a framework for interpreting the book's main results, the corresponding proofs are left for another book, currently in preparation.

The central topic of the book under review is a forcing construction called $\mathbb{P}_{\max}$. The conditions in $\mathbb{P}_{\max}$ are countable transitive models of a suitable fragment of ZFC, and the order is given by generic elementary embeddings with critical point ω_1. The consistency of modest large cardinals (daggers) is needed for the existence of $\mathbb{P}_{\max}$ conditions, but the full effect is achieved under the assumption that AD holds in $L(\mathbb{R})$. Assuming this, Woodin shows the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$ is maximal for Π_2 sentences over $H(\omega_2)$ using certain predicates, primarily the nonstationary ideal on ω_1, in the sense that all Ω-consistent statements of this type hold simultaneously in this extension. The following statement is denoted by $(*)$: AD holds in $L(\mathbb{R})$, and $L(\mathcal{P}(\omega_1))$ is a $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$. The theory ZFC + $(*)$ decides every sentence for $H(\omega_2)$ in Ω-logic, as does the corresponding statement for any variation of $\mathbb{P}_{\max}$.

One of the book's two primary ancestors is a result of Steel and Van Wesep lifting consequences of determinacy to models of choice. Under AD, the nonstationary ideal on ω_1 is saturated, and the second uniform indiscernible is ω_2. Starting from a strong determinacy hypothesis, Steel and Van Wesep constructed a forcing extension satisfying choice in which these two statements hold. Their construction was a precursor to $\mathbb{P}_{\max}$, and these statements hold in the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$ as well. The immediate inspiration for $\mathbb{P}_{\max}$ was the author's discovery that if the nonstationary ideal on ω_1 is saturated and there exists a measurable cardinal, then the second uniform indiscernible is ω_2, so in particular there is a definable counterexample to the continuum hypothesis. The proof of this fact and several generalizations are given in Chapter 3.

The study of $\mathbb{P}_{\max}$ begins in Chapter 4. The reader who wants to take the shortest path to working with $\mathbb{P}_{\max}$ should begin reading here, referring back to earlier parts of the book as necessary. He or she may also wish to replace the theory ZFC* with ZFC, which suffices for almost all arguments after Chapter 3. The key arguments rely on results of Martin and Steel on definable scales (JSL LVII 261), which give rise to conditions with certain second-order correctness properties (related to the properties used in defining Ω-logic) with respect to any given set of reals in $L(\mathbb{R})$. It is this fact that gives $\mathbb{P}_{\max}$ its strength, enabling arguments that diagonalize against any given $\mathbb{P}_{\max}$-name for a set of reals. The key corollary is that the structure $H(\omega_2)$ in the $\mathbb{P}_{\max}$ extension is the direct limit of the structures $H(\omega_2)$ from the conditions in the generic filter. As a result, showing that a given Π_2 sentence for $H(\omega_2)$ holds in the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$ requires showing only the one step, that one can do away with any given potential counterexample. The chapter culminates by applying this fact to obtain several absoluteness results. For example, if there is a Woodin cardinal smaller than a measurable cardinal, then any Π_2 fact for $H(\omega_2)$ that holds in V will hold in the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$. The significance of this fact is seen by noting that $\mathbb{P}_{\max}$ is homogeneous, so the theory of the $\mathbb{P}_{\max}$ extension is computed in $L(\mathbb{R})$. Adding these two facts to the absoluteness of the theory of $L(\mathbb{R})$, we have in the presence of a proper class of Woodin cardinals that all Π_2 sentences for $H(\omega_2)$ that can be forced hold simultaneously in the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$.

Chapter 5 completes the basic analysis of $\mathbb{P}_{\max}$ by showing that the cardinality of the continuum is $\aleph_2$ in the $\mathbb{P}_{\max}$ extension, and so choice holds there as well. This is shown by a certain form of stationary reflection. Several alternate presentations of $\mathbb{P}_{\max}$ are then given, all giving rise to the same model or very similar models, and each most convenient for certain types of arguments in the $\mathbb{P}_{\max}$ analysis. The analysis of the $\mathbb{P}_{\max}$ extension is then continued, giving a reformulation of the axiom $(*)$ in terms that make no mention of $\mathbb{P}_{\max}$ or determinacy.

In the $\mathbb{P}_{\max}$ extension, the nonstationary ideal on ω_1 is the direct limit of selected precipitous ideals from the conditions in the generic filter. Chapter 6 uses this fact to convert facts from arbitrary precipitous ideals on ω_1 to the nonstationary ideal. The primary result of the chapter is that assuming only that determinacy holds in $L(\mathbb{R})$ we can obtain a variation of $\mathbb{P}_{\max}$ (called $\mathbb{Q}^*_{\max}$) in whose extension the nonstationary ideal on ω_1 is ω_1-dense. Woodin had previously shown that ω_1-dense ideals on ω_1 could be produced by starting from from a huge cardinal, but currently the only known way of getting the nonstationary ideal to be ω_1-dense is through variations of $\mathbb{P}_{\max}$. The same key point is used to show that the saturation of the nonstationary ideal on ω_1 can be destroyed by forcing without adding a subset of ω_1.

Chapters 7 and 8 consider conditional variations of $\mathbb{P}_{\max}$, variations that produce maximal models relative to a given minimality assumption. Three examples are given, one concerning Suslin trees, one the Borel conjecture, and one variations of the principle $\clubsuit$. As shown by several authors, this approach is a useful means of obtaining consistency results. Chapter 9 considers the effect of forcing with $\mathbb{P}_{\max}$ over larger models of stronger forms of determinacy. One of the principal results of the chapter is that if $\mathrm{AD}_{\mathbb{R}}$ holds in $L(\mathcal{P}(\mathbb{R}))$ and Θ is regular there, then Martin's maximum (MM) holds in the corresponding $\mathbb{P}_{\max}$ extension for all posets of the size of the continuum, i.e., $\aleph_2$. As the chapter proceeds, the use of the analysis of strong forms of determinacy becomes more and more involved, culminating in a section producing consistency results for ideals on ω_2 which is one of the most technically difficult parts of the book. Chapter 10 considers the relationship between MM and axiom $(*)$. While it was shown by the reviewer that MM itself does not imply $(*)$, the question whether certain technical strengthenings of MM do imply $(*)$ appears to be very deep. As with the Ω-conjecture, the fundamental issue is whether everything that can hold in a suitably robust model can be obtained by forcing, and this in turn depends on the existence of certain iteration strategies. Coding techniques are introduced which are useful for showing that $(*)$ fails under certain circumstances, and a reformulation of $(*)$ is given in terms of a bounded form of MM. Finally, Ω-logic is introduced.

Several factors make the book a difficult read. First, the reader is presumed to be familiar with more set theory than that contained in the Jech (JSL XLVI 876) or Kunen (JSL LI 462) texts. Familiarity with more recent developments in descriptive set theory and large cardinals is required. Also, many theorems are stated without proof. In some cases, these are black boxes which would take another book to fill. For instance, the existence of $\mathbb{P}_{\max}$ conditions in the generality suitable for the main arguments of the book requires deriving inner models with Woodin cardinals from determinacy, the subject of an upcoming book from Woodin, Mathias, and Hauser. In some cases, the theorems are beyond the scope of the book, and are given as pointers towards further topics and future works—the theorems on Ω-logic, for instance. Also, due to length considerations, certain proofs (and indeed, entire chapters of the original manuscript) were omitted. Lastly, the results in the book are often stated as pieces of a larger framework, and the reader not familiar with this framework may miss the important points, one example being the absoluteness theorems in Chapter 4.

To be certain, the book reflects a monumental achievement, one that plainly strains the capacity of the medium in which it is presented. Its most apparent contribution is the introduction of the counterpoint to L for combinatorics on ω_1, the $\mathbb{P}_{\max}$ extension of $L(\mathbb{R})$. It also introduces a powerful new method for proving consistency results for the first two uncountable cardinals. But as discussed in the last chapter it also fits into a larger project whose significance has yet to be fully realized, concerning absoluteness, canonical models, and even truth in set theory.

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R. Dougherty and A. S. Kechris. *Hausdorff measures and sets of uniqueness for trigonometric series*. ***Proceedings of the American Mathematical Society***, vol. 105 (1989), pp. 894–897.

Alexander S. Kechris and Alain Louveau. *Covering theorems for uniqueness and extended uniqueness sets*. ***Colloquium mathematicum***, vol. 59 (1990), pp. 63–79.

Alexander S. Kechris. *Hereditary properties of the class of closed sets of uniqueness for trigonometric series*. ***Israel journal of mathematics***, vol. 73 (1991), pp. 189–198.

A. S. Kechris and A. Louveau. *Descriptive set theory and harmonic analysis*. ***The journal of symbolic logic***, vol. 57 (1992), pp. 413–441.

It is a difficult (and pointless) task to draw the boundary between logic and the rest of mathematics. Part of the difficulty is due to the field of descriptive set theory. This field, which is about a hundred years old, has its origins in analysis. The first descriptive set theorists were analysts, and in its early days, the field was a subfield of analysis. The name of the field derives from this era, when an analyst, Lusin, divided the theory of real functions into two parts, which he called the *metric theory* and the *descriptive theory*. Later, the field became a subfield of topology; indeed, the canonical reference to the field was Kuratowski's book, entitled ***Topology*** (Volume I, PWN and Academic Press, 1966; see JSL IV 136(17)). In the last third of the twentieth century, most descriptive set theorists have been logicians, and the field is now usually considered to be a subfield of mathematical logic. At the beginning of this third era, logicians studied descriptive set theory for its own sake, or for its connections with other parts of logic. That changed in the mid-eighties, when logicians began looking at connections between various branches of analysis and descriptive set theory (including "logical" aspects of descriptive set theory, such as recursion-theoretic concepts and strong set-theoretic axioms).

Descriptive set theory not only returned to its roots in analysis, but returned to the branch of analysis in which the theory of sets began: trigonometric series. Some of the most important work done on connections between descriptive set theory and analysis involves certain types of exceptional sets, that is, notions of smallness, which occur in the study of these series. Two types of exceptional sets are the *sets of uniqueness*, or U-sets, and the *sets of extended uniqueness*, or U_0-sets. The sets under consideration are subsets of the circle, $\mathbb{T}$. A set E is an M-set if there is a non-zero trigonometric series that converges to 0 outside E, and is a U-set otherwise. A set E is an M_0-set if E supports a non-zero measure μ with Fourier coefficients $\hat{\mu}(n)$ converging to 0 as $|n| \to \infty$, and is a U_0-set otherwise. All M_0-sets are M-sets, since $\Sigma\, \hat{\mu}(n)e^{inx}$ converges to 0 off the set. Most of the theory concerns closed sets. The closed U- and U_0-sets form two pointsets in the Polish space of compact subsets of $\mathbb{T}$, and the connection between descriptive set theory and exceptional sets in harmonic analysis begins with the following theorem of Kaufman and Solovay: Both pointsets are true $\mathbf{\Pi}^1_1$. Those with a serious interest in this topic should read the 1987 book, ***Descriptive set theory and the structure of sets of uniqueness***, by A. S. Kechris and A. Louveau (JSL LVI 344).

All four papers under review were written after the appearance of that book. The first three are technical mathematical papers, written for a reader who is reasonably familiar with that book's contents. These three papers will be discussed individually, below. The fourth paper is different. It is a survey/expository paper, a written version of a short course given by Kechris and Louveau at the 1989 European summer meeting of the Association for Symbolic Logic. There is a substantial overlap between this paper and the Kechris–Louveau book. In addition, this paper discusses work done after the appearance of the book, including results in the first three papers under review. This paper is an excellent survey, clearly written, comprehensive and up to date (in 1992); but in contrast to the book, complete proofs are not given.

All sets of positive Lebesgue measure are M_0-sets, but there exist M_0-sets of measure 0. We consider a strengthening of this latter fact. A *Hausdorff determining function* is any non-decreasing function $h : (0, \infty) \to (0, \infty)$ with $h(0^+) = 0$. A set $E \subset \mathbb{T}$ has *Hausdorff h-measure* 0 if for every $\varepsilon > 0$ there is a sequence $\{I_n\}$ of open arcs in $\mathbb{T}$ with $E \subset \bigcup_n I_n$ and $\Sigma\, h(\text{length}(I_n)) < \varepsilon$. Kaufman proved that for any h as above, any closed M_0-set has a closed subset which is an M_0-set of Hausdorff h-measure 0. He asked whether the same result holds for M-sets. In the first paper under review, the authors provide a negative answer.

The second paper is concerned not only with U- and U_0-sets, but with two additional classes of exceptional sets, called U_0'- and U_1'-sets, which will not be defined here. It was known that $U_1' \subset U_0' \subset U_0$, but that neither U_1' nor U is a subclass of the other. This paper is mainly about two covering theorems. First, it contains a new proof of the theorem of Debs and Saint Raymond that every $\mathbf{\Sigma}^1_1$ U_0-set can be covered by countably many closed U_0'-sets. Secondly, it contains the new theorem that every $\mathbf{\Sigma}^1_1$ set which includes no closed M-set can be covered by countably many closed U_1'-sets. The first of these results enabled Debs and Saint Raymond to prove that every U_0-set with the property of Baire is meager in $\mathbb{T}$, thereby solving a problem published in 1927. Their proof provided a beautiful example of an *application* of descriptive set theory, solving a problem whose statement makes no mention of descriptive set theory. Readers of this journal may be disappointed to learn that in this paper, Kechris and Louveau have eliminated the descriptive set theory and produced a proof which uses only harmonic analysis, functional analysis, and general topology.

The collection of closed U-sets is a σ-ideal, a subcollection of the σ-ideal of closed U_0-sets. By the Kaufman–Solovay theorem, neither σ-ideal is Borel. When $G \subset \mathbb{T}$ is G_δ, the σ-ideal I^G of closed subsets of G is Borel. By the Debs–Saint Raymond theorem, if G is dense in a neighborhood of $\mathbb{T}$, then I^G is not included in the σ-ideal of U_0-sets. The third paper contains a theorem that generalizes both the Kaufman–Solovay theorem and the Debs–Saint Raymond theorem: If I is a σ-ideal of closed U_0-sets and the singletons in I are dense in an M_0-set, then I is not Borel.

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Advances in modal logic, Volume 1, edited by Marcus Kracht, Maarten de Rijke, Heinrich Wansing, and Michael Zakharyaschev, CSLI lecture notes, no. 87, Center for the Study of Language and Information, Stanford 1998, also distributed by Cambridge University Press, New York, xvi + 392 pp.—therein:

Alberto Artosi, Paola Benassi, Guido Governatori, and Antonino Rotolo. *Shakespearian modal logic: a labelled treatment of modal identity*. Pp. 1–21.

Philippe Balbiani. *Terminological modal logic*. Pp. 23–39.

Patrick Blackburn and Jerry Seligman. *What are hybrid languages?* Pp. 41–62.

Lilia Chagrova. *On the degree of neighborhood incompleteness of normal modal logics*. Pp. 63–72.

Giovanna D'Agostino and Marco Hollenberg. *Uniform interpolation, automata and the modal μ-calculus*. Pp. 73–84.

Carsten Grefe. *Fischer Servi's intuitionistic modal logic has the finite model property*. Pp. 85–98.

Bernhard Heinemann. *Topological nexttime logic*. Pp. 99–113.

Oliver Lemon and Ian Pratt. *On the incompleteness of modal logics of space: advancing complete modal logics of place*. Pp. 115–132.

Larisa Maksimova. *Interpolation in superintuitionistic and modal predicate logics with equality*. Pp. 133–140.

Maarten Marx. *Mosaics and cylindric modal logic of dimension 2*. Pp. 141–156.

AIDA PLIUŠKEVIČIENĖ. *Cut-free indexed calculi for modal logics containing the Barcan axiom*. Pp. 157–172.

RICCARDO ROSATI. *Minimal knowledge states in nonmonotonic modal logics*. Pp. 173–187.

RENATE A. SCHMIDT. *Resolution is a decision procedure for many propositional modal logics*. Pp. 187–208.

VALENTIN SHEHTMAN. *On strong neighbourhood completeness of modal and intermediate propositional logics. (Part I)*. Pp. 209–222.

HIROYUKI SHIRASU. *Duality in superintuitionistic and modal predicate logics*. Pp. 223–236.

VLADIMIR V. SPANOPULO and VLADIMIR A. ZAKHAROV. *On the relationship between models of parallel computations*. Pp. 237–248.

TIMOTHY J. SURENDONK. *On isomorphisms between canonical frames*. Pp. 249–268.

DIMITER VAKARELOV. *Hyper arrow structures. Arrow logics III*. Pp. 269–290.

YDE VENEMA. *Atom structures*. Pp. 291–305.

ALBERT VISSER. *An overview of interpretability logic*. Pp. 307–359.

FRANK WOLTER. *Fusions of modal logics revisited*. Pp. 361–379.

This collection brings together twenty-one of the papers presented at the conference on Advances in Modal Logic held in Berlin in 1996. The articles cover areas of interest to philosophers, mathematicians, computer scientists, and linguists working in semantics.

Some of the papers prove results, such as Larisa Maksimova's *Interpolation in superintuitionistic and modal predicate logics with equality*, Renate Schmidt's *Resolution is a decision procedure for many propositional modal logics*, and Timothy Surendonk's *On isomorphisms between canonical frames*. Others, in particular Patrick Blackburn and Jerry Seligman's *What are hybrid languages?* and Albert Visser's *An overview of interpretability logic*, survey the then-current state of an area.

I will not attempt to summarize every paper in the collection. Rather I will choose a few to discuss in some depth. These choices are based on my own interests and abilities.

According to Blackburn and Seligman, a hybrid language is a language that contains variable binders as well as modal operators. Variables themselves range over worlds but can stand as formulae by themselves. I suppose that one rather conservative way to think of this is to consider a world to be a very specific proposition. The binders discussed include the usual quantifiers, but also include more exotic items such as $\downarrow$, which picks out the world at which a formula is being evaluated. Thus we have the truth condition

$$M, g, t \models \downarrow x\varphi \quad \text{iff} \quad M, g', t \models \varphi$$

where $g'(x) = t$ and g' agrees with g on the assignment of all values to variables except possibly x (p. 45). $\downarrow$ can be used to formulate a logic for indexicality. The authors also discuss the binder @, where $@x\varphi$ means that φ holds at the world x. They use these binders in combination with the usual quantifiers and F (future) from temporal logic. A past operator is then defined as is an until operator.

In his *An overview of interpretability logic*, Albert Visser introduces the reader to logics with a binary operator, $\rhd$ (following the work of Vítězslav Švejdar). For two theories U and V, $U \rhd V$ means that U can be interpreted in the theory V. More precisely, $\mathcal{M} : U \rhd V$ means that U can be interpreted in V relative to the translation $\mathcal{M}$. Thus, for example, arithmetic can be interpreted in set theory via a translation of the various numbers into sets (such as 0 translated to the empty set) and the various operations into set-theoretic operations. Visser uses a wide variety of theories to demonstrate the usefulness of a theory of interpretation, such as Gödel's interpretation of ZF + $V = L$ in ZF and the interpretation of two-dimensional elliptical geometry in three-dimensional Euclidean geometry. Interpretability logic (IL) is a propositional modal logic with the usual operators and $\rhd$. A Hilbert-style axiomatisation is provided. (Slightly more motivation for the axioms would be helpful, since it contains some odd axioms such as $\Diamond\varphi \rhd \varphi$.) A semantics (due to Frank Veltman) is presented and a variety of interesting theorems are proved about IL and alternative formulations of the logic of interpretation.

Oliver Lemon and Ian Pratt's *On the incompleteness of modal logics of space: advancing complete modal logics of place*, is an interesting survey of the literature on modal logics that formalise spatial notions like "near," "inside," "elsewhere," and "touching." They also present a semantics for spatial logics that takes as its indices regions in either $\mathbb{R}^2$ or $\mathbb{R}^3$. Thus, for example, if we want to formalise "touching," we have an accessibility relation R that connects regions if and only if they are touching (this relation will be symmetrical and serial). We then read $\Diamond p$ as saying "p in a region that touches the present one." The paper surveys incompleteness results for logics of "elsewhere," distance (where $\Diamond p$ is read as "p a distance d from here in a straight line"), and inclusion (where $\Diamond p$ is taken to mean "p somewhere inside this region"). They then develop a logic of "planar contact," in which regions are accessible if they have exactly one point in common. Lemon and Pratt describe an axiom system and a completeness theorem for their set of planar contact structures. They then go on to show how these structures can be used to formalise other spatial notions such as inclusion.

In addition to the survey articles, there are also a few articles that combine a presentation of a class of modal logics with technical results about the members of that class. Here, for example, we have Berhard Heinemann's *Topological nexttime logic*, which looks at a system that includes both the operator $\bigcirc$ (where $\bigcirc\alpha$ means "α at the next point in time") and a knowledge operator. The semantics for this system—a development of the subset space semantics of Moss and Parikh—and completeness is proved. Similarly, Philippe Balbiani's paper introduces terminological modal logic and uses Max Cresswell's subordination frame to prove completeness.

The more technical papers span areas such as logics for parallel computation, intuitionist logic, and arrow logics, as well as more standard modal logic.

This collection is a useful resource for anyone working in modal logic. It contains both interesting surveys and cutting-edge technical results.

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FRED SOMMERS and GEORGE ENGLEBRETSEN. ***An invitation to formal reasoning. The logic of terms.*** Ashgate, Aldershot, Burlington, Singapore, and Sydney, 2000, xvi + 260 pp.

This book is intended to be an introduction to the logic of terms. But it is much more. Authored by the two foremost authorities on this subject, it is an up-to-date definitive statement of terminist philosophy.

The logic of terms (also known as term logic or term-functor logic, TFL) has its roots in Aristotelian syllogistic and its medieval development, here referred to as "traditional logic."

Traditional logic bears an obvious correspondence to natural language and has an intuitive appeal that facilitates understanding. Unfortunately, it lacks the expressiveness necessary to reason about much of mathematics and science. Many attempts were made to remedy this, particularly when the need for sound foundations for mathematics became apparent. The most successful began with Frege and grew into today's modern predicate logic (MPL). Rather than extending traditional logic, MPL swept it aside, relegating it to a historical curiosity.

TFL is designed to extend traditional logic, giving it additional expressiveness, yet retaining the direct correspondence to natural language and the intuitive appeal of traditional logic. It is the authors' contention that neither TFL nor MPL should replace the other, but rather that each offers unique advantages that justify its use.

On the one hand, MPL excels as the language of mathematics, but has lost a direct correspondence to natural language and an intuitive appeal. TFL, on the other hand, retains a direct correspondence to natural language and an intuitive appeal. The book shows that TFL is equivalent in expressiveness to MPL. One can be translated quite simply to the other. This allows TFL to function as a "bridge" between natural language and MPL. The book uses this bridge to introduce the reader to MPL. The bridge also induces a more intuitive understanding of reasoning in MPL.

Moreover, TFL provides a more gentle introduction to formal logic. This can be especially important for students whose mathematical background is deficient and whose motivation is strongly tied to a perception of immediate relevance (a type of student frequently seen in colleges and universities today). The style of presentation is gentle as well. Rather than the usual "definition-theorem-proof" style, this book uses multiple examples and intuitive arguments to support the rules and principles.

To provide more detail, a bit of background is needed. Traditional logic regiments knowledge into four types of *categorical sentence*.

A. all X are Y	E. no X is Y
I. some X is Y	O. not all X are Y

The X and Y are *terms*. The terms are combined by a *functor*, for example, "all ... are ...." The functor may also include a *sign of quality*, as in "some X is not Y," and a *sign of judgment* as in "it is the case that some X is Y" or "it is not the case that some X is Y." (The positive judgment is usually elided.)

It is easy to see that traditional logic cannot express relations. To incorporate relations, it is necessary to employ variables or some surrogates to coindex the arguments of relations. For example, consider the sentence "Every boy likes some girl." In MPL, this sentence is construed:

$$\forall x_1(Bx_1 \rightarrow \exists x_2(Gx_2 \wedge Lx_1x_2)).$$

The logic of terms might employ variables explicitly as in restricted quantification logic

$$\forall(x_1 : Bx_1)(\exists(x_2 : Gx_2)Lx_1x_2)$$

or it might employ subscripted indices:

$$\text{all } B_1(\text{some } G_2L_{12}).$$

These term logic sentences are said to be in *dyadic normal form* because each subject expression is followed by its predicate expression. With appropriate conventions, the syntax can be loosened to bring it still closer to English:

$$\text{all } B_1(L_{12} \text{ some } G_2).$$

Sommers chooses to employ subscripted indices. He then goes much further. The term-combining functors are given algebraic representation: 'all' is represented by '$-$', 'some' by '$+$', 'not' (either as a sign of quality or as a sign of judgment) by '$-$', and parts of the verb 'to be' by '$+$'. Thus, the present example has the algebraic representation

$$-B_1 + (+G_2 + L_{12}).$$

This algebraic representation is introduced early in the book as an integral part of the logic of terms. It makes possible inclusion of propositional logic, since X **and** Y is shown to behave like $+X+Y$ and **if** X **then** Y is shown to behave like $-X+Y$. It follows by De Morgan's law that X **or** Y behaves like $-((-X)+(-Y))$, conventionally written $--X--Y$. Now both quantificational logic and propositional logic coexist in a logic that bears a close resemblance to traditional logic.

When appropriate rules are observed (these rules are developed in the book), formal reasoning is reduced to algebra. For example,

$$+[-X + Y] + [-Y + Z] = -X + Z.$$

Relational arguments also reduce to calculation. Consider the premisses "Every boy likes a girl" and "Every girl owns a dog." In TFL, these are construed:

$$+[-B_1 + (L_{12} + G_2)] + [-G_2 + (O_{23} + D_3)] = -B_1 + (L_{12} + (O_{23} + D_3)).$$

That is, these premisses imply "Every boy likes some owner of a dog." Note that because G in the second premiss is *distributed* (i.e., universal), it may be assigned any subscript. Of course, the way in which subscripts are assigned is non-deterministic and can affect the outcome of the calculation.

This example also illustrates the overloading of the operators $+$ and $-$. That is, '$+$' is the algebraic representation of both 'some' and '**and**'; '$-$' is the algebraic representation of both 'all' and '**not**'. While this overloading facilitates the reduction of reasoning to algebraic calculation, it also requires restrictions that are not found in arithmetic. For example, $-(-X + Y) = +X + (-Y)$, but $-(-X) + Y \neq +X + Y$.

It should be pointed out that the logic of terms does not mandate algebraic representation. Alternative development of the logic of terms based on the logic of restricted quantification is also possible. In this alternative formulation, the logical principle of monotonicity along with rules of passage for quantifiers are used in place of the algebraic calculation in this book. This alternative is developed, with similar emphasis on correspondence to natural language, in McKay's ***Modern formal logic*** (Macmillan, 1989).

The rather informal style of the book can be a source of frustration for readers who prefer formal presentation. For example, "term" and "dyad" are concepts central to the development of the logic of terms, but formal definitions are not to be found. The book covers the logic of terms (TFL), modern predicate logic (MPL), and semantic trees. But many of the basic results of mathematical logic such as compactness and Löwenheim–Skolem theorems are not covered.

The rather informal development may have contributed also to a lack of generality of some of the principles. Two cases in point will be mentioned: the principle of equivalence and the dictum de omni.

The principle of equivalence (PEQ) states (page 62): two sentences are logically equivalent iff they are covalent and equal. A sentence has positive valence if it asserts the existence of something. For example, "Some woman is a farmer" and "Some boy likes every dog" have positive valence. A sentence has negative valence if it denies the existence of something. For example, "No human is immortal" and "Nobody likes a poor loser" have negative valence. A rule for determining valence is given on page 57: "Look at the judgment sign and then look at the sign of quantity: if these signs differ, the valence is negative; if they are the same, the valence is positive." Two sentences are equal if they are algebraically equal as term logic sentences.

For example, PEQ shows that "Every president is a citizen" (in TFL, $-P + C$) is logically equivalent to "No noncitizen is a president" (in TFL, $-(+(-C) + P)$) because the two term logic sentences are algebraically equal and have the same (negative) valence. But page 143 contains a counterexample. By PEQ it would follow that "Anything that's either A or M is D" (algebraically, $-\langle - - A - -M\rangle + D$) is logically equivalent to "Every M is either not A or D" (algebraically, $-M + \langle -A - -D\rangle$). But it is easy to see that the Venn diagrams for the two sentences are different. Thus the principle of equivalence is not general.

The dictum de omni (DDO) is perhaps the centerpiece of the logic of terms. It generalizes Aristotle's dictum: what is true of every X is true of whatever is an X. The generalization to

TFL is most succinctly given in so-called "host-donor form":

$$E(+M),\ E^*(-M)\,/\,E(E^*)$$

where M is the *middle term*, $E(+M)$ is the *host sentence*, and $E^*(-M)$ is the *donor sentence*. E^* is the donor sentence with $-M$ deleted, i.e., the environment or context of $-M$ in the donor sentence. $E(E^*)$ is the host sentence with E^* substituted for M in the host sentence.

Application of the DDO can be very impressive, as illustrated by the following example (page 144). The host sentence is "Every woman kisses some child" (in TFL, $-W_6+K_{67}+C_7$). The donor sentence is "Some sailor is giving every child a toy" (in TFL, $+S_1+G_{123}-C_2+T_3$). With C as the middle term, $E(\cdots) = -W_6 + K_{67} + \cdots$ and $E^* = +S_1 + G_{123} + T_3$. Replacing '2' by '7' (permitted because C is universally distributed in the donor sentence) gives $E(E^*) = -W_6+K_{67}+(S_1+G_{173}+T_3)$, which is the algebraic representation of "Every woman kisses someone a sailor is giving a toy to."

Why DDO works can be seen as follows. Since M is universally distributed in the donor sentence, the rules of passage for quantifiers permit moving $-M$ left to the head of the donor sentence. Then the principle of monotonicity (also known as the "substitutivity of implication rule") permits substitution of the donor context (which is the consequent of the implication) for M in the host sentence.

But like the principle of monotonicity, to enjoy full generality, DDO must be defined recursively. The problem is shown by the following example. The host sentence is "Some man is father of some girl" (algebraically, $+M_1 + \langle F_{12} + G_2\rangle$), or in dyadic normal form, $+M_1 + \langle G_2 + F_{12}\rangle$. The donor sentence is "If a man[1] is father of a girl[2] then he$_1$ is parent of her$_2$" (algebraically, $-[M_1' + \langle F_{12} + G_2''\rangle] + [M_3' + \langle P_{34} + G_4''\rangle]$), which is equivalent (in TFL) to $-M_1 + \langle -G_2 + \langle -F_{12} + P_{12}\rangle\rangle$. With F as the middle term,

$$E(E^*) = +M_1 + \langle G_2 + \langle -M_1 + \langle -G_2 + P_{12}\rangle\rangle\rangle$$

which entails the obviously false $-M_1 + \langle -G_2 + P_{12}\rangle$, i.e., "Every man is parent to every girl." But the problem is avoided if DDO is defined recursively.

On balance, this book is stimulating and thought-provoking. It would be an interesting experience to use it to teach an introductory course in formal logic. In addition to the copious examples, the book contains after each section a set of exercises to test comprehension. The exercises are given in natural language form that will cultivate critical thinking skills. As a text, it is unfortunate that it has no index. It does, however, have a detailed table of contents and a reasonably complete table of rules, laws, and principles. The book contains a number of typographical errors, including disruption of some of the semantic trees. But these shortcomings as well as the lapses in PEQ and DDO could be removed easily in a subsequent edition.

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ALASDAIR URQUHART. *The complexity of linear logic with weakening*. ***Logic Colloquium '98, Proceedings of the annual European summer meeting of the Association for Symbolic Logic, held in Prague, Czech Republic, August 9–15, 1998***, edited by Samuel R. Buss, Petr Hájek, and Pavel Pudlák, Lecture notes in logic, no. 13, Association for Symbolic Logic, Urbana, and A K Peters, Natick, Mass., 2000, pp. 500–515.

There have been many surprises connected with propositional linear logic. First Lincoln, Mitchell, Scedrov, and Shankar (*Decision problems for propositional linear logic*, ***Annals of pure and applied logic***, vol. 56 (1992), pp. 239–311) showed how to simulate computations in linear propositional logic and thereby proved that it was undecidable. Later Kopylov (*Decidability of linear affine logic*, ***Proceedings, Tenth annual IEEE symposium on Logic in***

Computer Science, June 26–29, 1995, San Diego, California, IEEE Computer Society Press, 1995, pp. 496–504) proved the surprising result that linear logic with weakening was decidable. Lafont has given another proof of it (*The finite model property for various fragments of linear logic*, ***The journal of symbolic logic***, vol. 62 (1997), pp. 1202–1208).

The present paper investigates the complexity of linear logic with weakening, LLW, and gives the best results so far. The author proves that linear logic with weakening is exponential-space hard.

The method in the paper is to give a problem that is exponential-space complete and that can be simulated in propositional linear logic with weakening. This is done by going through the following systems: TUR, ESC, CSTS, LLW$^-$, LLW. Let us look at these in turn.

The starting point is a problem TUR which is exponential-space complete. This is the halting problem for Turing machines space-bounded by $2^{2^{O(x)}}$. Then this is simulated by ESC, the halting problem for 3-counter machines with similar space bound. All this is of course standard and is simply referred to in the paper. The next step is perhaps the crucial one. CSTS stands for commutative, contractive, semi-Thue systems, which are semi-Thue systems where we have both commutativity and contraction rules $(xy \to x)$. For the simulation of 3-counter machines in CSTS we have the problem of representing tests of 0. In the usual representation we can represent only positive information—that a counter contains something. This is similar to the key problem in proving the undecidability of linear logic. As there, we use a representation where the only successful derivations are those where the tests of 0 are all right. Then the derivations in CSTS are simulated by cut-free derivations in linear logic with weakening, LLW$^-$. The last step is to give a cut-elimination result for LLW.

As a side note we observe that we need a slightly strengthened cut-free calculus. We need a sequent calculus using some extra axioms and admitting cut elimination. This was also needed in proving the undecidability of linear logic. This is a common situation and has recently been cleared up by the work of Sara Negri and Jan von Plato (***Structural proof theory***, Cambridge University Press, 2001).

The author mentions some of his work in giving doubly-exponential space as an upper bound for the complexity of LLW. The decidability proof in Kopylov's paper cited gives a bound primitive recursive in the Ackermann function. The author mentions that this can be refined considerably, to give a doubly-exponentially space bound, but he has since withdrawn that claim.

From this paper we know more about the complexity of linear logics. It would of course be nice to get an exact bound, but so far we do not know how that should be done.

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RICHARD L. EPSTEIN and WALTER A. CARNIELLI. ***Computability. Computable functions, logic, and the foundations of mathematics.*** The Wadsworth & Brooks/Cole mathematics series. Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, Calif., 1989, xvii + 297 pp.

RICHARD L. EPSTEIN and WALTER A. CARNIELLI. ***Computability. Computable functions, logic, and the foundations of mathematics.*** Second edition of the preceding. Wadsworth, Belmont, Calif., etc., 2000, xii + 299 + 38 pp.

RICHARD L. EPSTEIN. *Computability and undecidability—a timeline*. Therein, pp. 1–38.

The main purpose of this book is to give an introductory presentation of the theory of computable functions (of the natural numbers into the natural numbers) and to develop the fundamental aspects of logic needed to prove incompleteness, undecidability, and non-provability of consistency of some systems of arithmetic. This is a textbook appropriate for an undergraduate course, but supplemented with some extra material, it could be used also in an introductory graduate course.

One thing often neglected in mathematics courses is encouragement for the students to go to the sources and read (at least parts of) the original texts and papers on the subject. This is unfortunate since usually the intuitive ideas of the creators of the subject remain the best motivating guides to understanding its development. Reasons for the neglect are that the original texts are not always easily available, and that usually the pace of the course does not allow the teacher to devote time to historical digressions. One of the many virtues of the book under review is precisely that it includes extensive excerpts from articles and books and other original sources that provide sound motivation and put the material in a historical and philosophical perspective.

The concern with historical aspects is enhanced in the second edition of the book with a "timeline" of the development of the theory of computable functions and the undecidability of arithmetic up to 1970, together with a corresponding bibliography. The second edition also corrects a few technical errors which found their way into the first edition.

The book is divided into four parts. Part I, *The fundamentals*, sets the general background on which the other parts of the book are based. It starts with the assertion that modern logic has developed as a response to paradoxes and problems in the foundation of mathematics, and goes on with a nice informal presentation of self-referential paradoxes and of Zeno's paradox. Whole numbers, functions, and proofs receive a brief introduction, taking the whole numbers as well as the operation of "adding one" as primitive undefined terms. Functions are viewed both as rules and as collections of ordered pairs, and the λ-notation, which is used extensively in the book, is introduced here. Concerning proofs, the authors say that "most mathematicians do not concern themselves with clarifying precisely the notion of proof ... they know intuitively what is a correct proof and what is not." They briefly describe some basic forms of argumentation used in mathematics: mathematical induction, proof by contradiction, proof by construction; with special comments on proofs of existence by contradiction. The treatment of functions and proofs is elementary, perhaps too elementary for a reader expected to be able to master the more advanced chapters.

The concepts of finite, infinite, countable, and uncountable are also dealt with, after which extensive extracts from the article *On the infinite* by David Hilbert (108***13***) are reproduced with the warning that the text is long and difficult to grasp on a first reading. The inclusion of this text is justified since Hilbert's concern with the uses of the infinite in mathematics is at the heart of the problems of decidability and consistency of arithmetic treated in the third part of the book.

Part II, *Computable functions*, and Part III, *Logic and arithmetic*, constitute the main mathematical portion of the text. Some general considerations about the concept of algorithm precede a chapter on Turing machines and Turing computability. Several examples of Turing machines are developed explicitly, and some familiar functions are shown to be computable by Turing machines. After a short chapter on Church's Thesis, primitive recursive functions are defined: the functions obtained from the initial functions (the constant zero function, the successor function, and the projections) by using the basic operations of composition and primitive recursion. After showing that many familiar functions of arithmetic are in this class, it is explained, via diagonalization, why the primitive recursive functions do not capture the intuitive idea of computable function. These concepts are developed further in two optional sections, one on the Grzegorczyk hierarchy of primitive recursive functions and the other on multiple recursion.

Partial recursive functions are introduced (adding to the basic operations the μ-operator or least search operator) and then three basic results are proved: a normal form theorem, the *s-m-n* theorem, and the recursion theorem. There is a short presentation of recursively enumerable sets, and a detailed proof that the Turing computable functions are exactly the partial recursive functions.

The third part, *Logic and arithmetic*, starts by recalling Hilbert's position regarding the use of infinite collections in mathematics. Hilbert believed their use was amply justified, but felt the need to prove their consistency with finitistic mathematics by using solely finitistic methods. With this purpose in mind, Hilbert set out to formalize mathematics, starting with arithmetic. The main goal of this part of the book is to develop an axiomatic system for (a fragment of) arithmetic arriving at the obstacles confronted by Hilbert's program, namely, Gödel's theorems on incompleteness and unprovability of consistency.

The first step taken in this direction is a presentation of propositional logic. Semantics is based on truth tables, and an axiomatization is given in a way that makes the proof of the completeness theorem an easy task.

The decidability of propositional calculus is proved by using Gödel numbers to define a recursive function e such that $e(n) = 1$ if and only if n is the Gödel number of a propositional tautology. The authors explain why the decision procedure obtained is unusable in practice for long formulas, and refer the reader to other sources for more information on complexity of algorithms. Gödel numbers are used again to present proving in the axiomatic system as a computable procedure.

First-order logic is not presented in a general setting; rather, a first-order theory for arithmetic is developed with decidability and incompleteness in mind. In consequence, the basic theorems of first-order logic are just briefly mentioned (the completeness theorem), or not mentioned at all (compactness and Löwenheim–Skolem, for example). The mathematical scope of the book is thus less ambitious than that of other successful texts on the subject like Enderton's ***A mathematical introduction to logic*** (JSL XXXVIII 340; second edition, 2001) or Yasuhara's ***Recursive function theory and logic*** (JSL XL 619). Instead, the reviewed book gives more prominence to the historical development of the subject and its philosophical motivations and implications.

The authors introduce a first-order system, called Q, which formalizes a sufficiently rich portion of arithmetic. Although Q is shown to be too weak to be considered a formalization of arithmetic, it is sufficiently strong so that the recursive functions are representable in the system. System Q is obtained adding to the logical axioms and rules seven arithmetical axioms: the basic properties of the successor function (that it is a one-one function whose range is all the natural numbers except zero) and the inductive definitions of addition and of multiplication. The recursive functions are exactly those functions representable in Q. Rather than showing directly that the functions representable in Q are closed under definition by recursion, the authors use a characterization of the partial recursive functions as the smallest class of functions containing the constant zero, the successor function, the projections, addition, multiplication, and the characteristic function of the equality relation, closed under composition and the μ operator. The essential use of Gödel's β function for coding finite sequences of natural numbers is pointed out. Using this, the authors then show that Q, if consistent, is undecidable, i.e., the set of (Gödel numbers of) theorems of Q is not recursive, and the same applies to any extension of Q, such as PA, Peano arithmetic, obtained adding to Q all first-order instances of the principle of induction. This part ends with the proofs of Gödel's incompleteness theorems.

The last part of the book, *Church's Thesis and constructive mathematics*, consists of two chapters of historical and philosophical character, composed mainly of quotes from a variety of authors. The first of these chapters discusses Church's Thesis and in particular whether the identification of the computable functions with the recursive functions (or the Turing computable functions) should be taken as a thesis or as a definition. The second presents several constructivist views of mathematics, most notably Brouwer's intuitionism and Bishop's constructivism, together with several criticisms thereof.

The book is very well written, with clear and careful explanations. Each chapter is supplemented by a collection of exercises and by suggestions for further reading. There is an ***Instructor's manual***, a separate booklet, containing brief comments on how to use each chapter, together with solutions for the exercises. As is frequently the case with good textbooks, this one admits several different uses, depending on which parts of the book are covered. It is a very nice text for an introductory course in computability and logic for students of mathematics, computer science, or philosophy.

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MARTIN DAVIS. ***Engines of logic. Mathematicians and the origin of the computer.*** Paperbound reprint with altered title of BSL VII 65. W. W. Norton & Company, New York and London 2001, xii + 257 pp.

This is a paperback re-issue of the author's recently published ***The universal computer***. The title was changed at the behest of the publisher, and a number of errors in the hardback original, including all those noted in my review in this BULLETIN (BSL VII 65), have been corrected.

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ANITA BURDMAN FEFERMAN. ***From Trotsky to Gödel. The life of Jean van Heijenoort.*** Paperbound reprint with altered title of JSL LVIII 1465. A K Peters, Natick, Mass., 2001, xv + 415 pp.

SOLOMON FEFERMAN. *Jean van Heijenoort's scholarly work, 1948–1986*. A reprint of JSL LVIII 1465(2). Therein, pp. 371–390.

This book was originally published in 1993 with the title ***Politics, logic, and love***. For a discussion of the contents of the book—unchanged in the reprint—see the review JSL LVIII 1465.

Jean van Heijenoort led several lives: three, according to the original title of this biography. The new title incorporates only two. It will, however, be recognized—which is what the publisher intended—as an echo of van Heijenoort's own ***From Frege to Gödel***.

By whatever name, the book remains a remarkable biography of an extraordinary logician.

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