THE BULLETIN OF SYMBOLIC LOGIC
Volume 7, Number 3, Sept. 2001

2000-2001 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Minneapolis Hyatt Regency, Minneapolis, Minnesota
May 3–4, 2001

A Spring Meeting of the Association for Symbolic Logic was held in Minneapolis, Minnesota on May 3-4, 2001, in conjunction with the annual meeting of the Central Division of the American Philosophical Association. The Program Committee consisted of Jeremy Avigad, Richard Heck, David McCarty and Michael Detlefsen (Chair). The program consisted of four symposia, each with three invited speakers, and two sessions of contributed papers, each also with three speakers.

The topics of the four symposia were: (i) Reverse Mathematics and Computability Theory, (ii) History and Philosophy of Mathematics, (iii) Formal Methods in Philosophy, and (iv) Proof Theory and Constructivism.

Speakers and topics in the Reverse Mathematics and Computability Theory Symposium (chaired by Wayne Richter, University of Minnesota-Twin Cities) were:

Jeffry Hirst (Appalachian State University), *Graphs, free sets, and reverse mathematics*.

Reed Solomon (University of Wisconsin-Madison), *Partial orders, ordered structures, and reverse mathematics*.

Stephen Simpson (Pennsylvania State University), *Why the recursion theorists ought to thank me*.

Speakers and topics in the History and Philosophy of Mathematics Symposium (chaired by Michael Detlefsen, University of Notre Dame) were:

James Tappenden (University of Michigan-Ann Arbor), *Proof styles in mathematics*.

Erich Reck (University of California-Riverside), *Categoricity and completeness: from Dedekind to Carnap and beyond*.

Colin McLarty (Case Western Reserve University), *Cantor's cardinals as category theory*.

Speakers and topics in the Formal Methods in Philosophy Symposium (chaired by William Hanson, University of Minnesota-Twin Cities) were:

Rohit Parikh (City University of New York), *Belief revision*.

Larry Moss (Indiana University-Bloomington), *Logic can change the world*.

Scott Weinstein (University of Pennsylvania), *Logic in the finite*.

Speakers and topics in the Proof Theory and Constructivism Symposium (chaired by Gabriel Uzquiano, University of Rochester) were:

Geoffrey Hellman (University of Minnesota-Twin Cities), *Constructivity and mathematical existence*.

Grigori Mints (Stanford University), *Epsilon substitution, foundations, applications*.

Richard Zach (University of California-Berkeley), *Hilbert's 'Ansatz' for the epsilon-substitution method and Ackermann's dissertation*.

1079-8986/01/0703-0010/$1.70

Speakers and topics of contributed papers were:

Kevin C. Klement (University of Massachusetts, Amherst), *Some difficulties with sense-functions in Church's logic of sense and denotation*.

Gabriel Uzquiano (University of Rochester), *Categoricity results for second-order set theory*.

Andrew Arana (University of Notre Dame), *Minimal consistency and independence over PA*.

Paul Bankston (Marquette University), *More on the model theory of continua*.

Natasha Dobrinen (University of Minnesota, Twin Cities), *A complete embedding of the Cohen algebra into the Galvin-Hajnal algebra*.

Wim Ruitenburg (Marquette University), *Classical model theory and Kripke models*.

Colin McLarty and Reed Solomon chaired the two sessions of contributed papers.

Abstracts of the invited and contributed talks follow.

For the Program Committee
MICHAEL DETLEFSEN

Abstracts of invited talks in the Symposium on Reverse Mathematics and Computability Theory

► JEFFRY L. HIRST, *Graphs, free sets, and reverse mathematics*.
Department of Mathematical Sciences, Appalachian State University, Boone, North Carolina 28608, USA.
E-mail: `jlh@math.appstate.edu`.

Reverse mathematics analyzes the strength of mathematical theorems by proving equivalence results in weak subsystems of second order arithmetic. This talk introduces the main subsystems and illustrates their application to statements of graph theory, with emphasis on the relationship between computability theory and reverse mathematics. Some open questions concerning the free set theory will be included.

► STEPHEN G. SIMPSON, *Why the recursion theorists ought to thank me*.
Department of Mathematics, Penn State University, USA.
E-mail: `simpson@math.psu.edu`.

Foundations of mathematics (f. o. m.) is the study of the most basic concepts and logical structure of mathematics. Gödel, Turing, Post and Kleene pioneered recursion theory (a. k.,a. computability theory) in an f. o. m. context. Over a period of forty years, recursion theory lost touch with its f. o. m. roots. More recently, reverse mathematics came to the rescue. Reverse mathematics is an explicitly foundational research program which aims to discover which axioms are necessary and sufficient to prove standard theorems of core mathematics. For recursion theorists, reverse mathematics represents a rich problem area where some (but probably not all) highly developed recursion-theoretic techniques can be applied to draw f. o. m. conclusions which are of general intellectual interest. Confronted with this opportunity, many leading recursion theorists react favorably, while others such as Robart I. Soare exhibit intense hostility. We comment on scientific and sociological aspects of the confrontation.

► REED SOLOMON, *Partial orders, ordered structures, and reverse mathematics*.
Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, Wisconsin 53706, USA.

E-mail: `rsolomonmath.wisc.edu`.

Reverse mathematics seeks to determine which set theoretic axioms are necessary to prove particular theorems of countable mathematics. The axiom systems used in reverse mathematics are weak subsystems of second order arithmetic. The most natural models of such systems are ω-models, that is, models in which the standard natural numbers form the first order part. Computability theory provides a useful array of tools for constructing ω-models, and hence for obtaining results in reverse mathematics. For example, to show that a theorem T is not provable in a subsystem S, it suffices to construct an ω-model in which the axioms of S are true, but T is not. In this talk, we examine several theorems about orderings to illustrate how techniques in computability theory can be used to obtain results in reverse mathematics. Part of this work is joint with Rod Downey, Denis Hirschfeldt, and Steffen Lempp.

Abstracts of invited talks in the Symposium on History and Philosophy of Mathematics

► COLIN MCLARTY, *Cantor's cardinals as category theory*.
Case Western Reserve University, USA.
E-mail: `cxm7@po.cwru.edu`.

Cantor's set theory did not have extensionality. He would not say what the elements of any set are, only how many it has. Zermelo criticized this, but it is now Mac Lane's approach to categorical foundations. Insofar as work on large cardinals, etc., is isomorphism invariant it does not use extensionality anyway. Cantor also mentions kinds of "sets" (Menge) with intrinsic structure, geometry, or order. This is Lawvere's viewpoint in categorical foundations. It poses the question of V = L very differently from ZF—asking not "how big" the universe is but "what part" of it is sets in the narrow sense.

► ERICH H. RECK, *Categoricity and completeness: From Dedekind to Carnap and beyond*.
Department of Philosophy, University of California, Riverside, California 92521-0201, USA.
E-mail: `reck@citrus.ucr.edu`.

Mathematicians and logicians have introduced several different, though related, notions of completeness for systems of mathematical axioms. We start by distinguishing these notions explicity, and by reviewing some basic facts about them. Then their historical origins in the development of the axiomatic method in nineteenth and early twentieth century mathematics are established. The focus here is on the contributions of four mathematicians: Dedekind, Hilbert, Huntington, and Veblen. This leads us, next, to some neglected metatheoretic works by Fraenkel and Carnap from the 1920s, in which the first systematic discussions of the interrelations between our notions of completeness can be found. Finally, we direct attention back to a remaining question, central to the work of Carnap (see[1]), concerning the relation between categoricity and one form of completeness. The question is this: Suppose T is a finitely axiomatizable theory in higher order logic. Does the semantic completeness of T, i.e., the assumption that all of its models satisfy the same higher-order sentences, imply the categoricity of T?

This is a report on joint work with Steve Awodey; see [2].

[1] S. Awodey and A.W. Carus, ***Carnap, completeness, and categoricity: The Gabelbarkeitssatz of 1928***, Erkenntnis 54:2, March 2001, pp. 145-172.

[2] S. Awodey and E.H. Reck, ***Completeness and categoricity: 19th century axiomatics to 21st century semantics***, Carnegie Mellon University technical report CMU-PHIL-130, June 2001.

► JAMES TAPPENDEN, *Proof styles in mathematics*.
Department of Philosophy, University of Michigan, Angell Hall, Ann Arbor, Michigan 48109, USA.
E-mail: tappen@umich.edu.

This paper explores the philosophical significance of certain differences in proof style, with special emphsis on the role of visual representation. The core case studies are Artin's Geometric Algebra and Hall's Ternary rings.

Abstracts of invited talks in the Symposium on Formal Methods in Philosophy

► LAWRENCE S. MOSS, *Logic can change the world*.
Department of Philosophy, Indiana University, Sycamore Hall, Bloomington, Indiana 47405, USA.
E-mail: lmoss@indiana.edu.

The work reported here is a contribution to epistemic logic that has both conceptual and technical sides.

On a conceptual level, we present a modeling of group-level epistemic actions and the resulting changes to the knowledge states of agents. The actions that we can model include things like honest announcements to groups, announcements intercepted by suspicious outsiders, etc. We work on op of the standard logic of knowledge, using the Kripke semantics. But our system enriches this with new operators for epistemic actions. The first people to mention an enrichment in this way are Jan Plaza and, independently, Jelle Gerbrandy.

On the technical side, we study logics with the new operators and also common knowledge operators. These are logic of knowledge and change, not knowledge and time. We have complete axiomatizations, the logics are decidable, and there are a number of expressive power results.

The work reported here is being applied (sometimes with interesting modifications) to the modeling of things like: dialog, probabilistic reasoning, games, and security. I hope to survey the applications of this area as well.

This talk is based on work done with Alexandru Baltag and Slawomir Solecki.

► SCOTT WEINSTEIN, *Logic in the finite*.
Department of Philosophy, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6304, USA.
E-mail: weinstei@linc.cis.upenn.edu.

A brief survey of results and problems in finite model theory.

Abstracts of invited talks in the Symposium on Proof Theory and Constructivism

► GEOFFREY HELLMAN, *Constructivity and mathematical existence*.
Department of Philosophy, University of Minnesota, Minneapolis, Minnesota 55455, USA.
E-mail: hellm001@tc.umn.edu.

A common way to resolve the apparent conflict between constructivism and classicism in mathematics is to point out that the two traditions attach very different meanings to the "logical vocabulary", reflecting interest in different universes of objects, e.g., constructive vs. non-constructive functions, etc. A challenge to this tidy resolution is posed by the views of the mathematician Fred Richman. The Richman constructivist claims to be speaking of the same objects in the same language as the classicist, but to have a better standard of proof, allowing for constructive interpretability of results ("logical generality"). Moreover, doubt is raised over the soundness of LEM, suggesting different usage, after all. In any case, a structuralist approach to mathematical objects presents special challenges to the constructivist. Structural axioms of interest may well imply LEM (e.g., Foundation, LUB, etc.). Yet, in scientific applications, classical structures, e.g., space-time continua, may actually exist and must not be ignored on a priori grounds. Certain celebrated theorems on causal structure of space-time are, moreover, apparently non-constructive, yet they yield insight. (Hawking-Penrose singularity theorems, for example [1].) And in quantum mechanics, unbounded operators have wider domains when treated classically than constructively, theoretically an advantage despite some non-computability, illustrating a competing kind of generality, i.e., of conclusions, also of importance in the sciences [2].

[1] G. HELLMAN, *Mathematical constructivism in space-time*, ***British Journal for Philosophy of Science***, vol. 49 (1998), pp. 25–39.

[2] ———, *Quantum mechanical unbounded operators and constructive mathematics—A rejoinder to Bridges*, ***Journal of Philosophical Logic***, vol. 26 (1997), no. 2, pp. 121–127.

► GRIGORI MINTS, *Epsilon substitution for impredicative systems*.
Philosophy Department, Stanford University, Stanford, California 94305-2155, USA.
E-mail: mints@csli.stanford.edu.

This is a survey of the present state of epsilon substitution method for analysis.

► RICHARD ZACH, *Hilbert's "Ansatz" for the ε-substitution method and Ackermann's dissertation*.
Group in Logic and the Methodology of Science, University of California, Berkeley, California 94720, USA.
E-mail: zach@math.berkeley.edu.

The central technical goal of Hilbert's Program in the foundation of mathematics was giving proofs of consistency of formal systems of analysis (and set theory). Early advances in this direction were made by Hilbert (and Bernays) in a series of lecture courses at the University of Göttingen between 1917 and 1923, and notably in Ackermann's dissertation of 1924 [1]. The main innovation was the invention of the ε-calculus, on which Hilbert's axiom systems were based, and the development of the ε-substitution method as a basis for consistency proofs. In his dissertation, Ackermann gave consistency proofs for primitive recursive arithmetic and analysis (without the axiom of choice), the latter using the substitution method. It is striking that these proofs use transfinite induction not dissimilar to that used in Gentzen's later consistency proof as well as non-primitive recursive definitions, and that these methods were accepted as finitistic at the time. Although it is now clear that Ackermann's consistency proof for analysis cannot be correct, it was believed until Gödel's results became known in 1930 that he had succeeded in proving consistency of at least first-order arithmetic.

[1] ———, *Begründung des "tertium non datur" mittels der Hilbertschen Theorie der Widerspruchsfreiheit*, ***Mathematische Annalen***, vol. 93 (1924), pp. 1–36.

Abstracts of contributed talks

► ANDREW ARANA, *Minimal consistency and independence over PA*.
Department of Mathematics, University of Notre Dame, 370 CCMB, Notre Dame, Indiana 46556, USA.
E-mail: aarana@nd.edu.

We say that a sentence φ in the language of first-order Peano Arithmetic (PA) is minimally consistent over PA if φ is consistent with PA and if there is no sentence ψ in the language of PA consistent with PA such that

$$\mathrm{PA} + \varphi \vdash \mathrm{Con}(\mathrm{PA} + \psi).$$

A natural question to ask is whether there is a minimally consistent sentence over PA. Toward answering that question, we ask whether there is an infinite sequence of sentence $(\varphi_i)_{i\in\omega}$ such that

$$\mathrm{PA} + \varphi_i \vdash \mathrm{Con}(\mathrm{PA} + \varphi_{i+1}).$$

If there were no such sentence, then for any maximally finite sequence the last term would be minimally consistent over PA. However, we show there is such a sequence. Our proof uses the Barwise-Kreisel Compactness Theorem.

► PAUL BANKSTON, *More on the model theory of continua*.
Department of Mathematics, Statistics and Computer Science, Marquette University, Milwaukee, Wisconsin 53201-1881, USA.
E-mail: paulb@mscs.mu.edu.

Our consuming interest is the model theory of compact Hausdorff spaces, especially of continua. Define a *lattice base* for a topological space to be a meet-dense sublattice of the closed set lattice for the space. Then it is not hard to show that the class of all bounded lattices that are isomorphic to lattice bases for continua is Π^0_2-definable. In this talk we discuss a version of the Chang-Łoś-Suszko theorem appropriate to the study of compact Hausdorff spaces, and prove that the class of indecomposable continua is Π^0_2-definable. (We have yet to discover a Π^0_2-axiomatization for this class.)

► NATASHA DOBRINEN, *A complete embedding of the Cohen algebra into the Galvin-Hajnal algebra*.
University of Minnesota, 206 Church St. SE, 127 Vincent Hall, Minneapolis, Minnesota 55414, USA.
E-mail: dobrinen@math.umn.edu.

In the late 1920s, von Neumann conjectured that measurable Boolean algebras can be characterized as those σ-algebras which satisfy the countable chain condition and the weak (ω, ω)-distributed law [2]. Although consistent counterexamples have been constructed, von Neumann's proposed characterization has not been disproved within ZFC. It is of interest, therefore, to investigate examples of c. c.,c., non-measurable, complete Boolean algebras to find out exactly how much semblance of weak (ω, ω)-distributivity, if any, they possess.

Galvin and Hajnal constructed a partial ordering $\mathbf{P}$ which satisfies the countable chain condition but not the σ-bounded chain condition [1]. The Boolean algebra r. o.$(\mathbf{P})$ is an example of a non-measurable Boolean algebra which satisfies the c. c. c., but not the weak (ω, ω)-distributive law. I have constructed a complete embedding of the Cohen algebra, r. o.$(\mathrm{Clop}(\omega^\omega))$, into r. o.$(\mathbf{P})$. Not only does the weak (ω, ω)-distributivity law fail in the Cohen algebra, but moreover, the Cohen algebra is in a sense the most non-infinitely distributive Boolean algebra. Thus, the complete embedding of the Cohen algebra into r. o.$(\mathbf{P})$ is the strongest result to date, regarding distributivity, as to why r. o.$(\mathbf{P})$ is non-measurable.

[1] W. W. COMFORT and S. NEGREPONTIS, ***Chain conditions in topology***, Cambridge University Press, Cambridge, 1982.

[2] D. MAULDIN, ***The Scottish book***, Birkhäuser, Boston, 1981, pp. 240–243.

► KEVIN C. KLEMENT, *Some difficulties with sense-functions in Church's logic of sense and denotation*.
Philosophy Department, 352 Bartlett Hall, University of Massachusetts, 130 Hicks Way, Amherst, Massachusetts 01003-9269, USA.
E-mail: klement@philos.umass.edu.

This paper discusses certain problems concerning the treatment of senses picking out functions in Church's Logic of Sense and Denotation. Church understands such senses to be "sense-functions", functions from sense to sense. However, the conditions he lays out under which a sense-function is to be regarded as a sense picking out another function allow for certain undesirable results given certain unusual or "deviant" sense-functions. When combined with the identity conditions he layws out for complex senses in his "Alternative(0)," certain absurdities result. E.g., an argument can be found for equating any two senses of the same type (including senses picking out different truth values, threatening the consistency of the system). An alternative treatment of senses picking out functions is discussed, and is thought to do better justice to Frege's original theory of sense. Senses that pick out functions should not be understood themselves to be functions (or, at least, not understood as functions in the same sense).

► WIM RUITENBURG, *Classical model theory and Kripke models*.
Department of Mathematics, Marquette University, P. O. Box 1881, Milwaukee, Wisconsin 53201, USA.
E-mail: wimr@mscs.mu.edu.

Part of this paper is derived from joint work with Mohammad Ardeshir and Saeed Salehi. We give illustrations of how classical model theory can be successfully employed in Kripke model theory. We illustrate how certain classical model theoretic results have natural generalizations to the Kripke model case.

► GABRIEL UZQUIANO, *Categoricity results for second-order set theory*.
Department of Philosophy, University of Rochester, 523 Lattimore Hall, Rochester, New York 14627, USA.
E-mail: uzquiano@ling.rochester.edu.

There is a well-known almost-categoricity result for second-order set theory. Two models of second-order ZFC are not necessarily isomorphic to each other, but at least one is isomorphic to an initial segment of the other. The situation is more subtle when we allow for urelements, but there is a recent categoricity result due to Vann McGee in [2]. Two models of second-order ZFCU satisfying the additional axiom that the urelements form a set with the same universe of discourse are not necessarily isomorphic to each other, but the pure sets of one are isomorphic to the pure sets of the other. One reason this result is of interest is that we can appeal to this theory to fix a determinate truth value for every sentence of pure set theory.

We investigate whether similar categoricity results obtain for weaker axiomatizations of set theory with urelements. In [1], George Boolos argued that the axioms of standard set theory are motivated by a mixture of two conceptions of set: The iterative conception of a set, on which sets are formed in stages of a cumulative hierarchy; and the limitation of size doctrine, on which some objects form a set if and only if they are not too numerous. We discuss whether there are categoricity results for second-order variants of Zermelo set theory with urelements which, unlike standard formalizations of this theory, are adequate axiomatizations of the iterative conception. A similar question arises for second-order axiomatizations of the limitation of size doctrine. We consider the case of New V, a modification of Frege's Basic Law V developed by Boolos in [1] that is in line with the limitation of size doctrine.

[1] G. Boolos, *Iteration again*, ***Philosophical Topics***, vol. 17 (1989), pp. 15–21.

[2] V. McGee, *How we learn mathematical language*, ***The Philosophical Review***, vol. 106 (1997), pp. 35–68.