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THE PROBLEM OF LOGICAL CONSTANTS

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Abstract. There have been several different and even opposed conceptions of the problem of logical constants, i.e., of the requirements that a good theory of logical constants ought to satisfy. This paper is in the first place a survey of these conceptions and a critique of the theories they have given rise to. A second aim of the paper is to sketch some ideas about what a good theory would look like. A third aim is to draw from these ideas and from the preceding survey the conclusion that most conceptions of the problem of logical constants involve requirements of a philosophically demanding nature which are probably not satisfiable by any minimally adequate theory.

§1. Introduction. There is among many philosophers of logic a feeling that a theory of logical constants is necessary, and a considerable number of theories have been proposed in reaction to that feeling. This paper does not intend to offer a full theory of logical constancy (although it contains some remarks in this direction), and one of its main aims is instead to clarify a conceptually prior question: what are the requirements that a good theory of logical constants ought to satisfy. Put somewhat rhetorically, I propose to ask here not so much what is a logical constant as what is the problem of logical constants. The reason why I think that illumination is needed in this area is that the question of what is the problem of logical constants, i.e., of what are the requirements that a theory of logical constants ought to satisfy to count as a good theory, is rarely dealt with in an explicit way by the authors who have worked on the problem. This tends to generate the impression that a common conception of this question has been assumed by all of them. But this is far from true. I think that even a relatively superficial examination of the literature should reveal that that question has been implicitly conceived of in disconcertingly different ways: there have been several disconcertingly different (often incompatible) implicit views as to what the criteria would be

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for calling a theory good. This paper distinguishes and clarifies these different conceptions of the problem of logical constants, and surveys the theories they have given rise to. The distinctions and clarifications proposed here are also used to draw the conclusion that under most of these conceptions, which impose philosophically demanding requirements on a good theory, the problem is probably unsolvable, and that perhaps only a less demanding conception of the problem admits of a correct, although philosophically not too significant answer.

Logic is primarily concerned with arguments, with finding ways of distinguishing correct arguments from incorrect ones, and with finding methods of effectively telling apart the correct from the incorrect arguments. The notions of argument, of correct argument, etc. are notions of logic, just as the notions of natural number and integral are notions of mathematics and the notions of substance and cause are notions of metaphysics. The problem of logical constants is not primarily the problem of demarcating which notions of this sort are the notions studied by logic, it is not the problem of what is logic. We have a reasonably clear, if vague, intuitive idea of what logic does and of what are the notions it studies, just as we have similar ideas concerning other disciplines. The problem of logical constants, under any of the potentially interesting ways of looking at it, is a more specific problem —although it is indirectly relevant to the problem of what is logic.

At least under most views, logic is concerned only with certain kinds of correct arguments, and in fact only with arguments whose correctness is due to the peculiar properties of the expressions in a certain set. 'Some Greeks are mortal; therefore some mortals are Greek' is a logically correct argument, due to the peculiar properties of 'some'. 'Some widows are musicians; therefore some females are musicians' is a correct argument, but it is not logically correct, since it is correct because of the peculiarities of 'widow' and 'female'; and these are not expressions that logic should deal with as directly responsible for the logical correctness of arguments. In the most general, least theory-laden conception of it that seems possible, the problem of logical constants is the problem of demarcating in some principle-based, non-arbitrary-looking way the set of expressions that logic should deal with as directly responsible for the logical correctness of arguments[1] —as distinguished from the set of expressions that logic actually deals with at any particular moment of its history.

Is there a principle-based intuition behind choices of expressions for logical study? I believe there is (or at least there has often been).[2] But I also

[1]To be sure, there is a common tendency, especially in recent times, to speak as if every expression had a "logic". From this point of view there is no problem of logical constants, since there is no real demarcation of the mentioned sort, let alone a principle-based one.

[2]Perhaps the tendency mentioned in the previous note is currently too common to think that these choices do not by now lack a principle-based motivation.

believe that the set of expressions logic should deal with is not (and has not been) determined within the logicians' minds by any *philosophically* substantive intuitions or principles (and in particular not any semantic, epistemological or mathematical intuitions or principles). However, some philosophically unloaded and largely pragmatic principle or principles seem to have guided logicians' choices of expressions as logical, either explicitly or (mostly) implicitly. Typically, logic has been thought of as a discipline that is concerned with general reasoning, with reasoning usable in all or at least a wide range of spheres where argument is employed.[3] Hence one principle that it is reasonable to think of as underlying choices of expressions for logical study is the principle that logic should deal with expressions usable in and relevant to general reasoning, expressions not specific to any of the spheres where argument is employed but common to all or a great number of them.[4]

This natural principle suggests just a necessary condition for an expression to be an expression that logic should deal with. Other desiderata seem to have been implicitly at play. Surely prepositions like 'for', 'in' and 'with' and verbs like 'add', 'include' and 'exclude' (in some of their acceptations), to name a few, are usable in and relevant to general reasoning, but they certainly don't belong to the set of expressions usually dealt with by logic and presumably the common view is that they should not be added to this set. Matters of degree and of specific utility must certainly be at play. Logicians may have implicitly applied the desideratum that the expressions they should deal with be *very* relevant to general reasoning, or that their study ought to be *helpful* in the clarification of particularly bothersome confusions or problems in reasoning, to mention just two possibilities.

And probably there are other guiding principles.[5] If this is the right picture, then there is a significant inherent complexity in the intuitive concept of an

[3] Thus Aristotle says: "All the sciences associate with one another in respect of the common items (I call common those which they use as demonstrating from them —not those about which they prove nor what they prove); and logic associates with them all, and so would any science that attempted to prove universally the common items" [1, A 11, 77a26-29; here 'logic' is an appropriate translation of 'dialektiké']; Frege says that "the most reliable way of carrying out a proof, obviously, is to follow pure logic, a way that, disregarding the particular characteristics of objects, depends solely on those laws upon which all knowledge rests" [9, p. 5]. A version of the idea is conspicuous in [29] as well.

[4] An author who has briefly enunciated a similar principle in the recent literature is Hanson, [13].

[5] In a recent paper, [38], Warmbrōd has defended the idea that choices of expressions for logical study are fundamentally determined by the systematic and communicative purposes of the sciences. According to his view, roughly, the expressions logicians choose (or have chosen) are those which are used in an essential way in deductive scientific reasoning (secondly, but it seems less fundamentally, logicians also study expressions that have a more restricted interest, which comes from the need to formalize certain semantic intuitions, like those giving rise to tense logic or modal logic). There is much in Warmbrōd's outlook that I agree with. In

expression that logic should deal with. The principles underlying this concept are subtle and numerous and reflect a variety of fundamentally pragmatic desiderata: general usability, degree of usability, level of utility ... [6] A logical constant may be just an expression which satisfies these desiderata, vague and complex as this characterization may be. Its vagueness and complexity don't suppress the fact that it is principle-based.

Philosophers of logic have typically sought to unearth something more substantive philosophically than the complex pragmatic principles just described, and this is so, in part, because for them the problem of logical constants has typically been something more substantive than the non-theory-laden problem of demarcation that I have mentioned. One makes a step towards a better understanding of the issues in this area when one realizes that the substantive problem of logical constants in the philosophy of logic rarely seems to be the problem of giving a conceptual analysis of some notion which is broadly speaking intuitive, or pretheoretical, such as the notion (or notions) associated with the mentioned pragmatic principles. It is not even the problem of merely extensionally characterizing the intended set of logical expressions in terms of a reasonably broad, philosophically unprejudiced conceptual apparatus.

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particular, it is clear that a strong impulse for the development of logical studies (and thus for choices of logical expressions) has always been the need to systematize scientific theories (Aristotle and Frege are clear examples). Nevertheless, it is no less clear that this impulse has typically been guided by a more general desideratum that logical principles should be relevant outside the sciences and across as wide as possible a range of areas of argument (as the quotations from Aristotle and Frege in a previous note also make clear). Besides, it is evident that other spheres where argument is characteristically employed, like political and legal argumentation, had in antiquity a particular genetic force in the development of logical studies, and they still have a force in that development. A further impulse may have come from the perception that the study of some expressions, for the most part expressions relevant to general reasoning, may help in the resolution of argumentative disputes within parts of philosophy or logic itself (the development of modal logic may be an example). Clearly it's not just science that counts; hence my statement in the main text of somewhat more general pragmatic principles than Warmbrōd's. But perhaps Warmbrōd's more specific pragmatic principles lead as a matter of fact to the same set of expressions as the more general principles. This is a relatively minor point. A more substantial difference between my point of view and Warmbrōd's is explained in the final section.

[6]These principles are less strict and more flexible than other intuitive principles sometimes claimed by philosophers to underlie logicians' choices of expressions. Thus, topic-neutrality or syncategorematicity are often claimed to provide a necessary and sufficient condition for logicality, but this is hard to reconcile with the facts. The natural view seems to be that the identity sign and the membership sign are definitely categorematic, and yet they have been often and decidedly taken to be expressions that logic should deal with. Also, expressions which seem topic-neutral like some prepositions are not intuitively logical, as pointed out in the text. (It should be noted that the sense of 'topic-neutral' in the philosophy of logic literature is an almost non-technical sense different from the more technical sense the term has in the philosophy of mind since Smart. In the non-technical sense, an expression is topic-neutral when it does not denote something specific to a subject-matter.)

The problems are always more theoretically charged. We will point out that in many authors of broadly logicist views the problem was, and is, that of using a notion of logical constancy as a theoretical concept with the help of whose *hypothesized* properties an explanatory theory of the *semantics* and *epistemology* of logic could be given. For other authors, inspired by Tarski's work, the problem was, and is, that of finding a notion whose extension is the traditional set of logical constants, which is characterizable in *mathematical* terms, and which can be used in a mathematical characterization of the notion of logical truth; they are after an explicated notion of logical constancy (and with its help, after an explicated notion of logical truth). In recent authors the problem is less definite than the explanatory and the explicatory problems, but it rarely takes the form of a pursuit of conceptual analysis of a reasonably well isolated pretheoretical notion, or the form of a search for an extensionally correct characterization in terms of concepts which are not required in advance to be taken from (or at least be relevant to) semantics, epistemology or mathematics.

The next section gives a historical survey of the conceptions of the problem of logical constants of a number of broadly logicist authors and of Tarski. Since the main aims of this paper are not historiographical, this section must be very brief.[7] Section 3 discusses critically a number of recent theories of logical constancy, noting that the conceptions of the problem underlying them are often somewhat confusing hybrids of earlier conceptions, and points out a number of serious difficulties facing these theories. The final section 4 offers some considerations in favor of the conclusion that some of these difficulties may be a sign that most conceptions of the problem of logical constants turn it into an unsolvable problem. The essence of these considerations is the claim that one ought not to expect that a set of expressions determined by application of a complex bunch of largely pragmatic principles can be characterized in terms of semantic, epistemic or mathematical properties.

§2. Early conceptions of the problem.

2.1. Logicist conceptions. The substantive philosophical conceptions of the problem of logical constants seem to originate in the work of philosophers of broadly logicist views, such as Bolzano, Russell and Carnap, thus philosophers with a natural interest in explaining the apriority of logic and mathematics through the analyticity of logic (and in explaining or illuminating the analyticity of logic in turn). They seem to be the first to have sought a philosophically substantive theory of logical constants.[8] These philosophers

[7] I hope to be able to publish a much expanded version elsewhere.

[8] This is not to say that philosophical reflections about peculiar classes of expressions of interest to logicians cannot be found earlier. The medieval distinction(s) between syncategorematic and categorematic words, mentioned in a previous note, are an example. But these

were also the first to clearly enunciate (and accept) the idea that a logical truth is just a truth such that all the propositions of the same form are true.

This idea is prominent in Bolzano, one of the first authors to question Kant's claim that all mathematical truths are synthetic and have the source of their apriority in spatio-temporal intuition. That all propositions of the same form as a given proposition are true means for Bolzano that all the results of uniformly replacing the *non-logical concepts* in that proposition are true propositions (see [3, §148]). And Bolzano claimed that "in order to appraise the analytic nature of [these propositions], no other than logical knowledge is necessary, since the concepts which form the invariable [non-replaceable] part of these propositions all belong to logic" ([3, §148, pp. 198-199]). We are going to see that versions of this claim about the peculiar semantic and epistemic properties of logical concepts are found as desiderata on logical constancy in later anti-Kantian authors, fond of the idea that a particular kind of knowledge called logical knowledge is responsible for our knowledge of mathematics; these later authors were of explicitly logicist views.

Russell was perhaps the first to use the expression 'logical constant' (in the first sentence of his *Principles of Mathematics* of 1903), putting forward the thesis that the apriority of mathematical truths follows from the fact that the only constants appearing in these truths (once they are "reduced" in the logicist fashion) are logical constants:[9]

> The fact that all mathematical constants are logical constants, and that all the premisses of mathematics are concerned with these, gives, I believe, the precise statement of what philosophers have meant in asserting that mathematics is *a priori* ([25, p. 8]).

Russell's thesis is that truths containing only logical constants (and variables) must be *a priori*. His implicit idea is that a true proposition containing only non-empirical notions with which we are intimately acquainted must be knowable non-empirically. The reasons for Russell's belief are close to those latent in Bolzano: if a proposition contains only notions (or "constants") of logic, plus variables (and such are the propositions of *pure* mathematics), then, if it is true, its truth must be recognizable through the special kind of knowledge involved in being cognitively acquainted with the logical notions; and this knowledge is presumably logical, certainly non-empirical (although

distinctions, as far as I can tell, bear little relation to any of the ways of understanding (or aiming at understanding) the distinction between logical and non-logical constants in logic.

[9]Russell uses the term 'constant' to refer to notions, not to expressions, although of course he could define an expression to be logical if it denotes a notion which is a logical constant. The reason for Russell's use of the term 'constant' to refer to a certain class of notions is that the notions in the complement of this class are *variables*. It is thus reasonable to conjecture that there cannot be much earlier uses of the expression 'logical constant', since the appearance of variables in the languages considered by logicians (as opposed to the use of implicit metalinguistic variables to signal the validity of arguments of certain forms) seems not to be earlier than Frege's *Begriffsschrift*, [9].

not based on *a priori* spatio-temporal intuition), and thus *a priori* knowledge. Notice that a logically analytic proposition in Bolzano's sense need not contain only logical concepts; but if it contains only logical concepts, then, as any other logically analytic proposition, it can be seen to be true through logical knowledge alone. The idea that true propositions containing only logical constants (and variables) must be *a priori* is thus shared by Russell with Bolzano.

Russell was also committed to the more general view that truths which remain true for all replacements of their non-logical constants, truths which are *formally true* (as we may call them), are *a priori*. This claim is a version of the Bolzanian hypothesis we saw earlier; it will be useful to have a labeled general statement of it:

> (*) if a proposition p is formally true, i.e., if it is true by virtue of its form, i.e., if all the results of replacing/reinterpreting uniformly its non-logical constants are true, then p is analytic/*a priori*.

(I write 'replacing/reinterpreting' and 'analytic/*a priori*' to indicate that particular versions of (*) may be about replacements or reinterpretations or both, and about analyticity or apriority or both).

The reason why Russell was committed to (a "replacing-*a priori*" version of) (*) is that if p is formally true then the result of replacing uniformly its non-logical constants with new variables and forming the universal closure must be a true proposition containing only logical constants (and variables). Since this universal closure must then be *a priori* and apriority is preserved under *a priori* known logical consequence, p must be *a priori*.

Russell does not typically speak about logically valid propositions which are not propositions of pure mathematics, but a view about these truths similar to Bolzano's is explicit (even if not as clearly as in Bolzano) in several places. Although logic itself does not affirm these propositions, it follows from logic that their validity consists in the fact that they are formally true; and this is a fact stated by logic, the discipline concerned with "formal reasoning":

> It seems to be the very essence of what may be called a *formal* truth, and of formal reasoning generally, that some assertion is affirmed to hold of every term; and unless the notion of *every term* is admitted, formal truths are impossible ([25, p. 40]).

Over the years following the *Principles*, the semantic and epistemological theories developed by Russell had as one of their essential aims to justify his idea regarding the epistemic properties of propositions containing only logical constants (and variables). Russell's theories on this point are all variations on a basic view according to which the notions of logic are objects of cognitive acquaintance, of a special (logical) non-empirical kind, and being acquainted with them is enough to make one able to know the truth of the true propositions that contain only these objects.[10] Sometimes these objects are referred to as universals (e.g., *The Problems of Philosophy* of 1912), sometimes it is denied that they are either universals or particulars (e.g., the 1913 *Theory of Knowledge* manuscript). But it is always claimed (until 1913 at least) that true propositions containing only these constituents must be *a priori*.

Later the thesis is rejected by Russell, in view of the existence of true propositions which, like the axiom of infinity of *Principia mathematica*, assert the existence of a certain number of individuals, contain only logical constants and variables, and yet do not appear to be *a priori*; see, e.g., [26, pp. 202-203].

Carnap and other members of the Vienna Circle embraced in its essentials Russell's view that mathematics is, or is reducible to, logic. More importantly, Carnap attempted to make precise Wittgenstein's claim that the truths of logic (and thus of pure mathematics) are tautologies, a claim which was supposed to explain their apriority. The name Carnap gave to his precise version of the notion of tautologicality adumbrated by Wittgenstein was 'analyticity'. In order to define it, in his *Logische Syntax der Sprache* of 1934 Carnap established a distinction between "logical" and "descriptive" terms, and gave definitions of these concepts in what he called "general syntax". With the help of these definitions and others given in terms of them, Carnap *defined* the notion of L-validity or analyticity in general syntax, in a way to which we will soon turn. "General syntax", for Carnap, included much that we now would not call syntax; in particular, it included virtually all resources of modern logic and mathematics.

Carnap's definitions of 'logical term' and 'descriptive term' in general syntax presupposed in turn the existence of a notion of "direct consequence" or of "set of transformation rules" for every particular language. For example, the class of logical terms of a language is defined by Carnap roughly[11] as the largest class of terms of the language such that every sentence which contains only members of this class (and variables) is determinately true or false on the basis of the transformation rules of the language alone.[12] The notion of transformation rule (and of a sentence being determinately true *on the basis of* transformation rules) was a notion lacking a definition generally applicable to arbitrary languages (it was, in fact, an unexplicated "primitive"

[10]The evolution of Russell's theories on this matter until the emergence of Wittgenstein's influence on him is usefully summarized in [6, pp. 124-128].

[11]The actual definition involves some complications inessential to our purposes (see [4, pp. 177-178]).

[12]Carnap's informal explanation of "the intended distinction" between logical and descriptive terms is that the former "have a purely logical, or mathematical, meaning" while the latter "designate something extra-logical —such as empirical objects, properties, and so forth" ([4, p. 177]).

of general syntax). The intuitive idea was that the transformation rules of a language would always give the truth conditions of its sentences.

Carnap's defined concept of analyticity (or L-validity) was devised in such a way that sentences that are intuitively analytic, but that contain constants other than logical constants, are analytic in the defined sense. Carnap observes that, if 'Q' is a descriptive predicate of a certain language that also contains numeral symbols (which are taken to be logical), then the sentence '$Q(3) \rightarrow (\neg Q(3) \rightarrow Q(5))$' (call it '$S_1$')

> is obviously true in a purely logical way, and we must arrange the further definitions so that S_1 is counted among the L-rules and is called (...) analytic (L-valid). (...) The example makes it clear that we must take the general replaceability of the A_d [i.e., the descriptive symbols] as the definitive characteristic of the L-rules ([4, p. 181]).

In other words, in the case of a sentence containing descriptive or non-logical symbols, the criterion for analyticity is that all the uniform replacements of those symbols with symbols of the same categories yield sentences that are "determined" to be true by the transformation rules of the language alone. Carnap's definition of analyticity is simply a precise codification of this version of the idea that logical truth is formal truth. (For Carnap's precise definition of analyticity or L-validity and related concepts, see [4, pp. 181-182].)

Under the conception of rules of transformation in general syntax, all the sentences containing only the traditional logical constants of, e.g., the language of *Principia mathematica*, must be determinately true or false on the basis of the transformation rules of its language alone (which, roughly, simply give the truth conditions of those sentences). Thus, when they are true, they are analytic, as 'analytic' is defined by Carnap. This is so simply because true sentences containing only logical constants do not have instances (nor, therefore, false instances) of replacement of descriptive symbols, as they do not have any descriptive symbols. These facts provide a justification of Carnap's definitions of 'logical term' and of 'analytic'. For they entail the idea (a special case of (*)) that all true propositions expressible only in terms of logical constants are analytic in Carnap's sense.[13]

The import of Carnap's explications of analyticity and logical term for the explanation of the apriority of logic and mathematics depended on the further Carnapian thesis that the transformation rules could be conventionally adopted by language users. Since the transformation rules determine in some sense the truth of all the analytic propositions of the language, a language

[13]Consistently, Carnap interprets his language II in such a way that true existential claims about the universe of individuals expressible with the help of only logical constants and variables of the language, like the axiom of infinity, are "determined" to be true by the transformation rules of language II, and thus analytic (see [4, pp. 140ff.]).

user who accepts the rules is supposed to be able to acquire *a priori* knowledge of those analytic propositions. Thus, Carnap quotes approvingly the first sentence of proposition 6.113 of the *Tractatus* ("It is the characteristic mark of logical propositions that one can perceive in the symbol alone that they are true"), but immediately makes it clear that the *a priori* knowledge Wittgenstein refers to is in fact possible because "syntactical" (transformation) rules for the language have been given in advance: "It is certainly possible to recognize from its form alone that a sentence is analytic; but only if the syntactical rules of the language are given" ([4, p. 186]).

Carnap's abandonment of general syntax in favor of Tarskian semantics (from about 1940 on) did not mean an abandonment of hypothesis (*), which he always accepted in some form. This Russellian belief in some version of hypothesis (*) was always accompanied in Carnap, as in Russell, by a philosophical theory of the semantics of the logical constants, and of how knowledge of the meanings of the logical constants (as given by certain conventional rules) is sufficient for *a priori* knowledge of logical truths. Carnap seems never to have questioned the essence of this theory (see [5, pp. 915ff.]), but of course the theory itself was never fully satisfactory as an explanation of analyticity, and increasingly lost favor in philosophical circles. (Quine, [23], offered the most influential discussion and critique of Carnap's views about logical truth.)

2.2. The Tarskian conception. In 1936 Tarski gave a precise version of the idea that a logical truth is a truth such that all the propositions of the same form are true, by means of his celebrated model-theoretic method of definition, whose *general description* (but not the particular definitions given rise to by the method) uses the notion of a logical constant: roughly, Tarski proposed to say that a sentence is logically true when it is true in all of a certain class of reinterpretations of its non-*logical constants*. Tarski did not think that his model-theoretic method for defining logical truth and logical consequence completely solved the problem of offering "a materially adequate definition of the concept of consequence" ([35, p. 418]). According to Tarski, perhaps the most important difficulty that remained towards solving that problem was created by the fact that "underlying our whole construction is the division of all terms of the language discussed into logical and extra-logical" ([35, p. 418]). Since this division is not based on a previous characterization of logical terms *generally applicable* to arbitrary languages, to that extent the method for defining logical truth is not fully general, and hence unsatisfactory. This situation is tolerable because, as Tarski says in a letter of 1944,

> it is clear that for all languages which are familiar to us such definitions [of 'logical term' and 'logical truth'] can be given (or rather: have been given); moreover, they prove fruitful, and this is really the most important. We can define 'logical terms', e.g., by enumeration ([33, p. 29]).

Thus, for Tarski a solution to the problem must consist in the discovery of a definition of ‘logical term’ generally applicable to all the languages he is concerned with (although the problem loses urgency because we can simply enumerate the logical terms of particular languages when we apply Tarski’s method to languages taken one by one).

Furthermore, if a satisfactory general definition were to be found, its application to a familiar logical language like the one of *Principia mathematica* should declare logical terms all the terms *traditionally considered logical* in the language. This is why in the final paragraph of Tarski [35] he says that a positive solution to the problem would “enable us to justify the traditional boundary between logical and extra-logical expressions” ([35, p. 420]). In fact, this is the boundary which for Tarski is “underlying our whole discussion”.

In addition to these two requisites, Tarski thought that a constraint on any acceptable definition of ‘logical term’ must be that it employ the apparatus of (scientific) semantics (or, at any rate, an equally innocuous apparatus). If we were to propose a definition of ‘logical term’ or ‘logical constant’ using concepts lacking precise characterizations or not deemed precise enough, Tarski’s project of defining the concepts of logical truth and logical consequence “in an exact form” ([35, p. 414]) would be vitiated, given the fact that ‘logical constant’ appears in Tarski’s description of his method. In particular, Tarski would obviously require that terms such as ‘analytic’ or ‘meaning’ (and also ‘*a priori*’, ‘necessary’, etc.) should not be used in a definition of ‘logical term’, for in this way unexplicated semantic concepts would be introduced in the definition of the semantic concepts of logical truth and logical consequence.

Related to this issue is the most important difference between Tarski’s viewpoint and that of the authors we have reviewed so far. According to Tarski, the problem of finding a general definition of ‘logical term’ is of importance “for certain general philosophical views” ([35, p. 419]). In the first place, “the division of terms into logical and extra-logical (...) plays an essential part in clarifying the concept ‘analytical’,” ([35, p. 419]); it is contextually clear that Tarski is referring to the concept “analytical” as defined in particular by Carnap in *Logische Syntax* (which, as we saw, featured in its definition a concept of “logical term”). In the second place, the concept of analytical truth is regarded by “many logicians” (that is, Carnap and the Vienna Circle) “as the exact formal correlate of the concept of *tautology* (i.e., of a statement which ‘says nothing about reality’)” ([35, pp. 419-420]). That is, the concept of analytical truth as defined by Carnap in *Logische Syntax* is taken by him and others to be *an exact formal correlate* of a philosophically important property of logical truths; since this exact formal correlate is defined in terms of the notion of a logical constant, a definition of this notion would further clarify that philosophically important property, thus throwing light on “certain general philosophical views”.

However, Tarski adds that he personally considers the philosophical notion of a tautology "rather vague". This means that there may not be a sharp, fixed notion of tautologicality, or intuitive analyticity; rather, these may be "relative concepts which must, on each occasion, be related to a definite, although in greater or less degree arbitrary, division of terms into logical and extra-logical" ([35, p. 420]). We see that, in a strict sense, Tarski favored the relativistic thesis that the question whether a term is logical might not have an entirely non-arbitrary answer.[14] In the case of the '$\in$' sign (Tarski's favorite example), some set-theoretic truths may be seen as logical or not, according as to whether we are working with '$\in$' as a logical constant in some formulation of the theory of types or with '$\in$' as a non-logical constant in first-order set theory. If Tarski's relativistic thesis is adopted, the immediate effect is that the explanatory desideratum on a theory of logical constants imposed by the authors we have reviewed is no longer necessary. If there is no sharp sense in which, e.g., some sentences containing only variables and the logical constants of *Principia* are analytic, then it is absurd to ask of a theory of logical constants that it explain or explicate an absolute analytic-synthetic or *a priori-a posteriori* distinction; these were for Tarski only relative distinctions.

But this does not mean that no other desiderata could or should be imposed on a definition of logical constancy. Thirty years after writing his paper on logical consequence, Tarski returned to the problem of the definition of the concept of logical term, and still some years later he advanced an attempt at a positive solution. Tarski's eventual definition satisfies two of the three requisites we spoke of above, and comes remarkably close to satisfying the other: (a) it is generally applicable to the languages to which his definitions of the semantic concepts are applicable, (b) it does not employ unexplicated semantic or epistemic concepts, but only general logical and mathematical concepts,[15] and (c) it generates the "traditional" extension of 'logical term' for some typical "logical languages", like that of *Principia* —although it does not do that for *all* languages, as we will see. On the other hand, Tarski will

[14]But he seems not to have been very informative about his reasons for that assumption. The '$\in$' example, described in the main text, does not by itself tell us much about any general views that Tarski may have had. Perhaps he had in mind some set of ideas related to the pragmatic principles mentioned in the Introduction, which, as we mentioned, seem to give rise to a somewhat vague characterization of logical constancy: the meaning of the characterization is compatible with a number of different extensions for the predicate 'logical constant'. But vagueness does not imply arbitrariness. '$\in$' may be a borderline case, but the characterization leaves many constants definitely out and keeps others definitely in.

[15]This ought to be qualified in a well-known way. In Tarskian semantics, including the definition of 'logical constant', one uses the concepts corresponding to the terms of the language for which one is defining the semantic concepts. Of course, these concepts need not be always logical or mathematical.

never make any attempt to explain the distinctive semantic and epistemic properties of logical truths with the aid of his proposed solution.

The beginnings of the proposed solution appear in Tarski's lecture "What Are Logical Notions?" of 1966 (published posthumously in 1986). Here Tarski gives a definition of 'logical notion'. A notion is an object appearing in some type in a *Principia mathematica*-like hierarchy of types generated by a universe of individuals. He proposes to define logical notions as those notions which are invariant under all one-one transformations of the universe of individuals onto itself (see [32, p. 149]). A one-one transformation of a class onto itself, also called a permutation, induces permutations of all the types in the hierarchy of types of "notions" determined by the class. Thus, a permutation of a domain of individuals D induces a permutation of the class of n-ary relations of elements of D, a permutation of the class of n-ary relations among m-ary relations of elements of D, etc. A notion or object O of a certain type t is invariant under all permutations of the universe of discourse if, for all permutations P of this universe, the permutations $\tilde{P}$ induced by P in the class of notions of type t are all such that $\tilde{P}\ (O) = O$.[16]

Tarski's definition of logical constancy appears in a book Tarski wrote in collaboration with Steven Givant, published in 1987, four years after Tarski's death. Tarski and Givant introduce informally the concept of a derivative universe of a given basic universe U. A derivative universe $\tilde{U}$ of a given basic universe U is the class of all notions of a certain type, generated from that basic universe U. Thus, the class of n-ary relations of elements of U, the class of n-ary relations among m-ary relations of elements of U, etc., are derivative universes of U. The definition appears in this passage:

> (**i**) *Given a basic universe U, a member M of any derivative universe $\tilde{U}$ is said to be logical, or a logical object, if it is invariant under every permutation P of U.* (Strictly speaking, since an object M can be a member of many derivative universes, we should use in (i) the phrase "*is said to be logical, or a logical object, as a member of $\tilde{U}$*".)

[16]In one version of the lecture (see footnote 6 in [32, p. 150]) Tarski indicated that the truth-functions and the denotations of the classical quantifiers can be constructed as certain objects in the type hierarchy that are invariant under all permutations, and, in this sense, are logical notions. For example, the truth-values "true" and "false" can be identified with the universe of discourse and the null set, respectively, and the truth-functions in turn with functions having (tuples of) these classes as arguments and values; and the denotations of the classical universal and existential quantifiers over a type of objects t can be identified with certain functions from the class of sets of objects of type t into the class of truth-values —identifying "true" with the universal set of objects of type t and "false" with the empty set of that type. (The denotation of a universal quantifier will assign "true" to the set of all objects of type t, and "false" to all other subsets of t; and the denotation of an existential quantifier will assign "true" to the non-empty subsets, and "false" to the empty subset.)

> ($\mathbf{ii}$) *A symbol S* (...) *is said to be logical, or a logical constant, if, for every given realization* **U** *of this* [language] *with the universe U, S denotes a logical object in some derivative universe* $\tilde{U}$ ([36, p. 57]).

($\mathbf{i}$) and the parenthetical comment that comes after it contain quite accurately the basic idea behind the definition of a logical notion offered in Tarski, [32], and ($\mathbf{ii}$) offers the definition of logical constancy.[17]

In order to distinguish perspicuously the Tarski-Givant defined concept of logical constancy from others to be considered later, let's recast their definition using a new notation. Let's use the notation '$den(C, U)$' to designate the denotation of a constant C in a domain U, and define

> (TLCt) A constant C is a (logical constant)^T (read: "a Tarskian logical constant") if, for every universe U and every permutation P of U, $\tilde{P}\ (den(C, U)) = den(C, U)$.

('(TLCt)' stands for 'Tarskian logical constancy'.) The concept "C denotes x in U" in (TLCt) (or "S denotes x in $\tilde{U}$" in Tarski and Givant's condition ($\mathbf{ii}$)) is understood to be definable using the methods of Tarski, [34]. C may be taken to be an individual constant, an n-place predicate constant naming an n-ary relation among individuals, etc.

Quite obviously, Tarski's definition of logical constancy is generally applicable, applicable to all the languages for which his definitions of the semantic concepts are especially intended. Also, it is given in terms of the apparatus of concepts acceptable in "scientific semantics". But the third Tarskian requirement, that a definition of logical constancy should generate the traditional extension of 'logical term' for all typical logical languages, is not satisfied by his definition. The reason will emerge in the next section.

§3. Recent conceptions and theories, and their difficulties.

3.1. (Broadly) Tarskian conceptions. Tarski's definition (TLCt) is related to, but not necessarily coextensional with, a definition based on a stronger requirement for logical constancy that suggests itself naturally. Given a bijection B between two universes U and V, let's use the notation '$\tilde{B}$' to designate the bijections between "isomorphic" derivative universes $\tilde{U}$ and

[17] We ought to mention that Tarski and Givant give their definition for symbols of a certain class of non-quantificational languages designed by them for the formalization of set theory. But there is no apparent obstacle to applying ($\mathbf{ii}$) to wider classes of languages than the class considered for special purposes of their investigation by Tarski and Givant. They themselves say that the usual logical constants of languages not in the class considered by them, like the symbols for the truth-functional connectives and quantifiers, "can also be subsumed under logical constants in the sense of ($\mathbf{ii}$)" ([36, p. 57]), presumably through some artifice in the style of the one mentioned in the preceding note. Also, at the very least Tarski clearly would want his definition to be applicable to some languages whose set of sentences is that of some formulations of the simple theory of types. Further, Tarski and Givant do not impose any restriction on the intended meanings of the symbols of the languages in the class for which they give their definition (as long as they are constants of certain syntactical kinds).

$\tilde{V}$ induced by B. Then we may define an alternative concept of logical constancy as follows.

> (MLCt) A constant C is a (logical constant)^M (read: "a Mostowskian logical constant") if, for all universes U and V of the same cardinality and all bijections B from U onto V, $\tilde{B}(den(C, U)) = den(C, V)$.

('(MLCt)' stands for 'Mostowskian logical constancy'.[18]) Put intuitively, (MLCt) requires that logical constants denote in every universe not merely an object invariant under permutations of that universe, but an object which is the same, up to isomorphism, in *all* universes of the same cardinality. If a constant is a (logical constant)^M then it is a (logical constant)^T, since any permutation of a universe U is a bijection between U and itself. But it is not necessarily the case that every (logical constant)^T is a (logical constant)^M. Suppose, for example, that *there were* a unary predicate constant of the second type C whose intended meaning was such that, in every universe, C denoted either the empty set or the universe itself, and it denoted the empty set in some universes of some cardinality κ but the full universal set in other universes of cardinality κ. C would be a logical constant in the sense of (TLCt) above, but not a logical constant in the sense of (MLCt).[19]

Some recent authors have adopted Tarski's defined concept of logical constancy, or the one inspired by the Mostowskian idea, essentially as we have presented these concepts. Some reasons for adopting these defined concepts are strong. Tarski and Lindenbaum, [37], had proved that given a basic universe U, all the notions in derivative universes of U which can be defined in the language of a standard formulation of the simple theory of types are invariant under all permutations of U.[20] That is, the individuals, relations of individuals, relations of relations of individuals, etc. which can be defined in the theory of types stay the same after any permutation of the universe of individuals. For example, no individual is logical, for every individual can be projected onto a different one in a permutation (of course, this is only true if the universe is of cardinality greater than one); the classes of individuals definable in the theory of types are the class of all

[18]A definition of a certain class of generalized first-order quantifiers as, essentially, those quantifiers which are (logical constants)^M, was given by Mostowski in a paper of 1957 (see [19, p. 13]). Mostowski did not use his condition of invariance under bijections to give a definition of logical constancy in general.

[19]In [28, p. 63] Sher asserts that "in the lecture "What are Logical Notions?" [32] Tarski proposed a definition of "logical term" that is coextensional with [(MLCt)]". This is somewhat inexact, for as we have seen Tarski did not propose a definition of 'logical term' in [32], and when he proposed one in [36], the proposed definition is not necessarily coextensional with (MLCt) (or, it is only coextensional with it under certain substantive assumptions about the possible meanings of constants).

[20]The converse is false, for if U is infinite there are non-denumerably many such invariant notions, but only denumerably many are definable in the language of the theory of types.

individuals and the empty class, and these are the only classes of individuals invariant under all permutations of the universe; the binary relations among individuals definable in the theory of types are the universal relation, the identity relation, the diversity relation and the empty relation, and these are the only binary relations of individuals invariant under permutations; all classes of classes of individuals which can be identified with "cardinality properties" (the class of two-membered classes, the class of finite classes, the class of infinite classes) are all invariant and hence logical; some relations among classes of individuals like inclusion are invariant and hence logical.

In general the Tarski-Lindenbaum theorem guarantees that all mathematical notions definable in the logicist fashion in the simple theory of types are logical notions no matter what the universe of individuals is taken to be. Since the theorem applies to every universe U supplying an interpretation of the language of the theory of types, the definition (TLCt) implies that all primitive symbols denoting notions in that language (e.g., quantifiers of all finite orders) are (logical constants)^T; also, if the definition were applicable to defined symbols, all these symbols would be (logical constants)^T. Such results agree well with (an important usage actually encountered in) traditional practice (for example, the practice of the logicists, but of others as well) according to which the constants of the language of the theory of types are logical constants.

When (TLCt) is applied to interpreted languages of mathematical theories with undefined mathematical primitives, it will generally yield the result that the notions denoted by these primitives are non-(logical constants)^T. Consider set theory formalized in first-order with a single primitive predicate for membership as a relation among elements of the universe. Obviously membership as a relation over a domain of individuals and sets is not invariant under all permutations of that domain, so it is not declared a logical notion of the language of set theory by Tarski's proposed definition.[21] Similarly, the class of all sets will be declared non-logical provided the class of individuals that are not sets is not empty. Under the definition of '(logical constant)^T', a predicate whose intended meaning is membership (among elements of the universe) is a non-(logical constant)^T (and hence a non-(logical constant)^M) simply because there is a universe in which it denotes a non-logical object (with respect to that universe); similarly, a predicate 'S' whose intended meaning is "is a set" (such as is used in some formalizations of set theory suitable for contemplating individuals other than sets in the universe) is a non-(logical constant)^T (and hence a non-(logical constant)^M), for there

[21]Compare the case of membership as an "intra-typical" relation among objects of contiguous types in the type hierarchy. It is a logical notion is Tarski's defined sense. This led Tarski to claim that his relativistic thesis about logical constancy mentioned above in the main text is at least partially (i.e., at least in the case of the membership sign) vindicated by his proposal (cf. [32, pp. 151-152]).

is a universe in which 'S' denotes a non-logical object (in such a universe, the class of individuals that are not sets must be non-empty). These results are again in agreement with actual usage, for example in the model theory of first-order set theory, where the membership predicate is taken as a non-logical constant.[22]

In a familiar first-order language suitable for the formalization of natural number arithmetic, the symbols 'N', '0', 's', '+', '·' with their intended meanings are all non-(logical constants)^T. The reason is that there are universes in which those symbols denote non-logical notions (it suffices to consider the universe of all natural numbers, except in the case of 'N', where we need to adduce a universe that properly includes a non-empty subset of natural numbers). Suppose now that we formalized some fragment of non-mathematical natural language by means of a first-order language with the predicate 'R', whose intended meaning is "is red" (languages such as this are common currency in the manuals of elementary logic). This predicate will be a non-(logical constant)^T (and hence a non-(logical constant)^M) in that language since there are universes that include properly a non-empty class of red things. To the extent that we find examples in usual practice in which such predicates as 'R' in first-order languages are treated as non-logical, Tarski's definition of logical constancy is in agreement with that usage.

These are strong reasons in favor of (TLCt) and (MLCt). But is Tarski's definition extensionally adequate, does it agree with the usage encountered in practice of *all* the formal languages for which it is given? The theorem by Tarski and Lindenbaum seems to provide a good justification for the claim that Tarski's definition gives a *necessary* condition of the traditional logical constants of classical quantificational languages, i.e., the traditional logical constants which are primitive constants of the higher-order logic of *Principia mathematica* or its fragments, or definable in some way in that logic.

[22]The class of all sets is not a set, so it cannot be the universe of discourse of a realization in the sense of Tarski and Givant; such universes are always taken to be sets by them (see [36, p. 15]). Therefore, in order to justify the claims just made in the main text we should appeal strictly speaking to the existence of *set-universes* in which '$\in$' denotes real membership among their members and S applies to sets. But the claims are justified more perspicuously by appeal to the natural universe of set theory. In fact, we could define, at least informally, the concepts of a logical notion and a logical constant envisaging universes which are not sets, but classes in general. Now by *notions* we would understand members of derivative universes of a class; these derivative universes would be hyper-classes entirely analogous to the derivative universes of sets. Similarly, we would extend the notion of a permutation to onto bijective correspondences whose domain may be a set or a proper class, and define invariance accordingly. Then given a basic class-universe U, we would define a member M of any derivative universe $\tilde{U}$ to be logical, or a logical object, as a member of $\tilde{U}$ if it is invariant under every permutation P of U; and we would define a constant C to be a logical constant if, for every class-universe U, C denotes a logical object in U. These informal definitions, given suitable principles governing the informal notions involved, will have the same consequences regarding the language of first-order set theory as (TLCt).

However, the converse problem, the question whether Tarski's definition provides a *sufficient* condition for logical constancy, even for the classical quantificational languages for which it is intended, seems to have a clear negative answer. A counterexample would be produced if, so to speak "by chance", some constants not typically treated as logical denoted notions invariant under permutations in all universes of discourse. Persuasive counterexamples of this kind result if we consider predicates not treated as logical and which are predicated *falsely* of all individuals in all universes. These predicates denote a logical notion in all universes, namely the empty set (often, they denote the empty set in all possible universes). 'Unicorn', 'heptahedron' and 'male widow' are good candidates; by 'heptahedron' I abbreviate 'regular polyhedron of seven faces'. (Since these predicates are (logical constants)^M, they also provide counterexamples to the extensional adequacy of (MLCt).) Examples such as these seem to constitute decisive refutations of the extensional adequacy of Tarski's definition of logical constancy (and of the analogous definition based on ideas of Mostowski), since these constants would not be treated as logical in any typical use of formalized languages.

Related counterexamples can be easily found or constructed higher up in the hierarchies of notions of a universe that Tarski had in mind. Consider the binary predicate 'is the same individual as', that is, the identity predicate. The binary predicate 'is the same as, if there are no heptahedra, and is not the same as, if there are heptahedra' has the same extension as the identity predicate. If it looks too long, or complicated, or if it bothers you that it is a complex predicate, abbreviate it with '$\underline{=}$' and think of it as a non-complex predicate with the already explained meaning. While the identity predicate is often taken to belong to the intended set of logical constants (and it was so taken by Tarski), '$\underline{=}$' certainly does not belong to that set. But it is a (logical constant)^T, since it denotes the identity relation in all universes, and this is always a notion invariant under permutations. (Another example of a (logical constant)^T not in the intended set of logical expressions is the sign '$\underline{\exists}$' of section 3.2 below.)

Gila Sher [28] has proposed a definition of logical constancy essentially identical to (MLCt), and she has considered an objection to her proposal essentially analogous to the one based on the existence of terms like 'unicorn', 'heptahedron', etc. In order to deny relevance to this objection, Sher says that under her proposal, a logical term is *identified* with the (class)-function that assigns to every (set)-universe the denotation of the term in the universe:

> logical terms are identified with their (actual) extensions, so that in the metatheory the definitions of logical terms are rigid. (...) Their (actual) extensions determine one and the same formal function over models, and this function is a legitimate logical operator. (...) ...we may say that the only way to understand the meaning of a

> term used as a logical constant is to read it rigidly and formally, i.e., to identify it with the mathematical function that semantically defines it ([28, pp. 64-65]).

This counterobjection is highly problematic. Suppose we had a primitive one-place logical predicate '$\emptyset$' in mathematical languages, whose meaning was given simply by the stipulation that it abbreviates the expression 'is not identical with itself'.[23] Under Sher's proposal, '$\emptyset$', 'unicorn', 'heptahedron', 'male widow', etc. are all the same term, and hence the sentences '$\forall x \neg$unicorn(x)', '$\forall x \neg$heptahedron(x)' and '$\forall x \neg$male widow(x)' must be as logically true (in the intuitive sense) as '$\forall x \neg \emptyset x$' (and '$\forall x \neg (x \neq x)$') would appear to be. Similarly, '$\forall x \forall y (x \underline{\equiv} y)$' would be as logically true as '$\forall x \forall y (x = y)$' appears to be. Since sentences like '$\forall x \neg$unicorn(x)', '$\forall x \neg$heptahedron(x)', '$\forall x \neg$male widow(x)' and '$\forall x \forall y (x \underline{\equiv} y)$' are *not* logical truths, in any traditional sense of 'logical truth', it seems clear to me that Sher's move is not acceptable as a way of meeting the obvious objection posed by terms like 'unicorn'.[24]

That neither (TLCt) nor (MLCt) will do as they stand has been recognized by other authors sympathetic with the idea underlying those definitions, such as Vann McGee (see [18, p. 578]) and Timothy McCarthy (see, e.g., [17, p. 411]).[25] Both of these authors propose modifications of (MLCt) destined to avoid the problem posed by terms like 'unicorn', modifications in

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[23]A primitive '$\neq$' for the *binary* diversity relation appears, and is considered a logical constant, in some languages used by Tarski [30] for the formalization of what he calls elementary geometry.

[24]That such problems arise for Sher's proposal is not surprising. Although it is common practice in mathematical logic since Gödel and Tarski to code (and hence, in a sense, to identify) expressions with mathematical entities, an obvious requisite of any such coding in all standard applications must be that different expressions are assigned different mathematical entities; otherwise, the risk arises that some mathematical entity will code two expressions with different properties relevant to the context of investigation. If we did not abide by the requirement when, say, devising a Gödel numbering for expressions, we often would end up with similar troubles: numbers that code both terms and non-terms, numbers that code both formulas and non-formulas, etc.

[25]The counterexamples actually considered by McGee and McCarthy are related to our '$\underline{\equiv}$' (and to the '$\underline{\exists}$' of section 3.2 below). Here is one of McCarthy's examples:

> Consider, for example, a quantifier symbol Q of [a language] L translated into English by the phrase "for some..., if P, and for all... if $\sim P$", where 'P' represents a contingent truth of English. Q extensionally coincides with $\exists$ on any (actual) domain. If Q were treated as a logical constant, then the formula $(Qx)A(x)$ would be a logical consequence of $(\exists x)A(x)$ in L. (...) But [it is not implied by it], assuming only that there exists a possible situation in which P fails and in which some but not all individuals fall under $A(x)$ ([17, p. 411]).

The examples unsuccessfully dealt with by Sher and referred to above, are similar first-order quantifiers inspired by McCarthy's examples. But notice that in place of the contingent truth P we put a necessary but (presumably) not logically true proposition in our definition of '$\underline{\equiv}$'; something analogous will happen in our definition of '$\underline{\exists}$' in section 3.2 below.

which unexplicated semantic or epistemic notions appear. In this way, they give up Tarski's project of mathematical explication, and the usefulness of their defined concepts of logical constancy as replacements of the unexplicated notion in Tarski's definitions of logical consequence and logical truth becomes doubtful. This kind of move might be worthwhile if the defined concepts were at least coextensional with the traditional notion. But I think it's quite clear that they are not.

McGee's characterization, which is merely enunciated by its author, is the following: "A [constant] is a logical [constant] if and only if it follows from the meaning of the [constant] that it is invariant under arbitrary bijections" ([18, p. 578]). In other words, this characterization says that a constant is logical when it follows from its meaning that it is a (logical constant)^{M}.The unexplicated semantic notion appearing in the characterization is the notion of (following from) meaning.

The most developed of the two characterizations is McCarthy's, which appears in [16] and [17] (an earlier characterization, different but in the same spirit, appears in [15]). The author begins his analysis by considering a characterization of logical constancy that is, essentially, the Mostowskian (MLCt). He then motivates the need for a stricter characterization especially by the desire to exclude exotic terms like 'unicorn' from the class of logical constants. To this effect he asks us to consider what he calls "modalities". A modality is a class of possible situations, in some sense of 'possible'. For example, intuitively, the modality of metaphysical necessity is the class of metaphysically possible worlds; the modality of apriority is the class of all the possible "worlds" or situations not excludable on *a priori* grounds (these situations need not be metaphysically possible), etc.

McCarthy then defines a notion of "rigid invariance" over a modality, which is essentially an extension of Mostowski's invariance property to pairs of universes from possibly different situations in a modality. Using our apparatus and notation, we can enunciate a definition close in all essential respects to McCarthy's:

> (r.i.) A constant C is *rigidly invariant over a modality* $\mathbf{M}$ if, for all "worlds" u and v in $\mathbf{M}$, all universes of the same cardinality U in u and V in v, and all bijections B from U onto V, $\tilde{B}(den(C, U)) = den(C, V)$.

(See [16, p. 426], or [17, p. 411]) Then the notion of a "logical constant over a modality" can be defined as follows:

> (l.c.m.) A constant C is an l.c.m.($\mathbf{M}$) (read: "a logical constant over the modality $\mathbf{M}$") if it is rigidly invariant over $\mathbf{M}$.

McCarthy considers in particular the notion of being l.c.m. over a very strong epistemic modality, the set of epistemically possible situations in the "intentional spaces" of speakers of natural language (see [17, pp. 411-412];

see also [15, pp. 517ff.]). Terms like ‘unicorn’ are perhaps not l.c.m.’s over this modality: there may be an epistemically possible universe V in which the denotation of ‘unicorn’ is a non-empty set; this non-empty set will not be the image of the empty set, which is the denotation of ‘unicorn’ in a universe U occurring in the “actual” epistemic situation, under a bijective correspondence induced by a bijection between U and V.

However, terms like ‘heptahedron’ and ‘male widow’ are presumably l.c.m.’s over McCarthy’s epistemic modality: there must not be epistemically possible universes in which the denotation of ‘heptahedron’ or ‘male widow’ is a non-empty set. And if it is claimed that there are, then it is clearly *ad hoc* to claim, as McCarthy would need, that there are not epistemically possible universes in which the traditional logical constants have denotations which are non-isomorphic to their denotations in universes (of the same cardinality) occurring in the “actual” epistemic situation. Equally clearly, ‘male widow’ is a logical constant in McGee’s defined sense, since it manifestly follows from its meaning (together with the minimal mathematical assumptions which are needed in any case if one wants to do any work with the characterization) that its denotation is invariant under arbitrary bijections.[26] It seems inevitable to conclude that these proposals inspired by Tarski, which do not meet his requirement of “conceptual innocuousness”, do not even meet the minimal requirement of extensional adequacy.

3.2. (Broadly) anti-Tarskian conceptions. Tarski’s remarks at the end of his 1936 paper “On the Concept of Logical Consequence”, that we reviewed in Section 2.2, are generally credited as the main original source for the formulation of the problem of logical constants in the more recent literature dealing with the problem (see, e.g., [20, p. 221], [12, p. 287], [16, pp. 424, 442]). For the most part, however, this recent literature does not attempt to offer solutions to the problem in the sense in which Tarski understood it and its possible solution in his 1966 lecture. Most of the literature rejects Tarski’s skepticism about the concepts of analyticity and apriority, and sees the problem precisely as that of offering a partial basis for rejecting that skepticism with the help of a theory of logical constants.

Thus, most post-Tarskian authors who accept some version of (*) do not postulate it, but try to derive it from their characterizations of logical

[26] Some scruples may arise in some readers because of the fact that ‘male widow’ is a complex predicate. But if we like we can again stipulate that it is a non-complex predicate with the indicated meaning. Although it appears not to be an uncontroversial thesis that there exist non-complex non-logical predicates in natural language with an analytically empty extension (an extension empty by virtue of their meaning), it seems to be a true thesis. For example, derogatory predicates like ‘dago’ are presumably analytically empty, in the sense that it is analytically true that no one is “contemptible because Italian or Spanish” (assuming that this gives the meaning of ‘dago’). At any rate, even if the thesis is in fact false, it can only be so accidentally.

constancy.[27] This generally leads to characterizations that use some unexplicated semantic or epistemic notion closely connected with the notions of analyticity or apriority. These are again characterizations which are unusable if one wants to eliminate the unexplicated notion of logical constancy from Tarski's definitions of logical consequence and logical truth in order to obtain non-semantic and non-modal explications of those notions. Again, this is a mildly important point which does not seem to have been sufficiently stressed, or stressed at all.[28]

But characterizations of logical constancy in terms of unexplicated semantic or epistemic notions may at least have some value if they are extensionally appropriate and, when they are extensionally appropriate, if they are coupled with an explanatory philosophical theory of the semantics and epistemology of the logical constants. The value of the existent theories seems doubtful even when judged by these standards.

A famous proposal that still receives some attention is that of Christopher Peacocke in [20] (later, Peacocke has turned to views close to the views of Ian Hacking that will be described below). Peacocke's characterization of logical constancy is the following:

> α is a logical constant iff α is noncomplex and, where the syntactic category of α is $\tau/\sigma_1,\ldots,\sigma_n$, for any expressions $\beta_1,\ldots,\beta_n$, of categories $\sigma_1,\ldots,\sigma_n$, respectively, given knowledge of
> (a) which sequences satisfy those β_i which have satisfaction conditions, and of
> (b) which object each sequence assigns to those β_i which are input to the assignment function, and of
> (c) the satisfaction condition or assignment clause for expressions of the form $\alpha(\gamma_1,\ldots,\gamma_n)$
>
> one can know a priori which sequences satisfy the expression $\alpha(\beta_1,\ldots,\beta_n)$ of category τ, or which object any given sequence assigns to $\alpha(\beta_1,\ldots,\beta_n)$, in particular without knowing the properties and relations of the objects in the sequences ([20, pp. 225-226]).

(The sequences mentioned in this definition are assignments of suitable objects drawn from a universe U to the variables of the language, as used in a Tarskian definition of truth.) Peacocke's intuitive idea is reasonably clear, and his precise version of it simply tries to make it somewhat rigorous:

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[27] Among the post-Tarskian authors already discussed, McCarthy offers proofs of this kind (see [16, pp. 432ff.]).

[28] It is perhaps good to point out that, if we are allowed to use unexplicated semantic, epistemic and modal notions in our explications, then we can use them directly in order to supplement the *definiens* in a Tarskian characterization. We may then say that a logical truth is a truth which remains true in all reinterpretations of its non-logical constants, and which is analytic, and *a priori*, and necessary, and whatever else we may care to add. Of course, proofs of versions of (*) corresponding to notions of logical or formal truth defined in this way are trivial and of doubtful value.

something in the nature of the logical constants makes someone acquainted with their meaning able to obtain a certain kind of *a priori* knowledge. By a process of composition, someone who makes use of this ability a sufficient number of times, applying it to more and more complex subparts of a logical truth, will eventually obtain *a priori* knowledge of that logical truth. Thus Peacocke says:

> the notion of validity naturally generated by this account is the standard model-theoretic conception of validity (...). For since in model theory one considers arbitrary domains and arbitrary assignments of sets of n-tuples of objects to n-place atomic predicates, but holds constant the interpretation of the logical constants, the model-theoretically valid sentences will be precisely those sentences which one can know a priori to be satisfied (absolutely) by all sequences (that is, true) regardless of which sequences satisfy the atomic nonlogical predicates, given knowledge of the satisfaction conditions of the logical constants (on this criterion) ([20, p. 230]).

Leaving aside the question of what proof Peacocke may have had for his claims, it is clear that he believes that it follows from his characterization that all the model-theoretically valid sentences are *a priori*, and this is just a version of condition (*).

Peacocke's characterization generates a number of intrinsic difficulties, on which there is a certain amount of literature, and it is fair to say that these difficulties have done enough to discredit the proposal (see, e.g., the discussion in [27, pp. 321ff.]). Especially noteworthy among these difficulties is the following problem: I may know which sequences satisfy the formula $A(x)$, and which object each sequence assigns to 'x', and also know the satisfaction condition or assignment clause for expressions of the form $\forall x\phi(x)$, but this does not make me able to know *a priori* the truth value of $\forall xA(x)$. For this, I also need to know that the sequences about which I know are all the sequences there are. Peacocke is aware of this problem, and postulates a way of dissolving it which is rather *ad hoc*, and which comes down to adding to his theory the extra hypothesis that some imaginary knower has all my knowledge about sequences, plus all the knowledge I lack, and that the imaginability of *this* knower is enough to make '$\forall$' a logical constant. I think this is an obviously unsatisfactory way of meeting the problem, since it is not clear that such knowers are imaginable or even possible, especially in cases in which '$\forall x$' ranges over huge domains of quantification.

Even if by some chance Peacocke's theory (including *ad hoc* hypotheses about imaginary knowers) were extensionally correct, I think that a serious objection to it would be that it does not come together with an explanatory theory of why expressions with the meaning of the logical constants have the complex epistemic properties attributed to them by Peacocke's characterization. In particular, there is no explanation in the theory of why acquaintance with the meaning of the logical constants puts one in the position of attaining certain kinds of *a priori* knowledge, and in particular *a priori* knowledge of logical truths.[29]

We will now review another group of characterizations of logical constancy in the recent, and not so recent, literature. These characterizations do offer a certain kind of explanation of the acquaintance with the meaning of the logical constants and of how this acquaintance produces *a priori* knowledge. However, they have problems satisfying the minimal requirement of extensional adequacy.[30]

In 1947 Karl Popper claimed to have

> established (...) that formative signs can be defined in terms of a concept of deducibility which does not assume any formative signs in turn, [which] opens a way to applying Tarski's concept [of logical consequence] without difficulty. I have in mind the difficulty, mentioned by Tarski, of distinguishing between formative ("logical") and descriptive signs. This difficulty seems now to be removed ([21, p. 203n.]).

Clearly, Popper failed to grasp the idea that Tarski's difficulty arose (in part) from the lack of a definition of the *class* of logical symbols. Instead Popper spoke of defining *each* logical (or formative) sign in terms of rules of inference:

> If we have an artificial model language with signs for conjunction, the conditional... etc. (we have called them "formative signs" of the language in question) then the meaning of these formative signs can be exhaustively determined by the rules of inference in which these signs occur; this fact is established by defining our definitions of these formative signs explicitly in terms of rules of inference[31] ([21, p. 220]).

[29]Similarly, there is no explanatory theory of this kind in McCarthy's work, even though, as we mentioned in a previous footnote, McCarthy offers proofs that all Tarskian logical truths (when for 'logical constant' in the Tarskian definition of 'logical truth' one substitutes McCarthy's defined concept of logical constancy) are *a priori*.

[30]Also, at least some of them (perhaps all) are given in terms of unexplicated semantic notions, and hence they cannot be used to eliminate the unexplicated notion of logical constancy from the Tarskian theory of logical truth and logical consequence.

[31]The source of these ideas of Popper can probably be traced ultimately to Gentzen, who had spoken of the introduction rules for logical expressions in his natural deduction calculi as "definitions", and of elimination rules as "no more, in the final analysis, than the consequences of these definitions", going on to suggest that "by making these ideas more precise it should be possible to display the elimination-inferences as unique functions of their corresponding introduction-inferences, on the basis of certain requirements" ([10, pp. 82-83]). It must be pointed out that Gentzen was not attempting to offer a definition of the class

Popper's confusion aside, a way of applying this idea to obtain a definition of the class of logical constants immediately suggests itself, and we find it a few years later in [14] (with credits to Popper): "formal (or logical) signs are those whose full *sense* can be given by laying down rules of development for the propositions expressed by their help" ([14, pp. 254-255], my emphasis).

This set of ideas came under criticism in Prior's famous [22]. Prior characterized a binary connective 'tonk', whose

> meaning is completely given by the rules that (i) from any statement P we can infer any statement formed by joining P to any statement Q by 'tonk' (...), and that (ii) from any 'contonktive' statement P-tonk-Q we can infer the contained statement Q ([22, p. 39]).

As is easily seen, the rules that give the "meaning" of 'tonk' allow us to infer any proposition from any other (provided only that the relation of inference is transitive). Presumably such inferences are not analytical implications, although it would seem that it is a consequence of Popper's and Kneale's views that they are, since the only thing necessary to draw them is to know the sense or meaning of 'tonk', which by hypothesis is "given" by the rules. The moral drawn by Belnap in his reply to Prior ([2]) was that not every set of rules we may come up with can fix a meaning for an expression, in the same way that not every statement containing a new symbol constitutes a licit definition of that symbol in a mathematical theory.[32] He suggested adding the further requirement that the rules be conservative with respect to a background set of correct inferences: any statement inferable by means of the rules for an expression but not inferable without those rules must contain that expression. Acceptable definitional extensions of mathematical theories are conservative, but the rules for 'tonk' are not conservative (unless the background set of inferences is already inconsistent) since, if P is a statement previously inferable and Q is a statement not previously inferable not containing 'tonk', they allow Q to be inferred.

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of logical constants. He was probably trying to emphasize some formal similarities between, on the one hand, the set of introduction and elimination rules for each logical constant in his system, and on the other, the acceptable definitions of new symbols in mathematical theories. Belnap also stressed some of these formal similarities (see below).

[32] Another possible, more defeatist reaction to Prior's remark, that would be favored by the skeptic about analyticity, would consist in accepting that the notion of analytical implication is such that every proposition is analytically implied by every proposition, taking Prior's example to have shown that there is an expression whose meaning is given by the rules of 'tonk', namely, 'tonk' as introduced through stipulation by Prior. The immediate conclusion would be that the notion of analytical implication is paradoxical or at least suspicious, for surely we at the same time believe that not every proposition is analytically implied by every proposition.

The best known characterization of logical constancy incorporating a Belnap-type suggestion to a Kneale-type characterization was offered by Ian Hacking:

> *...a logical constant is a constant that can be introduced, characterized, or defined in a certain way.* What way? My answer is about the same as Kneale's: a logical constant is a constant that can be introduced by operational rules like those of Gentzen. The question becomes, "like" in what respects? Different answers will mark off different conceptions of logic. My answer is that the operational rules introducing a constant should (i) have the subformula property, and (ii) be conservative with respect to the basic facts of deducibility [rules for the reflexivity, transitivity and monotonicity of the relation of deducibility] ([12, pp. 303-304]).

(It will not be necessary to enter into a discussion of the role of the requirement involving the subformula property.) Observe the expressions 'introduced', 'characterized' and 'defined' that appear in Hacking's definition.

Since Hacking treats 'introduced' as synonymous with 'defined', to introduce an expression must be to give its meaning. And not just a meaning introduced by stipulation, but its antecedent meaning. Hacking's definition should obviously be applicable to expressions already in use, not just to expressions "introduced" in a literal sense. Otherwise, Hacking's definition would have no interest as a boundary between logical and non-logical constants of languages already in use. That an expression *in use* is able to be "introduced" (by means of a set of Gentzenian rules of the kind Hacking has in mind) has to mean that those rules "completely give its meaning", i.e., its antecedent meaning, a meaning the expression has independently of the formulation of rules for it. Thus, a notion of the meaning of an expression is employed in the characterization.

But, what notion of meaning is at stake? And what does it mean 'to characterize' or 'to define' an expression? Can Hacking have meant by 'meaning' something like the set of conceptual notes that one associates, or should associate, with an expression? The set of aspects of use of an expression relevant to a full mastery of it? If he meant something like this set of aspects, which we might call the *sense* of an expression, then by 'characterizing' or 'defining' an expression he must have meant to completely list those notes or those aspects. And, if this is so, then I think there can be little doubt that his proposed characterization of logical constancy doesn't even offer a necessary condition of many expressions usually taken to be logical constants. Mark Sainsbury has pointed out, for example, that

> an important type of reasoning involving "all"-sentences, inductive reasoning from instances of a generalization to the generalization itself, might be argued to be partially constitutive of the

> meaning of "all", and it is simply not obvious whether this part would be captured by rules of "all" elimination and introduction ([27, pp. 316-317]).

Not just inductive reasoning involving 'all' seems to form part of the sense of this expression, but also the practice of going from a complete survey of the number of instances of a finite generalization to the generalization itself. Similar points might be made about the material conditional. Surely not all meaning-constituting uses of the sign for the material conditional appear in derivations in which a rule for conditional-introduction has been applied after a logical deduction of the consequent from the antecedent; one may directly assert a material conditional legitimately and in accord with its sense if one is ready to assert that there is, for example, a causal connection between antecedent and consequent.[33]

It is clear, however, that the notion of meaning that Hacking had in mind was not a notion of sense, and that when he spoke of "characterizing" an expression what he meant was something like this: offering some aspects of the *sense* of the expression which, through a certain procedure, suffice to determine its *extension* (or, equivalently, suffice to determine its contribution to the truth-conditions of sentences in which it appears).[34] As Hacking puts it,

> one is in a certain sense able to read off the semantics of the logical constants from the operational rules. Given the underlying notions of truth and logical consequence, the syntactic rules determine a semantics. (...) I claim (...) that the operational rules "fix the meanings of the logical connectives" in the sense of giving a semantics ([12, p. 300]).

The semantics that Hacking is talking about is an extensional semantics which gives instructions for assigning truth values to sentences dominated by logical constants, instructions determined by a procedure applied to the introduction and elimination rules for those constants (see [12, pp. 312ff.]). A technical evaluation of the merits of Hacking's procedure is out of place in this paper. It suffices to say that he claims that the procedure determines for the logical constants the extensional semantics that we antecedently attribute to them. In this sense, according to Hacking, the Gentzenian operational rules for the logical constants, when viewed as sense-constituting, "characterize" the logical constants or "fix their meaning".[35]

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[33]Related points are made in [7, ch. 12].

[34]Cf. [12, p. 317]: "The operational rules display certain features of logical constants. These constants do have a "meaning", *aspects of which* are displayed by these rules" (my emphasis; notice that in this context Hacking puts 'meaning' inside scare-quotes).

[35]The question whether the theories we are talking about provide explicated concepts of logical constancy which could be used as substitutes in a Tarskian definition of logical consequence or logical truth is not entirely transparent. If a theory of this kind speaks of the

Hacking's theory of logical constancy is intended to be compatible with a classical formal view of logical consequence as truth preservation, and thus of logical truth as truth in all interpretations of the non-logical constants[36] (see [12, p. 311]: "a set of sentences Θ is a logical consequence of the set of sentences Γ if no matter what values are assigned to the members of Γ, Θ, some member of Θ is true when every member of Γ is true"). He claims to have proofs that if a truth is a logical truth in this sense (with 'logical constant' understood in his sense) then it is derivable using only Gentzenian rules for the logical constants. This sort of derivability is supposed to imply that the derivable truth is analytic, and thus can be known *a priori* by someone who makes the derivation. Hence we again have a proof of a version of condition (*).

Thus Hacking's theory and similar theories give a characterization of the logical constants which, if correct, would provide a potentially explanatory theory of the semantics and epistemology of the logical constants. Aspects of the sense of the logical constants would be given by simple rules, and acquaintance with this sense would be attained by language-users when learning to use the rules. Then acquaintance with these rules could be used in the derivation of logical truths by language-users, who by means of those derivations could perhaps be said to know those logical truths *a priori*. A careful assessment of the explanatory value of these theories is out of the scope of this paper. But it seems fair to say that, as they stand, theories of this kind face problems when it comes to explaining how acquaintance

Gentzenian operational rules as fixing the *sense* of the logical constants (as Kneale's theory seems to do), then it is moderately clear that it does not provide a concept of logical constancy defined in terms of explicated semantic concepts. If the theory is one like Hacking's, in which fixing the meaning is fixing the extension, the situation may or may not be better. As we said, it seems that what Hacking must have meant is that the rules, viewed as "aspects of the sense", determine the extension. First, what does 'determine' mean? It ought to mean that the claim that the extension is such and such follows (perhaps in Tarski's explicated sense of 'follows') from a certain claim about the rules. But then this latter claim must be at least in part a claim about the rules not being merely syntactical expressions or patterns of use, for otherwise the claim can hardly be said to "determine" or have as a consequence any claim about extensions. If what Hacking meant is that this claim attributes a certain *sense* to the rules then quite obviously his defined concept of logical constancy cannot be a substitute for the unexplicated concept of a logical constant in the general Tarskian definition of logical truth and logical consequence, since the unexplicated semantic notion of sense will appear in the *definiens* of Hacking's general characterization. On the other hand, if what he meant is that the relevant claim attributes a certain "extensional" property to the syntactic expression of the rules then the situation may be different; but it is unclear that this second option does not directly suppress the value of Hacking's theory for an explanation of the apriority of logical truths (see below in the main text).

[36]The same is not true of certain projects of an intuitionistic persuasion, that attempt to make plausible the idea that the meaning of traditional logical constants must be given by certain rules of proof, rules which must satisfy somewhat stricter requirements than Belnap's or Hacking's.

with the *rules*, viewed as syntactic expressions or patterns of use, produces acquaintance with some property of expressions that determines their *denotations*. The basic problem is going to be that it will be mysterious how (acquaintance with) an expression or a pattern of use can determine by itself (acquaintance with) a denotation. And of course, if what the theory claims is that language-users, when learning to use the rules, get acquainted with some denotational or (extensional) semantic property of the rules, then the theory will not even have the *appearance* of clearing up a mystery.

However, it does not seem that Hacking's or any similar characterization of logical constancy can be extensionally correct, even after exegesis has made clear what seem to be its assumptions about meaning and semantics in general. Exegesis has helped us realize that the objection to the adequacy of these characterizations described above, based on the inability of Gentzenian rules to fully exhaust the sense of the logical constants, is founded upon a misunderstanding. (It seems correct, however, and even devastating, in cases where what has been meant by some author in Hacking's line is that Gentzenian rules fully characterize the sense of the logical constants. I think it's clear that Kneale meant precisely this, despite Hacking's remark about the similarity of their theories.) But it seems clear that Hacking's theory is extensionally inadequate for other reasons, related to the reasons why 'unicorn' poses problems for Tarski's theory and Sher's theory, 'heptahedron' poses problems for McCarthy's theory and 'male widow' poses problems for McGee's theory.

Consider the first-order quantifier 'not for all not..., if all are not male widows, and for all not..., if not all are not male widows'. It has the same extension as the usual first-order existential quantifier. If it looks too long, or complicated, or if it bothers you that it is a complex predicate, abbreviate it with '$\underline{\exists}$' and think of it as a non-complex predicate with the already explained meaning. Unlike the usual first-order existential quantifier, '$\underline{\exists}$' does not belong to the intended set of logical constants. But it is a logical constant according to Hacking's criterion. This is so because the *same* typical Gentzenian operational rules for the usual first-order existential quantifier hold for '$\underline{\exists}$'. (They can even be plausibly seen as constituting aspects of its *sense*, for surely whoever is acquainted with the sense of 'male widow' will know that all are not male widows, hence that '$\underline{\exists}$' is analytically coextensional with 'not for all not', etc.) Thus, if Hacking's procedure for "reading off" the semantics from the rules is right, then '$\exists$' and '$\underline{\exists}$' have the same extension. Their denotation is in both cases "fixed" by the rules, in the sense of Hacking.[37]

[37] Sainsbury, [27, pp. 315-316], considers a related problem for a theory of logical constants as expressions whose *sense* is fixed by Gentzenian rules. Specifically, the problem is created by the fact that the rules for '&' also hold for a connective '$\underline{\&}$', where 'A$\underline{\&}$ B' abbreviates 'not(not A or not B)', and this would seem to mean that '&' is not a logical constant according to the

§4. A diagnosis of the difficulties. Let me sum up the main lessons of the foregoing critical discussion. The substantive philosophical problems of logical constants were born in the work of some logicist authors, interested in giving an explanatory theory of the analyticity and apriority of logic. These authors agreed that the concept of logical constancy should satisfy some version of hypothesis (*), and saw the problem as that of giving a theory of the semantic and epistemic properties of the logical constants which could serve to ground that hypothesis. But this project was never carried out to the satisfaction of all concerned, because no satisfactory theory of the semantics and epistemology of logical constants was ever produced.

Tarski asked for a characterization of logical constancy because he wanted to eliminate the usual concept from his mathematical explication of logical truth and logical consequence. But he was not after an explanatory theory of the semantic and epistemic properties of logical truths, and in fact he was strongly skeptical about the possibility of such a theory. He only required an extensionally correct characterization of the traditional set of logical constants, but he required it to be given in terms of logical and mathematical concepts.

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theory, since its rules do not help "determine" its sense as distinct from that of '$\underline{\&}$'. Sainsbury suggests correcting the theory, in two steps: first, define a primitive logical constant to be a constant whose sense is "the least specific sense" which includes the Gentzenian rules; second, define a logical constant to be a constant which can be defined in terms of primitive logical constants. According to Sainsbury, '&' is a primitive logical constant; '$\underline{\&}$' is not, although it is a logical constant; in this way they can be distinguished. A similar maneuver using the notion of sense might be tempting in order to protect Hacking's "denotation" theory from my objection in the main text. Define a primitive logical constant to be a constant which satisfies Hacking's criterion for some Gentzenian rules *and* whose sense is "the least specific sense" which includes those rules; and define a logical constant to be a constant which can be defined in terms of primitive logical constants. If this is to serve any purpose, then '$\exists$' ought to be a primitive logical constant and '$\underline{\exists}$' ought not to be either a primitive logical constant or a logical constant. Unfortunately, it's unclear what to make of this idea before having an adequate understanding of Sainsbury's notion of "less specific sense", which is certainly not a familiar notion from mathematics, epistemology or the philosophy of language (Sainsbury doesn't provide much by way of an explanation). If he were to claim that the sense of '$\exists$' is less specific than the sense of '$\underline{\exists}$' he presumably ought to mean that the first expression is semantically less complex than the second in some natural order of complexity. (He could not mean that '$\underline{\exists}$' has as a matter of fact been introduced by definition using an expression which, unlike '$\exists$', is complex. Nothing in principle seems to preclude the possibility that '$\exists$' has been introduced (even as a matter of fact) by means of a complex definition into our language.) It's not implausible to think that '$\exists$' is semantically less complex than '$\underline{\exists}$' in some natural order of complexity, and this may even be a necessary consequence of the principles that determine choices of constants as logical. But there is nothing to make us think that the corrected characterizations of this note will provide sufficient conditions for intuitive logical constancy –as emphasized in the next section of this paper. Regrettably, the notion of "less specific sense" is too unclear to allow us to even consider the possibility of constructing counterexamples to the corrected characterizations.

We have seen that the Tarskian demand of a characterization has been taken up by later authors, who nevertheless (tacitly influenced by the logicist tradition) often give their characterizations in terms of unexplicated semantic and epistemic concepts, thus making the characterizations unusable for the Tarskian project. The relation of these post-Tarskian proposals to the project of some logicist authors is also tenuous, at least in that many of the post-Tarskian authors do not care much about offering explanatory theories of the semantics and epistemology of logical constants. (Nevertheless, most post-Tarskian authors accept versions of condition (*), and often offer proofs that these versions follow from a Tarskian characterization of logical truth in which the unexplicated notion of logical constancy has been replaced by their defined notions;[38] these proofs of course do not amount to explanatory theories.) Thus post-Tarskian authors often seem to have failed to clearly tell apart the logicist and Tarskian objectives. As a result, they have often engaged in an enterprise of a hybrid nature, that of offering a (non-explanatory) characterization of the intended set of logical expressions in terms of unexplicated semantic and epistemic properties of those expressions. Others have offered proposals more definitely in the spirit of the logicist and Tarskian projects. But all these authors, even those who care about an explanatory theory of the semantics and epistemology of logic (like some proponents of certain loosely proof-theoretic characterizations) and those who respect the restrictions of the Tarskian project of explication, have failed to delimit notions having as their extension the intended set of logical expressions.

We thus see that the implicit conceptions of the problem of logical constants are many, and that they have often emerged out of reactions to earlier conceptions or mixtures of them. Calling attention to the philosophical differences between these conceptions, to the fact that not just one but several different (and sometimes incompatible) philosophical purposes underlie the feeling that a theory of logical constancy is necessary, may lead to a better use of the philosophical effort employed in thinking about the concept of a logical constant.

One of the philosophical conclusions that seem to me to receive strong support from the preceding discussion is that most (perhaps all) of the substantive philosophical conceptions of the problem of logical constants may

[38]A recent author who rejects even a relatively weak and not obviously incorrect version of (*) is John Etchemendy; he rejects even the claim that all Tarskian logical truths in classical quantificational languages are analytic –when one fixes the logical constants of these languages by means of the traditional list (see [8, *passim*]). The merits of this view are discussed critically in several papers (of which see [11] and [13]). Etchemendy also argues that no division of expressions among logical and non-logical is compatible with the claim that intuitive logical truth and Tarskian logical truth are coextensional notions (see [8, ch. 9]). This argument is based on a conflation of logical truth and analytic truth; its flaws are aptly criticized in [24].

have created unsolvable versions of the problem. The search for a characterization of the intended set of logical expressions, which was inherited largely from Tarski's theoretical needs and which forms an essential component of nearly all post-Tarskian conceptions of the problem, may be a hopeless project if it is required (as it is) that the characterization be given in terms of mathematical concepts (Tarski and some post-Tarskians) or unexplicated semantic and epistemic properties (most post-Tarskian authors, influenced by the logicist tradition). If the project is hopeless, then all the versions of the problem generated by these conceptions will be unsolvable.

Why should these problems be unsolvable? As was pointed out in the Introduction, a natural view about the expressions in the usual set of logical constants is that they have probably been selected by implicit application of some very complex, largely pragmatic principles. If this is so, it is unreasonable to expect that a condition given in terms of mathematical, semantic and epistemic properties, and which doesn't use other less purified resources, may provide both necessary *and* sufficient properties of the constants in the intended set.

It can be reasonably expected that some mathematical, semantic and epistemic properties can be isolated which constitute *necessary* properties of the logical expressions, or of a group of them (for example, of the few traditionally logical expressions of classical quantificational languages). In fact, some of the theories reviewed here can plausibly be taken to offer necessary properties of (groups of) logical expressions –the Tarski-Lindenbaum theorem can even be seen as a *proof* of a claim of this kind. That all this can be reasonably expected may even follow from the pragmatic principles mentioned in the Introduction. The general idea behind characterizations like Tarski's and Hacking's is that the (extensional) semantics of logical constants ought to be "simple", obey some simple (in fact mathematical) semantic laws. In the case of other characterizations the "simple" laws invoke epistemic properties as well. And it is not unreasonable to expect that expressions usable in general reasoning ought to have a semantics and an epistemology of lower complexity than the semantics and epistemology of more specialized vocabulary. But then it is unreasonable to expect that *all* expressions with these "simple" mathematical, semantic and epistemic properties will turn out to "look like" expressions that logic should deal with, i.e., that these properties will constitute *sufficient* conditions for membership in the intended set of logical expressions. For it is unreasonable to expect that *all* these expressions will satisfy the principles that do constitute the intuitions about the concept of a logical constant. I think that these ideas are confirmed very accurately by the data yielded by the preceding discussion. None of the characterizations we have reviewed seems to give sufficient conditions for membership in the intended set of logical expressions.[39]

[39] I haven't cared to argue for this claim in those cases in which a characterization didn't even seem to give necessary conditions for membership in that set —the cases of Peacocke's

Ken Warmbrōd (in [38, section 1]) has recently claimed that the search for a characterization of logical constancy giving both necessary and sufficient conditions may be a hopeless project. This view should be distinguished from mine. Warmbrōd's view is summarized in this passage:

> there is nothing inherent in the idea of a set of necessary and sufficient conditions for constancy which guarantees an answer to the critical question, namely, why should terms satisfying the criterion, and only those terms, have their meanings held constant while the meanings of other terms vary [in a Tarskian test for logical truth or logical consequence]. (...) Assuming that no obvious catastrophe results from assigning a fixed meaning to a new term, consideration surely must be given to the benefits achieved by treating the new term as a constant and whether, indeed, such treatment furthers the fundamental purposes of logical theory[40] ([38, p. 511]).

These "fundamental purposes" are the ones I explained in note 5 of the present paper: in particular, the purpose of deductively systematizing scientific theories in satisfactory ways. Now, I take it that it is almost vacuously true that if the expressions that logic *should* study are chosen following some principle-based pragmatic intuition then this intuition provides not only necessary but also sufficient conditions for membership in the intended set of logical expressions. For example, suppose one accepts Warmbrōd's view that logical expressions are those whose study "furthers the fundamental purposes of logical theory", as he understands these purposes. Then, obviously, with this he intends to give a necessary condition for an expression to be logical, but it's hard to see how he could claim that he's not giving a sufficient condition too: are there any expressions whose study "furthers the fundamental purposes of logical theory" and yet are not logical expressions? If so, Warmbrōd's view of logical expressions (of the intended logical expressions, the expressions logic *should* deal with) must be incomplete, and must leave out some pretheoretic intuition about the idea of an expression that logic should deal with.

A view like Warmbrōd's, that a characterization *tout court* is impossible, is plausible if what one is attempting to characterize is the set of expressions

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and Kneale's characterizations. These characterizations are somewhat obscure and it is difficult to decide when a particular expression satisfies them, certainly more difficult than to decide that a particular expression doesn't; and one needs to show that particular expressions satisfy them in order to claim that the characterizations don't give sufficient conditions for membership in the intended set of logical constants.

[40] It ought perhaps to be recalled that the historical reason for speaking of 'constants' when talking about "the problem of logical constants" is not that logical constants have a meaning held "constant" in a reinterpretation test for logical truth, but rather that people have not typically wondered whether variables are logical or non-logical. (There is nothing wrong in the idea of a non-logical constant.)

logic *actually* deals with at some particular moment of its history. This set is presumably determined not just by application of pragmatic principles of the kind with which both Warmbrōd and I are sympathetic, but also by historical accident. And it is unlikely that one can separate in a principle-based way what is sufficiently determined by historical accident (and insufficiently determined by the intentions of logicians) from what is not. It is obvious, however, that philosophers of logic have not attempted to characterize merely the actual set of logical constants, but the intended set, in some normative sense of 'intended'.

I think Warmbrōd has misidentified the source of the difficulties facing typical characterizations of logical constancy. The problem is not that a principle-based characterization is impossible *tout court*. In fact, I think both Warmbrōd and I must be committed to the view that a characterization in terms of complex pragmatic principles is correct as an analysis (in terms of both necessary and sufficient conditions) of the subtle tacit intuitions that have often lain behind choices of expressions for logical study —the subtle intuitions determining the intended set of logical expressions. I mentioned in the Introduction the suggestion that a logical constant may be just an expression which satisfies some appropriate basically pragmatic principles. The complexity of the principles, and also their somewhat vague nature, does much work to avoid any easy refutation of a proposal of this sort —either of the claim that it provides necessary conditions for an expression to be logical, or of the claim that it provides sufficient conditions. The problem with typical characterizations is that they have no place for that sort of complexity, as they try to unearth necessary and sufficient *mathematical*, *semantic* and *epistemic* properties for an expression to be logical; it is a characterization of this sort that seems hopeless.

That to expect an extensionally correct mathematical, semantic or epistemic characterization is unreasonable doesn't mean that the logicist project of giving an explanatory theory of some instances of analyticity and apriority by means of a theory of the peculiar properties of some expressions is hopeless. Even less hopeless is the project of giving some sort of proof or argument for relatively weak versions of (*), such as the thesis, mentioned in note 38, that all Tarskian logical truths in classical quantificational languages (under the traditional selection of logical constants) are analytic. In order to be carried out, these projects do not actually require a characterization of logical constancy in terms of necessary and sufficient conditions, but simply a theory based on semantic and epistemic postulates related in appropriate ways to the relevant logical constants. The reason is simple. In essence, both projects can be seen as aiming at establishing that the logical constants have certain properties, even if they don't have them *because* they are logical constants: roughly, properties that imply the analyticity and apriority of some logical truths and which thus make (versions of) (*) true. No characterization of the set of logical expressions is needed as a premise in arguments for those claims.

Of course I don't intend to deny that the intended set of logical constants might be characterizable using only semantic, epistemic or mathematical properties. But I do think that the chance of this happening is quite small. There is often some strong connection between the explicatum a philosopher proposes for an intuitive concept and the intuitive concept itself (even if the explicatum and the explicandum are different concepts and are characterized in terms of a philosophically distant conceptual apparatus). Take, as a relevant example, the case of the Tarskian explicatum of logical truth. The intuitive concept of logical truth has an intimate connection with the idea of formal truth, or of validity: the idea that a logical truth is a truth that stays true after all substitutions or reinterpretations (as the work of many of the authors reviewed here attests). The Tarskian explicatum domesticates this idea by making precise the notion of truth in an interpretation and by restricting the range of interpretations quantified over to a still vast but mathematically more manageable class of interpretations (set-theoretic structures, in the common version of Tarski's ideas). Or take the case of the usual explicata for the intuitive notion of effectively computable function. This intuitive concept has intimate connections with the ideas that a computation must end in a finite number of steps and that a computation can result of a finite composition of computations. And all of the mathematical explicata of effective computability make use of these ideas in some way or other. Of course the intimate connections do not guarantee success, but they do make success likelier.

But a strong connection with the intuitive concept(s) of logical constancy associated with the pragmatic principles seems to be absent from all reviewed characterizations of logical constancy. If, as seems natural to conjecture, the subtle pragmatic principles or some similar principles or intuitions underlie the choices leading to the usual set of logical expressions, then it doesn't seem reasonable to expect that the corresponding intuitive concept(s) can be characterized in terms of a purified philosophical apparatus.

If this is accepted, reflection on the notion of logical constancy is likely to take forms substantially different from the forms it has taken so far. The need for a characterization is likely to cease to be felt, or at least, it is likely to cease to be seen as a pressing problem in the philosophy of logic. If a characterization is sought, it will be good to have in mind the idea that this characterization may only be possible in terms which are relatively uninteresting from a philosophical point of view. (This is not to deny that characterizations in terms of complex pragmatic principles may have philosophical interest, but certainly it will be a different kind of interest from that which the desired mathematical, semantic or epistemic characterizations would have.) More importantly, if what is sought is a philosophical justification of the analyticity or apriority of some logical truths —whether in the form of an explanatory theory or merely in the form of an argument for some version of (*)—, then efforts will be directed at finding the only thing that this enterprise requires: semantic and epistemic postulates which allow that justification to proceed and which can be plausibly argued to hold of the relevant logical constants.

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