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2001 ANNUAL MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of Pennsylvania, Philadelphia, PA
March 10–13, 2001

The Annual Meeting of the Association was held on the campus of the University of Pennsylvania, March 10–13, 2001. The program included the Eleventh Annual Gödel Lecture, a Retiring Presidential Address, eleven invited talks, four special sessions, ten contributed talks, and two abstracts presented by title. The members of the Program Committee were J. D. Hamkins, D. Haskell, P. Kremer, A. Scedrov (Chair), and R. Shore. The members of the Local Organizing Committee were A. Scedrov, V. Tannen, and S. Weinstein (Chair). There were 111 registered participants at the meeting, including 27 graduate students. Generous financial support was provided by the University of Pennsylvania. The ASL hosted a welcoming reception.

The Eleventh Annual Gödel Lecture, *Recursion theory*, was presented by Theodore A. Slaman (University of California, Berkeley), and the Retiring Presidential Address, *The maxim "maximize" in set theory*, was delivered by Donald A. Martin (UCLA). The hour-long invited addresses included:

Joan Feigenbaum (Yale University), *Incentive-compatible distributed algorithms*.

Haim Gaifman (Columbia University), *Contextual logic and its application to vagueness*.

Jean-Yves Girard (Institut de Mathématiques de Luminy), *Locus solum*.

C. Ward Henson (University of Illinois at Urbana-Champaign), *Model theory in functional analysis*.

Denis Hirschfeldt (University of Chicago), *Structure theorems and non-structure theory in computable structure theory*.

Carl G. Jockusch, Jr. (University of Illinois at Urbana-Champaign), *Ramsey's theorem, computability, and second-order arithmetic*.

Saul Kripke (Princeton University), *Fulfillability: A survey*.

Salma Kuhlmann (University of Saskatchewan), *Ordered exponential fields*.

John C. Mitchell (Stanford University), *Adventures in computer security*.

Ernest Schimmerling (Carnegie Mellon University), *Core models*.

Philip Welch (Institut für Formale Logik, Vienna, and University of Bristol), *The beginning of the inner model hierarchy*.

There also were special sessions (with organizers given in parentheses) on Geometric Model Theory and Applications (M. Laskowski and A. Pillay), Large Cardinals (E. Schimmerling and P. Welch), on Linear Logic (M. Kanovich), and on Reverse Mathematics (S. Simpson), with a total of twenty-three speakers.

Abstracts of talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
ANDRE SCEDROV

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Abstracts of invited plenary talks

► JOAN FEIGENBAUM, *Incentive-compatible distributed algorithms*.
Computer Science, Yale University, P. O. Box 208285, New Haven, Connecticut 06520, USA.
E-mail: jf@cs.yale.edu.

In designing efficient, distributed algorithms and network protocols, computer scientists typically assume either that users are cooperative (i.e., that they follow the protocol) or that they are Byzantine adversaries (i.e., that they may deviate from the protocol in arbitrary ways that harm other users, even if the deviant behavior does not bring them any obvious tangible benefits). In contrast, game theorists design market mechanisms in which it is assumed that users are neither cooperative nor adversarial but rather selfish: They respond to well-defined incentives and will deviate from the protocol only for tangible gain. Until recently, computer scientists ignored incentive compatibility, and game theorists ignored computational efficiency.

The emergence of the Internet as a standard, widely used distributed computing environment and of Internet-enabled commerce has drawn computer scientists' attention to incentive-compatibility questions in distributed computation. In this talk, I will review what is known in this new field of "algorithmic mechanism design" and will pose some basic open problems. Special attention will be paid to questions about algorithmically efficient mechanisms for network-infrastructure problems such as routing, caching, and multicasting.

► HAIM GAIFMAN, *Contextual logic and its application to vagueness*.
Philosophy Department, Columbia University, New York, New York 10027, USA.
E-mail: hg17@columbia.edu.

Contextuality is usually relegated to the domain of informal pragmatics. Indeed, barring indexicals, contextuality does not, usually, lend itself to more than informal accounts, which appeal to common sense and to paradigmatic examples. Even in such situations it is often useful to consider a context operant [ ], which operates on sentences or, in formal settings, on wffs. For a wff A,

$$[c]A$$

is a wff. Here c is a parameter ranging over contexts and $[c]A$ reads, roughly, "A in the context c". Specific systems are obtained by deciding what should count as context, how it is syntactically represented and what is the semantics of the language.

The contextual approach proves efficient in explaining and resolving two ancient paradoxes, the Liar and the Sorites. They are formally resolved in systems where contextuality is explicitly treated. The treatment of the self referential semantic paradoxes has been given in my works on pointers, most recently in "Pointers to propositions"; though that system does not include a context operant, it achieves the desired effect by equivalent devices.

The context operant plays a pivotal role in the treatment of vagueness, in particular, the Sorites. To take a simple example, let the language contain, among other things, a monadic vague predicate P. Then an application of the context operant gives rise to wffs of the form:

$$[t_1, \ldots, t_n]A$$

where n can vary and the ts denote objects which are supposed to be classified into Ps and non-Ps.

The idea is that the sharp boundary between P and non-P may depend on the set of objects that have to be classified; the truth-value of the wff can change accordingly. I consider a full-fledged first-order language, in which the context operant can be applied recursively (interspersed with connectives and quantifiers) and contain quantifiable variables. I provide a semantics, based on plausible intuitions concerning vague predicates, and how they behave under change of context. I give a formal deductive system which is both sound and complete.

► JEAN-YVES GIRARD, *Locus solum*.
Laboratoire de Mathematiques, Equipe Logique de la Programma, 163 Avenue de Luminy Case 930, 13288 Marseille Cedex, France.
E-mail: girard@iml.univ-mrs.fr.

In contrast to the dominant realist (or dualist) paradigm, the procedural approach explains logic as the logic-of-itself, i.e., the logic of its own rules. This view is implicit in Brouwer: intuitionistic disjunction is a disjunction performed on the logical rules. This view has been pushed further up to accommodate negation: linear negation corresponds to a negation (i.e., orthogonal) performed on logical rules.

As soon as logical rules interact with logical rules, things start to be different; the usual "spiritual" logic appears as the result of systematic delocations, necessary to avoid interference. But in the absence of delocations, the new "locative" logic happens to be better behaved than the familiar one.

► C. WARD HENSON, *Model theory in functional analysis*.
University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, Illinois 61801, USA.
E-mail: henson@math.uiuc.edu.

Since the late 1960s, model theoretic ideas have played a significant role in many important results in functional analysis, especially in the geometry of Banach spaces. The main vehicle for these applications has been an ultraproduct construction for Banach spaces that was introduced in the PhD thesis of Jean-Louis Krivine and used by him in papers with Dacunha-Castelle and Bretagnolle. A parallel development, which occurred at the same time, was Luxemburg's introduction of the nonstandard hull construction in nonstandard analysis; along with the Loeb measure construction, this is one of the key contributions of nonstandard analysis to analysis. Now it is understood that the Banach space ultraproduct and the nonstandard hull of an internal Banach space have an essentially model theoretic character and can best be understood via the introduction of an analogue of first order logic for many sorted structures based on metric spaces. This talk will give a quick survey of the most important of these applications, a presentation of the framework within which they can be seen as connected to first order logic, and a sketch of the research program to which all of these ideas naturally lead.

► DENIS HIRSCHFELDT, *Structure theorems and non-structure theory in computable structure theory*.
Mathematics Department, University of Chicago, 5734 University Avenue, Chicago, Illinois 60637, USA.
E-mail: drh@math.uchicago.edu.

There has been increasing interest over the last few decades in the study of the effective content of mathematics. One field whose effective content has been the subject of a large body of work, dating back at least to the early 1960s, is model theory. The study of properties of computable structures has formed an important and fertile branch of computable model theory.

In this talk, I will introduce some of the basic concepts of computable structure theory, and will discuss some of the surprising properties that computable structures can have. As an example, let us consider the computable dimension of a computable structure.

In model theory, we identify isomorphic structures. From the point of view of computable model theory, however, two isomorphic structures might be very different. For example, under the standard ordering of the natural numbers, the successor relation is computable, but it is not hard to construct a computable linear ordering of the same type as the natural numbers in which the successor relation is not computable. Thus, for the purposes of computable model theory, studying structures up to isomorphism is not enough. Instead, we study structures up to computable isomorphism. This leads naturally to the idea of a computable presentation of a structure, which is, roughly speaking, a computable copy of this structure, and to the notion of the computable dimension of a structure, which is the number of computable presentations of the structure up to computable isomorphism.

It is easy to give examples of structures of computable dimension 1 (e.g., the rationals as a linear ordering) or infinite (e.g., the natural numbers as a linear ordering), but much harder to find structures of finite computable dimension greater than 1. Nevertheless, as shown by Goncharov, such structures do exist. In this talk, I will discuss this and related phenomena, both in the general setting and in restricted ones, where it is sometimes the case that some of the phenomena that can occur in general are no longer possible. For example, there are no computable linear orderings or Abelian groups of finite computable dimension greater than 1. (This was shown by Goncharov and Dzgoev and, independently, Remmel for linear orderings and by Goncharov for Abelian groups.)

I hope to explain a currently emerging viewpoint, stemming in part from recent joint work with Bakhadyr Khoussainov, Richard A. Shore, and Arkadii M. Slinko, which sees the question of whether certain computable structure theoretic phenomena can occur in a particular setting as a dichotomy between structure theorems and a nonstructure theory based on the concept of reduction common (in different guises) to several branches of mathematical logic and computer science.

► CARL G. JOCKUSCH, JR., *Ramsey's theorem, computability, and second-order arithmetic*.
Mathematics Department, University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, Illinois 61801, USA.
E-mail: jockusch@math.uiuc.edu.

This talk will deal with the analysis of the infinite form of Ramsey's theorem from two different, but related, points of view. (For our purposes, Ramsey's theorem is the assertion that, for all n, $k \in \omega$, for every k-coloring of the n-element subsets of ω there is an infinite set $A \subseteq \omega$ which is homogeneous in the sense that all n-element subsets of A have the same color.) The first aspect is the complexity of the infinite homogeneous sets for computable k-colorings P of the n-element sets of natural numbers. In particular, I will discuss recent joint work with Peter Cholak and Theodore Slaman showing that for every computable k-coloring P of the n-element sets of natural numbers there is an infinite homogeneous set $A \subseteq \omega$ with A'' (the second Turing jump of A) computable from $0^{(n)}$. The second aspect is the strength of Ramsey's theorem as a statement in second-order arithmetic. Here the emphasis will be on exponent 2, i.e., Ramsey's theorem for pairs, where the previously mentioned result gives an infinite low_2 homogeneous set. The same authors have obtained the related result that every Π^1_1-statement in the language of second-order number theory provable from $\mathrm{RCA}_0 + \mathrm{I}\Sigma_2 + \mathrm{RT}^2_2$ is provable from just $\mathrm{RCA}_0 + \mathrm{I}\Sigma_2$. (Here RCA_0 is the standard weak base theory for reverse mathematics based on Δ^0_1 comprehension and Σ^0_1 induction, $\mathrm{I}\Sigma_2$ is the Σ_2 induction scheme, and RT^2_2 is Ramsey's theorem for 2-colorings of unordered pairs.) They also showed that RT^2_2 is strictly weaker that $\mathrm{RT}^2_{<\infty}$ over RCA_0. (Here $\mathrm{RT}^2_{<\infty}$ is $(\forall k)\ \mathrm{RT}^2_k$ where RT^2_k is Ramsey's theorem for k-colorings of pairs.) Many of the results to be discussed are in the paper "On the strength of Ramsey's theorem for pairs," by P. Cholak, C. Jockusch, and T. Slaman, to appear in the *Journal of Symbolic Logic*. Related results of the speaker, S. Simpson, D. Seetapun, L. Harrington, J. Hirst and others will also be discussed.

► SALMA KUHLMANN, *Ordered exponential fields*.
Department of Mathematics and Statistics, University of Saskatchewan, 106 Wiggins Road, Saskatoon, Saskatchewan, Canada S7N 5E6.
E-mail: skuhlman@math.usask.ca.

The study of o-minimal expansions of the reals has witnessed a remarkable progress in the last decade. The notion of o-minimality was first introduced by van den Dries [2] (further developed in [10]) while studying Tarski's problem. In 1991 Wilkie [12] proved the o-minimality of $(\mathbb{R}, \exp)$; the expansion of the reals by the exponential function (and more recently by all Pfaffian functions [13]). This was a major breakthrough and has motivated further research on this topic. Van den Dries ([3]) observed that the definable sets in an o-minimal expansion share many of the nice geometric properties of semi-algebraic sets, and the quest for analogies between the semi-algebraic case and the o-minimal case has often guided this research. The progress and the goals have spread in several directions. For example, new important expansions of the reals by functions were shown to be o-minimal (e.g., [5], [6], [13]). The asymptotic behaviour of definable functions, and the Hardy fields associated to o-minimal expansions were subjects of detailed analysis (e.g., [7], [1], [11]). Constructions of models (e.g., [4], [7], [9]) and understanding their algebraic (valuation theoretic) properties ([7], [8]) was another fascinating and challenging problem.

In this talk, I will focus mainly on this last problem, presenting what is achieved to date and what remains to be done. In particular (for polynomially bounded + (exp) expansions) I will describe the non-archimedean models and their associated Hardy fields. The basic tool for this is valuation theory, and I will explain how one can develop an exponential analogue for all important notions and methods of the theory of real places and real closed fields. I will show how to use this abstract machinery to give concrete constructions of models with interesting algebraic and model theoretic properties. These constructions use power series fields, and I will discuss both the power and limitations of these methods.

[1] M. Aschenbrenner and L. van den Dries, *H-fields and their Liouville extensions*, to appear in ***Mathematische Zeitschriften***.

[2] L. van den Dries, *Remarks on Tarski's problem concerning* $(\mathbb{R}, +, \cdot, \exp)$, in ***Logic colloquium '82***, 1984, pp. 97–121.

[3] ———, *O-minimal structures*, in ***Logic: From foundations to applications***, European Logic Colloquium, Oxford, 1996, pp. 137–185.

[4] L. van den Dries, A. Macintyre, and D. Marker, *Logarithmic-exponential power series*, ***Journal of the London Mathematical Society***, vol. 56 (1997), pp. 417–434.

[5] L. van den Dries and P. Speissegger, *The real field with convergent generalized power series*, ***Transactions of the American Mathematical Society***, vol. 350 (1998), pp. 4377–4421.

[6] ———, *The field of reals with multisummable series and the exponential function*, ***Proceedings of the London Mathematical Society***, to appear.

[7] S. Kuhlmann, *Ordered exponential fields*, The Fields Institute Monograph Series, vol. 12, American Mathematical Society Publications, 2000.

[8] F.-V. Kuhlmann and S. Kuhlmann, *Valuation theory of exponential Hardy fields*, submitted June 2000, 35 pages.

[9] F.-V. Kuhlmann, S. Kuhlmann, and S. Shelah, *Exponentiation in power series fields*, ***Proceedings of the American Mathematical Society***, vol. 125 (1997), pp. 3177–3183.

[10] J. Knight, A. Pillay, and C. Steinhorn, *Definable sets in ordered structures II*, ***Transactions of the American Mathematical Society***, vol. 295 (1986), pp. 593–605.

[11] J. M. Lion, C. Miller, and P. Speissegger, *Differential equations over polynomially bounded o-minimal structures*, preprint, 2000.

[12] A. J. Wilkie, *Model completeness results for expansions of the real field*, ***Journal of the American Mathematical Society***, vol. 9 (1996), pp. 1051–1094.

[13] ———, *A general theorem of the complement and some new o-minimal structures*, ***Selecta Mathematica***, New Series, vol. 5 (1999), pp. 397–421.

► DONALD A. MARTIN, *The maxim 'maximize' in set theory*.
Mathematics Department, UCLA, Los Angeles, California 90095-1555, USA.
E-mail: dam@math.ucla.edu.

One sometimes hears it alleged that various set theoretic axioms and hypotheses are motivated or even justified by a maximization principle demanding something like: "Adopt theories that yield the most sets." Among the assertions said to be supported by such a principle are the Axiom of Infinity, large cardinal axioms, and certain forcing axioms.

I will present a mostly critical discussion of maximization principles. Some of the topics I will consider are: (1) maximization principles governing choice of theories versus maximization principles governing choice of concepts; (2) the difficulties of formulating a maximization principle that has the intended applications; (3) attempts to overcome these difficulties; (4) the question of whether maximization principles can be motivated from a non-platonistic viewpoint.

► JOHN C. MITCHELL, *Adventures in computer security*.
Department of Computer Science, Stanford University, Stanford, California 94305, USA.
E-mail: mitchell@cs.stanford.edu.

Computer security is the branch of computer science concerned with the protection of computer systems and digital information. While cryptography is a well-developed area with a sound mathematical basis, most practical computer security problems involve failures of software design and implementation, not failures of cryptography. The areas of network security, access control, intrusion detection, and system security are full of informal practices waiting to be given precise definitions and analyzed. This talk will use network security protocols to illustrate some general themes in computer security and to identify and explore applications for methods from mathematical logic to computer science.

► ERNEST SCHIMMERLING, *Core models*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890, USA.
E-mail: eschimme@andrew.cmu.edu.

Core models are generalizations of Gödel's Constructible Universe L. Another well-known core model is $L[U]$, the minimal transitive proper class model with a measurable cardinal. The hierarchy of core models has levels between L and $L[U]$, for example, $L[0^{\#}]$. There are also levels beyond $L[U]$.

In this talk, which is intended for non-specialists, I will attempt to give some intuition about what core models are and why they are important in diverse parts of set theory. One of the key links that I will discuss is Jensen's combinatorial principle $\Box$, which he showed holds in L. If κ is a cardinal, then $\Box_\kappa$ is local combinatorial principle that follows immediately from $\Box$. Early on, Solovay proved that if κ is a κ^+-supercompact cardinal, then $\Box_\kappa$ fails. Recently, Zeman and I have characterized those κ for which $\Box_\kappa$ holds in a given core model M based on a large cardinal property of κ in M. This solves a problem with a long history, which I plan also to discuss.

► PHILIP WELCH, *The beginning of the inner model hierarchy*.
Institut für Formale Logik, Währingerstr. 25, A-1090 Wien, Austria.
E-mail: welch@logic.univie.ac.at.

We shall, in a survey talk, look at some of the features of the Inner Model Hierarchy that occur very low down in the wellordering of such models. Some of the phenomena that occur here are below the traditional classification of models by embeddings or even older fashioned small "large cardinals". In this arena some of the classification appears best done using consistency strength relating to purely descriptive set theoretic properties. The concepts involved here for example include, those of fragments of determinacy in certain pointclasses, weak forms of Chang Conjecture, and the universal Baire property.

Abstracts of invited talks in the Special Session on Geometric Model Theory and Applications

► GREGORY CHERLIN, *Simple groups of finite Morley rank of characteristic 2 type*.
Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903, USA.
E-mail: cherlin@math.rutgers.edu.

It has been conjectured that all simple connected groups of finite Morley rank are algebraic. We consider simple groups of finite Morley rank whose proper definable connected simple sections are algebraic. In this case the structure of the connected component of a 2-Sylow subgroup is known; it is either trivial (in which case the group is a counterexample to the conjecture), divisible abelian (in which case the group is hopefully algebraic in characteristic not 2), or of bounded exponent (in which case the group is almost certainly algebraic in characteristic 2); the analysis in the last of these three cases is essentially complete, and while modeled on the corresponding analysis in the finite case, takes advantage of the amalgam method as used by Delgado and Stellmacher, which produces a drastic reduction in the complexity of the arguments in the present context. The proof relies on the work of a broad range of model theorists; those most actively involved in the characteristic 2 type classification project as such have been Altinel, Berkman, Borovik, Cherlin, Corredor, and Jaligot; and work on BN pairs by Kramer, Tent, and van Maldeghem is invoked to complete the recognition phase.

► HANS SCHOUTENS, *Cell decomposition in tame topologies*.
Department of Mathematics, Rutgers University, Hill Center – Busch Campus, 110 Frelinghuysen Rd., Piscataway, New Jersey 08854-8019, USA.
E-mail: hschoute@math.rutgers.edu.

I will present a generalization of o-minimality, termed t-minimality, based on the same underlying principle: provided we have a structure M with a definable topology (in [Pillay87] these are called 'first order topological structures') with the extra property that each definable subset of M admits a 'simple' description (as a finite union of points and basic opens), then the definable sets in the Cartesian powers M^n exhibit 'tame' behavior. More explicitly, under some fairly general model theoretic conditions (strong t-minimality and definable Skolem functions) I show that the following properties hold: (i) each definable set is a finite union of cells (Cell Decomposition); (ii) each definable set has a dimension; and, under some additional, topological, conditions (iii) every definable function is piecewise continuous; (iv) every definable set admits finitely many connected (definable) components. Two instances in which this applies are: (1) o-minimal expansions of fields, and, with some minor modifications, (2) finite extensions of p-adic fields. In both bases, Cell Decomposition is known (by van den Dries, Pillay and Steinhorn for (1) and by Denef for (2)), but by slightly different arguments. Therefore, the present method provides a unified treatment.

► CHARLES STEINHORN, *On properties that yield close-to-tame expansions of real closed fields*.
Department of Mathematics, Vassar College, Poughkeepsie, New York 12604, USA.
E-mail: steinhorn@vassar.edu.

This is a report on joint work in progress of C. Miller and the author that has its origins in a paper of Miller and P. Speissegger [1] on the open core of an expansion of a dense linear order. The current work seeks to identify topological and/or model theoretic conditions which imply that an expansion of a (non-archimedean) real closed field exhibits good model-theoretic behavior.

[1] C. Miller and P. Speissegger, *Expansions of the real line by open sets: o-minimality and open cores*, ***Fundamenta Mathematica***, vol. 162 (1999), pp. 193–208.

Abstracts of invited talks in the Special Session on Large Cardinals

► JAMES CUMMINGS, *Squares*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, Pennsylvania 15218-3890, USA.
E-mail: `jcumming+@andrew.cmu.edu`.

I will discuss various problems connected with the existence of square sequences.

► MIRNA DZAMONJA, *Weak reflection*.
School of Mathematics, University of East Anglia, Norwich NR 47T, United Kingdom.
E-mail: `m.dzamonja@uea.ac.uk`.

Weak reflection, as the name suggests, is a version of stationary reflection that is weaker from the ordinary stationary reflection. It was introduced for a different reason, but turned out to be useful in proving that various instances of stationary reflection are independent of each other. We comment on these results and offer a further discussion of independence results associated with stationary reflection.

► MATTHEW FOREMAN, *Simultaneous reflection of stationary subset of $P_{\omega_1}(\lambda)$*.
Department of Mathematics, University of California, Irvine, Irvine, California 92717, USA.
E-mail: `mforeman@math.uci.edu`.

In joint work with S. Todorcevic, we show that if you can simultaneously reflect any four stationary subsets of $P_{\omega_1}(\lambda)$ then the singular cardinals hypothesis holds. Moreover, you can always reflect to sets of uniform cofinality, and if the CH holds, to internally approachable sets. The critical lemma is the stationarity of a particular set in $P_{\omega_1}(\lambda)$. The proof of this lemma uses "Namba Combinatorics" and club guessing.

► MENACHEM KOJMAN, *Plane geometry and forcing*.
Department of Mathematics, Ben Gurion University of the Negev, P. O. B. 653, Be'er-Sheva 84105, Israel.
E-mail: `kojman@cs.bgu.ac.il`.

Two dual notions measure non-convexity of a set S in the Euclidean plane:

(a) the least cardinality of a convex cover of the S, denoted $\gamma(S)$;
(b) the maximal cardinality of $P \subseteq S$ so that no 3 points from P belong to a single convex subset of S, denoted $\alpha(S)$.

It will be shown that for a closed planar set S,

(1) the parameter $\alpha(S)$ satisfies $\alpha(S) \leq \aleph_0$ or $\alpha(S) = 2^{\aleph_0}$
(2) the parameter $\gamma(S)$ satisfies $\gamma(S) \leq \aleph_0$ or $\gamma(S) = 2^{\aleph_0}$ or $\gamma(S)^+ = 2^{\aleph_0}$.

So there is no apparent duality: two or more values of γ can correspond to $\alpha = \aleph_0$.

Duality is true, though, if one looks also at forcing extensions:

Theorem. *For a closed planar S, $\alpha(S) \leq \aleph_0$ if and only if in some forcing extension $\gamma(S) < 2^{\aleph_0}$. Even better: if $S \subset \mathbb{R}^2$ is closed and $\gamma(S) = c$ holds in all forcing extensions of the universe, then S contains a perfect P so that $|P \cap C| < 3$ for every convex $C \subset S$.*

The proofs reduce $\gamma(S)$ to homogeneity numbers for continuous pair-colorings of the Baire space.

Joint work with: S. Geschke, W. Kubis and R. Schipperus.

► PAUL LARSON, *Bounding by canonical functions, with CH*.
Department of Mathematics, University of Toronto, Toronto, Ontario, Canada M5S 3G3.
E-mail: larson@math.toronto.edu.

In joint work with Sarahon Shelah, we show that the continuum hypothesis is consistent with the statement that every function from ω_1 to ω_1 is bounded on a club by a canonical function for an ordinal less than ω_2. The key technical idea is to generalize the notion of α-properness to semi-proper forcing.

Abstracts of invited talks in the Special Session on Linear Logic

► RICHARD BLUTE, *Hopf algebras and models of noncommutative logic*.
Department of Mathematics and Statistics, University of Ottawa, 585 King Edward, Ottawa, Canada K1N 6N5.
E-mail: rblute@mathstat.uottowa.ca.

We give a definition of categorical models for the multiplicative fragment of non-commutative logic, which we call entropic categories. We will focus on methods of building entropic categories using the representation theory of Hopf algebras. Hopf algebras provide an excellent framework for modelling a number of variants of multiplicative linear logic, such as commutative and cyclic linear logic. We extend these ideas to the entropic setting by developing a new type of Hopf algebra, which we call entropic Hopf algebras. We show that the category of modules over an entropic Hopf algebra is an entropic category, (possibly after application of the Chu construction). Several examples are constructed and the Tannaka-Krein reconstruction theorem is extended to the entropic setting.

► ESFANDIAR HAGHVERDI, *From geometry of interaction to denotational semantics, a categorical perspective*.
Department of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6395, USA.
E-mail: esfan@math.upenn.edu.

The Geometry of Interaction (GoI) program was introduced by Girard in late 80s in a series of influential papers. GoI is a new kind of semantics which captures the dynamics of computation (cut-elimination). This is in contrast to the extant denotational semantics which is static and syntax which is blurred with unnecessary bureaucracy.

In this review talk we look at the role played by category theoretical approaches to GoI. In particular, we discuss the importance of the notion of trace in a categorical setting. Our emphasis will be on the class of partially additive categories of Manes and Arbib and their extensions. We follow this path to show that the use of compact closure and double glueing constructions provides us with denotational models for the multiplicative fragment of linear logic, hence establishing a connection between two different approaches to the semantics of programming languages.

► DOMINIC HUGHES, *Reinventing MALL proof nets*.
Department of Computer Science, Gates Building, Stanford University, Stanford, California 94305, USA.
E-mail: dominic@cs.stanford.edu.

Multiplicative linear logic has a beautiful theory of proof nets. An extension to multiplicative additive linear logic (MALL) was given by Girard in 1995 in “Proof nets: the parallel syntax for proof-theory”. However (quoting from the paper), “although this notion remains the only serious candidate, it is to be noted that it is far from being absolutely satisfactory.”

In this talk I shall propose a new definition of MALL proof net, with the following features:

- The sequentialisation theorem has been verified for all MALL formulae of up to 10 literals.
- Two highly desirable properties of MLL proof nets are preserved:
  (a) every proof defines a unique proof net.
  (b) the class of proof structures is natural. (I shall explain this in the talk.)
  These properties were sacrificed in the original 1995 definition of MALL proof net.

Hence there is considerable hope that this may be the "right" definition of MALL proof net. Work in progress is the proof of the full sequentialisation theorem.

► P. J. SCOTT, *What are full completeness theorems?*
Department of Mathematics, University of Ottawa, 585 King Edward, Ottawa, Ontario, Canada.
E-mail: phil@mathstat.uottawa.ca.

Full Completeness Theorems are completeness theorems at the level of proofs (rather than provability). Under the formulas-as-types view of logic, this says that a model M for a logic L is fully complete if functions in the model between definable types say $[[A]] \rightarrow [[B]]$ are themselves the interpretation of (unique) proofs of entailments $A \vdash B$. In categorical logic, thinking of L as a free structured category, full completeness says the unique interpretation functor of L into M is a fully faithful modelling of the proof theory of L. The goal is to find mathematically interesting fully complete models M for fragments of Girard's linear logic (which has a rich structure of interesting subtheories).

In general, the known fully-complete models (for various fragments of linear logic) fall into five classes: (i) game-theoretic models, (ii) traditional mathematical models (e.g., reflexive topological vector spaces, spaces equipped with continuous actions of Hopf algebras, reflexive topological abelian groups via Pontrayagin duality, etc.) (iii) models based on Girard's GoI program, (iv) coherence and hypercoherence space models, (v) generalizations of (iv) via Chu spaces and relational double-glueing models.

We shall survey a selection of recent results in the area and discuss some of the different general methods of such proofs. If time permits, we shall discuss current work with R. Blute, M. Hamano, and myself on building fully complete models for MALL starting from Erhard's hypercoherences.

Abstracts of invited talks in the Special Session on Reverse Mathematics

► JEREMY AVIGAD, *Weak theories of nonstandard arithmetic and analysis*.
Department of Philosophy, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA.
E-mail: avigad+@andrew.cmu.edu.

In this talk, I will discuss weak higher-type theories of nonstandard arithmetic. Such theories provide a natural setting for formalizing nonstandard arguments in analysis and combinatorics, while an explicit forcing translation allows one to intepret them in their standard counterparts. This enables us to formalize interesting portions of mathematics in conservative extensions of primitive recursive arithmetic, elementary recursive arithmetic, and possibly even polynomial-time computable arithmetic.

[1] J. Avigad, *Weak theories of nonstandard arithmetic and analysis*, to appear in ***Reverse mathematics 2001*** (S. Simpson, editor).
[2] J. Avigad and J. Helzner, *Transfer principles for intuitionistic nonstandard arithmetic*, to appear in ***Archive for Mathematical Logic***.

► DOUGLAS A. CENZER, *Degrees of difficulty of* Π^0_1 *classes*.
Department of Mathematics, University of Florida, 358 Little Hall, P. O. Box 118105, Gainesville, Florida 32611-8105, USA.
E-mail: cenzer@math.ufl.edu.

The degrees of difficulty of a class are given by the partial ordering where P is Medvedev reducible to Q if and only if there is a computable function F which maps Q into P. If P and Q represent the sets of solutions to two different problems, then we can use F to compute a solution $F(x)$ to the first problem from a solution x to the second problem. Thus the degree of difficulty of computing a solution to the first problem is less than that of the second problem. The connection with reverse mathematics is that, speaking informally, the computable function F can be applied to turn a proof that the second problem has a solution into a proof that the first problem has a solution. The Medvedev degrees compose a lattice with the least element being the family of classes with a computable element. We consider the restriction of this lattice to (recursively bounded) Π^1_0 classes, where there is now also a greatest element. The main results are that this restricted lattice has no minimal element and that for any class P of positive measure, there exists a class Q which is not below P, and thus another class which is strictly above P. The class Q may be taken to be the class of separating sets of some recursively inseparable pair of computable enumerable sets.

► RICHARD A. SHORE, *Invariants, Boolean algebras and ACA*$^+$.
Department of Mathematics, Cornell University, Ithaca, New York 14853, USA.
E-mail: shore@math.cornell.edu.

The existence or nonexistence of invariants generally says something about the complexity of certain problems. It therefore seems to be an appropriate area for research from the viewpoint of reverse mathematics. As the statement that there are invariants for some class of (countable) structures is one of third order arithmetic, the first issue is formulating the problem in second order arithmetic. Once this is done, one can investigate the complexity of such statements in the hierarchy of axiom systems of reverse mathematics. One easy example is vector spaces. Here the existence of number invariants for isomorphism is equivalent (over RCA) to ACA.

Boolean algebras supply more interesting results. The isomorphism relation for arbitrary Boolean algebras is Σ^1_1 complete so no simply defined invariants for isomorphism can exist and the existence of invariants implies $\Pi^1_1-\mathrm{CA}$. However, it is a classical result of Tarski that there are number invariants for elementary equivalence of Boolean algebras. The existence of any such invariants is equivalent (over RCA) to ACA$^+$. This is the first real example of a theorem known to be equivalent to ACA$^+$ (the system corresponding to the existence of the ω-jump of A, for each set A). These invariants are, of course, isomorphism invariants for the class of saturated Boolean algebras. These algebras can be described algebraically and are called the dense Boolean algebras. The existence of number invariants for isomorphism for this class of Boolean algebras is also equivalent to ACA$^+$.

► STEPHEN G. SIMPSON, *An overview of reverse mathematics*.
Department of Mathematics, Pennsylvania State University, McAllister Building, University Park, Pennsylvania 16802, USA.
E-mail: simpson@math.psu.edu.

Foundations of mathematics is the study of the most basic concepts and logical structure of mathematics as a whole. Reverse mathematics is the foundational program of discovering which set existence axioms are needed to prove known theorems in core mathematical areas such as algebra, analysis, geometry, countable combinatorics. It turns out that a large number of theorems fall into a small number of equivalence classes with respect to provable equivalence over a weak base theory. (See my book ***Subsystems of Second Order***

Arithmetic, Springer-Verlag, 1999.) The equivalence classes appear to reflect well known philosophical/foundational programs such as constructivism (Bishop), finitistic reductionism (Hilbert), predicativism (Weyl, Feferman), predicative reductionism, and impredicative analysis. In this talk I comment on the mathematical, philosophical, and foundational significance of reverse mathematics as it has developed from the early 1970s to the present. I also comment on the significance of reverse mathematics for the principal subdivisions of mathematical logic: model theory, set theory, recursion theory, proof theory.

Abstracts of contributed talks

► ANDREW ARANA, *Arithmetic and independence*.
Department of Mathematics, University of Notre Dame, 370 CCMB, Notre Dame, Indiana 46556, USA.
E-mail: aarana@nd.edu.

In this talk we will discuss some new independence results. We will then apply these independence results to some computability-theoretic questions regarding models of first-order Peano arithmetic (PA).

► ALEXANDER BERENSTEIN, *The group configuration in simple stable homogeneous models*.
Department of Mathematics, University of Notre Dame, Notre Dame, Indiana 46556, USA.
E-mail: berenstein.1@nd.edu.

In [2], Pillay defined simplicity and stability for existentially closed structures. In [1], Buechler and Lessmann introduce simplicity and stability for homogeneous models. We study the notion of germ in the stable existentially closed structures and in the stable homogeneous models. We define a group configuration in this setting and show how to produce a group of germs from it.

[1] S. Buechler and O. Lessmann, *Simple homogeneous models*, preprint, 2000.
[2] A. Pillay, *Forking in the category of existentially closed structures*, preprint, 1998.

► LEONA F. FASS, *Gedanken-experiments on cyberspace*.
P. O. Box 2914, Carmel, California 93921, USA.
E-mail: lff4@cornell.edu.

As a theoretician interacting with cyberspace, we cannot help but analyze the structure and behavior of complex software systems, or the knowledge that complicated informational domains convey. We first considered such issues within our inductive inference research, seeking "optimal" models of knowledge and theoretical machines/systems, from specifications of how they should (or should not) behave. As an *experimenting user* of actual systems and machines, we are gratified to find that our theoretical results can have "real world" utility.

In our inference research, constraints imposed upon a knowledge domain enabled us to determine *perfect* behavioral realizations. Components of a behavioral model corresponded to categories of knowledge, and *if* a finite model existed, it could be constructed effectively, from a sample of knowledge. A specific finite realization might also be found adaptively, by iteratively modifying a potential model until its behavior tested as "correct".

This satisfying "verification" result may not apply to actual machines and systems but, rather, to their mathematical models [5, 7]. Theoretical constraints may not be imposed on "real" cyberspace, and "correctness" may be difficult, if not impossible, to completely specify. Hence we accept modeling of knowledge sub-domains; establishment that a sub-system is "approximately correct"; or, in the spirit of [2], detection of incorrectness and then, finding a better "less incorrect" system [4, 5].

With this approach our behavioral-modeling work has been applicable to cyberspace problems, most notably in model-based software design and in the use of model-checkers to detect specification/system flaws [4, 5, 6]. Our techniques for discovering model components, constructively or adaptively, are useful in the design of intelligent agents that decompose domain problems and solve them: autonomously, interactively, or reactively [1, 4]. Lastly, our determination of knowledge categories and the behaviors they define is closely related to the cyberspace process of link analysis [3]. There our research has had demonstrable success in determining a predictive stockmarket sub-domain model, and a locally-restricted model for detecting voter fraud.

[1] M. BIENKOWSKI, *A reader's guide to agent literacy*, ***SIGART Bulletin***, vol. 9, no. 2, (Fall 1998), pp. 23–29.

[2] J. C. CHERNIAVSKY, *Computer systems as scientific theories: a Popperian approach to testing*, ***Proceedings of the fifth Pacific Northwest software quality conference***, Portland, Oregon, October 1987, pp. 297–308.

[3] L. F. FASS, *Inductive inference and link analysis*, ***Papers from the AAAI fall symposium on artificial intelligence and link analysis***, Orlando, Florida, October 1998, AAAI Press FS 98-01, pp. 35–37.

[4] ———, *Electronic commerce is an intriguing domain for AI learning theory*, ***Papers from the AAAI-99 workshop on artificial intelligence for electronic commerce***, Orlando, Florida, July 1999, AAAI Press WS 99-01, pp. 115–116.

[5] ———, *Establishing software 'correctness' by logical and algebraic means*, ***Proceedings of the fifth joint conference on information sciences***, Atlantic City, New Jersey, February 27–March 3, 2000, vol. I, pp. 639–642.

[6] L. Khatib and C. Pecheur, editors, ***Papers from the AAAI spring symposium on model-based validation of intelligence***, Stanford University, March 2001, AAAI Press, SS01-04.

[7] J. S. MOORE, *Machines reasoning about machines*, Invited Plenary Lecture, AAAI 2000 National Conference, Austin, Texas, August 2, 2000; and private communication, September 2000.

► JEFFRY L. HIRST, *Reverse mathematics of ordinal arithmetic*.
Department of Mathematical Sciences, Appalachian State University, Boone, North Carolina 28608, USA.
E-mail: jlh@math.appstate.edu.

This talk will survey those portions of ordinal arithmetic that have been analyzed in the framework of reverse mathematics. Almost all statements of ordinal arithmetic are either provable in the base system RCA_0 or are equivalent to the system ATR_0. There are interesting exceptions to this rule. In particular, the equivalence of Cantor's definition of well orderings in terms of least upper bounds and the usual definition in terms of descending sequences is equivalent to ACA_0.

► SHAMIL ISHMUKHAVETOV AND GUOHUA WU, *A high d. c. e. degree isolated by a low c. e. degree*.
Department of Mathematics, Ulyanovsk State University, 42 Tolstoy St., 432700, Ulyanovsk, Russia.
E-mail: ishm@vens.ru.
School of Mathematical and Computing Sciences, Victoria University of Wellington, P. O. Box 600, Wellington, New Zealand.
E-mail: wu@mcs.vuw.ac.nz.

Let $\mathbf{D}_2$ be the set of all d. c. e. degrees, and $\mathbf{R}$ be the set of all c. e. degrees. Then $\mathbf{R} \subset \mathbf{D}_2$. Say that a d. c. e. degree $\mathbf{d}$ is isolated by a c. e. degree $\mathbf{a}$ if $\mathbf{a} < \mathbf{d}$ is the largest c. e. degree below $\mathbf{d}$. One would have the impression that if $\mathbf{d}$ is isolated by $\mathbf{a}$, then $\mathbf{d}$ should be "close" to **a**, since there is no c. e. degree between **d** and **a**. Most recent results in this area seem to support this hypothesis. For example, the isolated degrees are dense in the c. e. degrees, see [2]. However, in [3], Wu showed that there is a high d. c. e. degree **d** which can be isolated by a low$_2$ c. e. degree. We improve this result by showing that the isolating degree involved can be low, not merely low$_2$.

[1] S. B. Cooper and X. Yi, *Isolated d. c. e. degrees*, University of Leeds, Department of Pure Mathematics, 1995, Preprint series, no. 17, 25 pages.

[2] G. L. LaForte, *The isolated d. r. e. degrees are dense in the r. e. degrees*, ***Mathematical Logic Quarterly***, vol. 42 (1996), pp. 83–103.

[3] G. Wu, *Isolation and the jump operator*, School of Mathematical and Computing Sciences, Victoria University of Wellington, Research report 00-18, 15 pages.

► STEFFEN LEMPP, *Computability-theoretic and proof-theoretic aspects of partial and linear orderings*.
Departmnt of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706.
E-mail: lempp@math.wisc.edu.

This talk reports on joint work with Rod Downey, Denis Hirschfeldt and Reed Solomon.

Szpilrajn's Theorem states that any partial order $\mathcal{P} = \langle S, <_P \rangle$ has a linear extension $\mathcal{L} = \langle S, <_L \rangle$. This is a central result in the theory of partial orderings, allowing one to define, for instance, the dimension of a partial ordering. It is now natural to ask questions like "Does a well-partial ordering always have a well-ordered linear extension?" Variations of Szpilrajn's Theorem state, for various (but not for all) linear order types τ, that if $\mathcal{P}$ does not contain a subchain of order type τ, then we can choose $\mathcal{L}$ so that $\mathcal{L}$ also does not contain a subchain of order type τ. In particular, a well-partial ordering always has a well-ordered extension.

We show that several effective versions of variations of Szpilrajn's Theorem fail, and use this to narrow down their proof-theoretic strength in the spirit of reverse mathematics.

► RUSSELL MILLER, *Definable incompleteness and Friedberg splittings*.
Department of Mathematics, Cornell University, Ithaca, New York 14853, USA.
E-mail: russell@math.cornell.edu.

We define a property $R(A_0, A_1)$ in the partial order $\mathcal{E}$ of computably enumerable sets under inclusion, and prove that R implies that A_0 is noncomputable and incomplete. Moreover, the property is nonvacuous, and the A_0 and A_1 which we build satisfying R form a Friedberg splitting of their union A, with A_1 prompt and A promptly simple. We conclude that A_0 and A_1 lie in distinct orbits under automorphisms of $\mathcal{E}$, yielding a strong answer to a question previously explored by Downey, Stob, and Soare about whether halves of Friedberg splittings must lie in the same orbit.

[1] R. Downey and M. Stob, *Friedberg splittings of recursively enumerable sets*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 175–199.

[2] L. Harrington and R. I. Soare, *Post's program and incomplete recursively enumerable sets*, ***Proceedings of the National Academy of Sciences of the USA***, vol. 88 (1991), pp. 10242–10246.

► REED SOLOMON, *The group of computable automorphisms of a countable dense linear order*.
University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, Wisconsin 53706, USA.
E-mail: rsolomon@math.wisc.edu.

The group of all automorphisms of a countable dense linear order has been extensively studied. In this talk, we compare and contrast group theoretic properties of this group with group theoretic properties of the group of all computable automorphisms of a computable dense linear order. In particular, we consider divisibility, commutators, conjugation, and normal subgroups.

► EVGUENI VASSILIEV, *Homogeneous rank 1 structures with a generic automorphism*.
Department of Mathematics, University of Notre Dame, Notre Dame, Indiana 46556, USA.
E-mail: evassili@hilbert.helios.nd.edu.

We consider an analogue of the generic automorphism expansion of first order structures [2, 3] in the case of homogeneous models [1]. Under some technical assumptions, we show the existence of a generic automorphism σ for a homogeneous minimal structure M (i.e., a homogeneous supersimple SU-rank 1 structure with a unique large 1-type over any small set), and homogeneity of the resulting expansion (M, σ). Then we follow the ideas of [3, 4] to prove supersimplicity of (M, σ) in the sense of [1].

[1] S. Buechler and O. Lessmann, *Simple homogeneous models*, preprint, 2000.

[2] Z. Chatzidakis and A. Pillay, *Generic structures and simple theories*, ***Annals of Pure and Applied Logic***, vol. 95 (1998), pp. 71–92.

[3] A. Pillay, *Forking in the category of existentially closed structures*, preprint, 1998.

[4] ———, *Lecture notes on strongly minimal sets (and fields) with a generic automorphism*, preprint, 2000.

► DAN E. WILLARD, *The cut-free version of the second incompleteness theorem extends to* $\mathrm{I}\Sigma_0$.
University at Albany Computer Science Department, Albany, New York 12222, USA.
E-mail: dew@cs.albany.edu.

Let d denote any cut-free method of deduction, such as Semantic Tableaux, Herbrand Deduction or the Cut-Free version of the Sequent Calculus. Let us say that an axiom system α satisfies the *Conventional Deciphering Property* if and only if there exists some *finite subset* of its axioms capable of recognizing the Gödel numbers of all of α's axioms. For each cut-free deductive method d, we have proven that every consistent axiom system α that contains all $\mathrm{I}\Sigma_0$'s axioms and satisfies the Conventional Deciphering Property is unable to prove a theorem asserting the non-existence of a proof of $0 = 1$ from itself.

Prior to establishing this new result, we had published an article [1] discussing how a similar Second Incompleteness Effect was applicable almost down to the level of Robinson's Arithmetical System Q. Our prior result had shown that each cut-free deduction method d could be associated with a corresponding Π_1 sentence, called V_d, such that if α is any consistent extension of $Q + V_d$ satisfying the Conventional Deciphering Property then α cannot prove the non-existence of a proof of $0 = 1$ from itself.

Part of the intuition behind our new theorem is that it can be established that each of the above sentences V_d is a theorem of $\mathrm{I}\Sigma_0$. However, we must do some additional work, precluding a super-exponential growth effect, for our generalization of the Cut-Free Second Incompleteness Theorem to apply to $\mathrm{I}\Sigma_0$. In particular, it is well known that if W is a theorem of α and T is a theorem of $\alpha + W$ then a cut-free proof of T from α can be super-exponentially longer than the comparable proof from $\alpha + W$. The heart of our generalization of the Second Incompleteness Theorem rests on showing that this super-exponential growth is precluded in the particular case where α corresponds to $\mathrm{I}\Sigma_0$ and W corresponds to V_d.

[1] D. Willard, *The semantic tableaux version of the second incompleteness theorem extends almost to Robinson's artithmetic Q*, in ***Automated reasoning with semantic tableaux and related methods***, Lecture Notes in Artificial Intelligence, vol. 1847, Springer-Verlag, 2000, pp. 415–430.

Abstracts presented by title

► JOHN CORCORAN, *Omega arguments in the 1931 Gödel formalism*.
Department of Philosophy, University of Buffalo, SUNY, Buffalo, New York 14260-1010, USA.
E-mail: corcoran@acsu.buffalo.edu.

A (premise-conclusion) argument is an ordered pair consisting of a set of sentences, its premise-set, and a single sentence, its conclusion. In a (logically) valid argument the conclusion is a (logical) consequence of its premise-set. Informally, an omega argument has as its conclusion a universal sentence 'every number has (property) *P*', whose numeral instances, '0 has *P*', '1 has *P*', '2 has *P*', and so on, make up the premise set. This paper treats omega arguments in the formalism of the 1931 Gödel incompleteness paper where the (natural) numbers are exhaustively denoted by the numerals '*o*', '*so*', '*sso*', etc., '*s*' being the symbol for 'the successor of'. In this formalism, PMI, the Principle of Mathematical Induction, is (logically) equivalent to the sentence (translated) 'every property that belongs to every inductive number belongs to every number', called Eliminative Mathematical Induction, EMI. As usual, 'x is inductive' abbreviates 'x has every property belonging to zero and to the successor of every number to which it belongs'. Three closely related observations frame the whole treatment. First, an omega argument with conclusion 'every number has *P*' is valid if and only if the corresponding instance of EMI is tautological, i.e., if and only if the conditional sentence 'if every inductive number has *P* then every number has *P*' is logically true. Second, every omega argument is transformed into a valid argument by adding as a new premise the corresponding instance of EMI, which of course is not in general tautological. Third, the infinite premise-set of an omega argument concluding 'every number has property *P*' is logically equivalent to 'every inductive number has *P*', which is the antecedent of the corresponding instance of EMI. Since EMI is a second-order sentence the above observations do not apply when all considerations are restricted to first-order; but in this case analogous observations are made. Throughout the treatment interesting and sometimes surprising examples are given. The goal is to remove the mystery from the topic of omega arguments without destroying its charm.

► STEVEN LINDELL AND SCOTT WEINSTEIN, *The role of decidability in first order separations over classes of finite structures*.
Haverford College, 370 Lancaster Avenue, Haverford, Pennsylvania 19041-1392, USA.
E-mail: slindell@haverford.edu.

We establish that the decidability of the first order theory of a class of finite structures C is a simple and useful condition for guaranteeing that the expressive power of FO + LFP properly extends that of FO on C, unifying separation results for various classes of structures that have been studied. We then apply this result to show that it encompasses certain constructive pebble game techniques which are widely used to establish separations between FO and FO + LFP, and demonstrate that these same techniques cannot succeed in performing separations from any complexity class that contains DLOGTIME.