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2000-2001 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Sheraton New Orleans Hotel, New Orleans, Louisiana
January 12–13, 2001

A Winter Meeting of the Association for Symbolic Logic was held January 12–13, in New Orleans, Louisiana, at the Sheraton New Orleans Hotel in conjunction with the annual joint meetings of the American Mathematical Society, the Mathematical Society of America, and the Society of Industrial and Applied Mathematics. The Program Committee consisted of James Cummings, Marcia Groszek, and Dave Marker (chair). The program consisted of nine invited 50-minute lectures, a special session of three 25-minute talks in Set Theory, and ten contributed talks. In addition, on January 11 there was an AMS Special Session in Model Theory organized by Stephen Buechler and Sergei Starchenko.

The following invited 50-minute talks were presented at the meeting:

Peter Cholak (University of Notre Dame), *The latest news about the computably enumerable sets*.

Lou van den Dries (University of Illinois at Urbana-Champaign), *Kronecker's problem*.

Denis R. Hirschfeldt (University of Chicago), *The structure of the Solovay degrees of computably enumerable reals*.

Tamara J. Hummel (Allegheny College), *Ramsey's theorem and computability theory: A survey*.

Alexander Kechris (California Institute of Technology), *Logical aspects of the classification problem for torsion-free abelian groups of finite rank*.

Paul Larson, *Canonical theories for the first two uncountable cardinals*.

Thomas Scanlon (University of California at Berkeley), *Model theory of compact complex manifolds*.

John Steel (University of California at Berkeley), *The core model induction*.

Shmuel Weinberger (University of Chicago), *Computability, complexity, and riemannian variational problems*.

The following talks were presented in the special session in Set Theory.

Itay Neeman (UCLA), *Determinacy and large cardinals*.

Slawomir Solecki (University of Indiana), *Translation invariant ideals*.

Jindrich Zapletal (University of Florida), $\mathfrak{p}$.

Ilijas Farah (CUNY) was invited to participate in the Set Theory special session, but was unable to attend.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
David E. Marker

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Abstracts of invited talks

► PETER CHOLAK, *The latest news about the computably enumerable sets.*
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As partially suggested by the title this talk will focus on the latest (exciting) results of Leo Harrington and the speaker on the computably enumerable sets.

► LOU VAN DEN DRIES, *Kronecker's problem.*
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green St., Altgeld Hall, Urbana, Illinois 61801, USA.

Given any polynomials $f_0(X), f_1(X), \dots, f_k(X) \in \mathbb{Z}[X]$, $X = (X_1, \dots, X_n)$, when do there exist $g_1(X), \dots, g_k(X) \in \mathbb{Z}[X]$ such that $f_0 = g_1 f_1 + \cdots + g_k f_k$?

This is a key problem in Kronecker's "finite type" mathematics. It seems to be much more difficult than the analogous ideal membership problem for polynomials over fields. Nevertheless, a decision procedure has been known for about 40 years. More recently, the method of Gröbner bases has led to a procedure whose number of steps could be explicitly bounded in terms of the size of the coefficients, degrees of the f_is, and number n of variables. While for fixed n this upperbound is primitive recursive, as a function of n it involves the notorious Ackermann function, and is thus not primitive recursive.

A completely different *p-adic* approach to this and several related problems will appear in the thesis by M. Aschenbrenner. This method leads to much better (primitive recursive) bounds, and a good understanding of dependence on parameters from a logical-arithmetic viewpoint. I will touch on the history of the problem, and then discuss Aschenbrenner's work.

► DENIS R. HIRSCHFELDT, *The structure of the Solovay degrees of computably enumerable reals.*
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This talk concerns computably enumerable reals and their relative randomness. A real is c. e. if it is the limit of a computable increasing sequence of rationals; all reals mentioned below are assumed to be c. e. A real is random if its dyadic expansion forms a random infinite binary sequence (in the sense of, for instance, Martin-Löf). Chaitin's number Ω, the halting probability of a universal self-delimiting computer, is a standard random c. e. real.

Solovay discovered that several important properties of Ω (whose definition is model-dependent) are shared by a class of reals with a model-independent definition. To define this class, he introduced the concept of domination or Solovay reducibility, notated $\leq_S$. Informally, $\beta \leq_S \alpha$ if an approximating sequence for α can be used to generate one for β.

It is not hard to show that if $\beta \leq_S \alpha$ then the initial segment complexity of α is at least that of β. Solovay defined a c. e. real to be Ω-like if it dominates all c. e. reals. Recently, Slaman proved that a c. e. real is random if and only if it is Ω-like.

Solovay reducibility gives rise to a degree structure on the c. e. reals. The Solovay degrees form a distributive uppersemilattice (with join given by addition) and have many attractive features. However, their structure has remained largely unknown. Coles, Downey, and LaForte have shown that this structure is very complicated by proving that it has an undecidable first order theory. After an introduction to the basic concepts mentioned above, this talk will focus on joint work of the speaker with Rod Downey and André Nies, in which we investigate this theory more closely. First we show that the Solovay degrees are dense. To do this we divide the proof into two parts, first proving that every *incomplete* Solovay degree splits over any lesser degree, and then proving that the Solovay degrees are upwards dense. The proof of the first part is of particular technical interest, as it is a priority construction in which the control of the injuries is directly tied to the approximation of Ω.

In light of these results, and further motivated by the general question of how randomness can be produced, it is natural to ask whether the complete Solovay degree can be split, or in other words, whether there exist nonrandom c. e. reals whose sum is random. We give a negative answer to this question, thus providing a new characterization of the random c. e. reals.

► TAMARA J. HUMMEL, *Ramsey's theorem and computability theory: A survey*.
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An infinite version of Ramsey's theorem states that for every coloring of $[\omega]^n$ (the set of all n-element subsets of ω) by finitely many colors, there is an infinite set A which is homogeneous for that coloring, i.e., all elements of $[A]^n$ have the same color. We present a survey of results and open questions concerning the complexity of infinite homogeneous sets of effectively given (computable or computably enumerable) colorings. We also consider n-cohesive and n-r-cohesive sets, infinite sets which are almost homogeneous for every computably enumerable (respectively, computable) coloring of $[\omega]^n$ by two colors.

► ALEXANDER KECHRIS, *Logical aspects of the classification problem for torsion-free abelian groups of finite rank*.
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The torsion free abelian groups of rank (at most) n are simply the subgroups of $(\mathbb{Q}^n, +)$. A satisfactory classification, up to isomorphism, of rank 1 groups has been found by Baer in the 1930s but it has been a long standing open problem to find a satisfactory classification of rank 2 or higher groups. In this talk, I will give a general introduction to a theory of complexity of classification problems, such this one, and describe work of G. Hjorth, S. Adams-A. Kechris, and S. Thomas, which aims at quantifying the difficulty of such a classification. An interesting aspect of this work is its use of a mixture of methods from algebra, descriptive set theory, and the ergodic theory of discrete and linear algebraic groups.

► PAUL LARSON, *Canonical theories for the first two uncountable cardinals*.
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We consider some issues in extending the work on canonical models for $L(\mathbb{R})$ and $L(\mathcal{P}(\omega_1))$ to ω_2. Large cardinals have given us a canonical theory for $L(\mathbb{R})$, and forcing axioms, notably the $\mathbb{P}_{\max}$ axiom $(*)$ have given canonical theories for $L(\mathbb{P}(\omega_1))$ which are in a certain sense complete. What sort of principles might resolve the theory at the next level is unclear. In some ways, the maximizing intuitions at ω_1 and ω_2 contradict one another. We will discuss some test questions at ω_2 and some possible approaches.

► THOMAS SCANLON, *Model theory of compact complex manifolds*.
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While there have been brilliant successes with the model theory of real and p-adic analytic functions, complex analysis has resisted a model theoretic treatment. The zero sets of entire complex analytic functions can be quite pathological. For example, the integers are definable in $(\mathbb{C}, +, \cdot, \exp)$ so that at the very best one might hope to understand its theory relative to the theory of true arithmetic. Boris Zilber has proposed a satisfactory solution to the model theory of the complex exponential function, but his program depends upon conjectures in transcendental number theory and diophantine geometry which are extremely unlikely to be resolved in the near future. On the other hand, if one attempts to simplify the theory of complex analytic functions (as one does for real analytic functions) by considering only restrictions of analytic functions to closed disks, then the theory is already bi-interpretable with a theory of the real field with some analytic functions.

However, while complex analytic *functions* do not admit a good model theory, compact complex analytic *manifolds* do. As was noted by Zilber about a decade ago, a compact complex manifold considered as a first-order structure in the language having an n-ary relation for each closed analytic subvariety of its nth Cartesian power has finite Morley rank. Moreover, he noticed that the theory of Zariski geometries (which is instrumental in the fine structure of separably closed fields, differentially closed fields, and difference fields) applies to compact complex manifolds. In my talk, I will describe how Zilber's observations have been exploited over the past year and a half to explain some phenomena of compact complex manifolds and to classify the definable groups.

While the fact that compact complex manifolds are stable implies that they cannot interpret the reals, they are themselves interpretable in certain o-minimal expansions of the real field. I will explain how this intepretation (especially through o-minimal algebraic topology) plays a role in the analysis of compact complex manifolds.

► JOHN STEEL, *The core model induction*.
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The use of core model theory to produce consistency-strength lower bounds for interesting set theoretic propositions generally involves an induction in the case one is obtaining a lower bound strictly greater than the consistency of one Woodin cardinal. In such a "core model induction" one uses the fact that over every set there is a canonical inner model which agrees with the full universe of sets on the truth values of certain statements, together with the "interesting proposition", to construct inner models which are correct for even more statements. We shall attempt to explain how such an induction can be organized, and why it is needed. Among the interesting propositions we shall consider are the Proper Forcing Axiom and Martin's Maximum.

► SHMUEL WEINBERGER, *Computability, complexity, and riemannian variational problems*.
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(Joint work with Alex Nabutovsky.) Since the sphere cannot be recognized algorithmically among smooth manifolds, (say hypersurfaces in Euclidean space defined by polynomials with integer coefficients) any attempted method for doing so will be obstructed: this implies that many variational problems have lots of solutions (places where flowing along a gradient must get stuck).

This talk will explain this idea, develop it more quantitatively (how many solutions must there be? what do they "look like"?, etc.) and, finally, "geometrize" it as well, i.e., deduce geometric properties of the space of isometry classes of Riemannian metrics on manifolds.

Abstracts of invited talks in the Special Session on Set Theory

► ILIJAS FARAH, *Gaps and separation principles*.
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In the early days of set theory, Hausdorff has constructed a gap in $P(\mathbb{N})/\mathrm{Fin}$, the structure of all sets of integers ordered by the inclusion modulo finite. A 'Hausdorff gap' consists of two increasing σ-directed families, A and B, that are orthogonal (the intersection of any element of A with any element of B is finite) but no set separates A from B modulo finite.

Hausdorff's original gap has had an additional feature: both A and B had the order type equal to ω_1. Years later Kunen proved that, assuming a small amount of reflection at the level of $P(\mathbb{N})$, (ω_1, ω_1)-gaps are the only linear Hausdorff gaps in $P(\mathbb{N})/\mathrm{Fin}$. This result was later reformulated by Todorcevic in terms of Ramsey theory for the uncountable. Todorcevic went on to prove a separation principle for $P(\mathbb{N})/\mathrm{Fin}$, implying that there are no 'definable' Hausdorff gaps. (Here 'definable' can be taken either as 'analytic' or 'belonging to $L(\mathbb{R})$, assuming large cardinals'. Note that by the Kunen-Martin theorem an analytic gap cannot be linear.) This separation principle also strengthens separation principles of Hurewicz and Kechris-Louveau-Woodin from Descriptive Set theory.

A while ago, I have constructed an F_σ Hausdorff gap in a quotient over an F_σ P-ideal I. In particular, Martin's Axiom implies the existence of a (c, c)-gap in $P(\mathbb{N})/I$. At the time all this seemed to indicate that Kunen-Todorcevic results do not generalize to gaps in quotients over analytic P-ideals in any reasonable way. In this talk I will show that it is not so, by classifying gaps in quotients over analytic P-ideals using OCA and giving a separation principle for such quotients similar to Todorcevic's.

► ITAY NEEMAN, *Determinacy and large cardinals*.
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I discuss the long games determinacy corresponding to large cardinals in the region of Woodin limits of Woodin cardinals.

► SLAWOMIR SOLECKI, *Translation invariant ideals*.
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The talk will be about distinguishing between countable and uncountable subsets of groups. We present a general procedure for extending translation invariant ideals on groups. We employ it to show that Haar measure on a locally compact group has a translation invariant extension for which each uncountable subgroup has a measurable selector for the family of its cosets. Vitali's construction of a nonmeasurable set shows that this is never possible for countable subgroups. This extends a result of Nowik. Another application of the same general procedure answers a question of Seredynski on the possibility of finding a translation invariant σ-ideal of subsets of the reals such that countable subsets are characterized as those sets which can be translated into the complement of each set from the σ-ideal.

► JINDRICH ZAPLETAL, $\mathfrak{p}$.
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I will point out several syntactical properties of the cardinal invariant $\mathfrak{p}$.

Abstracts of contributed talks

► MATTHIAS ASCHENBRENNER, *Ideal membership in polynomial rings over the integers*.
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In this talk, we will present a novel approach to the ideal membership problem for polynomial rings over the integers: given $f_0(X), f_1(X), \ldots, f_n(X) \in \mathbb{Z}[X]$, $X = (X_1, \ldots, X_N)$, are there $g_1(X), \ldots, g_n(X) \in \mathbb{Z}[X]$ such that $f_0 = g_1 f_1 + \cdots + g_n f_n$? We will discuss three aspects of this problem: (1) *existence of bounds* for the degrees and coefficients of $g_1, \ldots, g_n$ (in terms of the degrees and coefficients of $f_0, \ldots, f_n$); (2) *decidability* of ideal membership by a primitive recursive algorithm; (3) *definability*: dependence on parameters. In particular, our method yields bounds which improve the previously known bounds obtained by Gallo and Mishra [1].

The talk will complement the talk of Lou van den Dries and provide some more technical details.

[1] Gallo and Mishra, *A solution to Kronecker's problem*, ***Applicable Algebra in Engineering, Communication and Computing***, vol. 5 (1994), pp. 343–370.

► L. R. GALMINAS AND TIMOTHY H. MCNICHOLL, *Speed-up properties and bounded query computation on computably enumerable sets*.
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Department of Mathematics, University of Dallas, Irving, Texas 75062, USA.

We investigate the relationship between the speed-up properties introduced by Blum and Marque and bounded query computation on c. e. sets. In particular, we examine the compatibility of speedability, levelability, effective speedability, and effective levelability with the bounded query properties of n-correctability and "n-incorrectability." First, we show that every nonzero c. e. degree contains a c. e. set that is 1-incorrectable (hence n-incorrectable for all n). Then we extend the work of Blum, Marque, Soare, Bauerle, Remmel, and others by showing that there exist 1-incorrectable hypersimple sets of all possible speed-up characteristics (i.e., nonspeedable, levelable of arbitrary degree, effectively levelable, and effectively speedable yet effectively nonlevelable). Likewise, we show that there exist 2-correctable (hence n-correctable for all n) hypersimple sets with all possible speed-up characteristics.

► JEFFRY L. HIRST, *Reverse mathematics and the Heine/Borel Theorem*.
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One of the earliest results in reverse mathematics was Friedman's theorem on the equivalence of WKL_0 and the Heine/Borel theorem for the unit interval [1]. In June 1999, he asked if the restriction of the Heine/Borel theorem to countable closed subsets implied WKL_0. We will show that this is indeed the case, even when the countable subsets consist entirely of rational numbers.

[1] Harvey M. Friedman, *Systems of second order arithmetic with restricted induction*, ***Journal of Symbolic Logic***, vol. 41 (1976), no. 2, pp. 557–559.

► KEJIA HO, *High and low Kolmogorov complexity of computably enumerable sets*.
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Given a set A, we consider $\chi_A \upharpoonright m$, the initial segment of length m of the characteristic sequence of A; and study what properties of A would guarantee the Kolmogorov complexity of $\chi_A\restriction m$ high or low. For the high end, we show that there exist a 2^x-c. e. set A and a constant c such that for infinitely many m, $c(\chi_A\restriction m) \geq m - c$. This is the highest complexity any set can reach and A is thus random in the sense of Martin-Löf; see [1, Theorem 2.5.5]. This result is sharp because for any $b < 1$, no 2^{bx}-c. e. set has such property; see [2, Corollary 7.12]. For the low end, we introduce the concept of *arbitrarily low Kolmogorov complexity* and use it to classify classes of computably enumerable sets. In addition, we define *Kolmogorov reducibility* and show that it is properly between weak truth-table reducibility and Turing reducibility.

[1] M. Li and P. Vitányi, ***An introduction to Kolmogorov complexity and its applications***, Springer-Verlag, Berlin, 1997.

[2] B. Schaeffer, *Abstract complexity theory and the degrees of unsolvability*, ***Ph. D. thesis***, University of Illinois at Urbana-Champaign, 1998.

► JOSÉ IOVINO, *On the problem of explicitly defined normals in Banach space theory*.

Devision of Mathematics and Statistics, The University of Texas at San Antonio, San Antonio, Texas 78249-0664, USA.
E-mail: iovino@math.utsa.edu. The following problem has been embedded in the folklore of Banach space theory for decades, but acquired special prominence after the historical breakthroughs of the late 20th century.

In all examples of Banach spaces known before the mid 1970's, the norm is defined explicitly, i.e., by a single formula. This changed with Tsirelson's epochal paper [1]. The norm of Tsirelson's space is given by an equation, but the peculiarity is that the norm appears on both sides of the equation, so at first sight the definition appears to be circular; one needs to prove that such a norm exists and is unique. This type of norm definition is referred to in Banach space theory as an *implicit definition*.

The goal of Tsirelson's original construction ("the first truly nonclassical Banach space", see [2]) was to provide an example of a space which does not contain any of the classical sequence spaces ℓ_p and c_0. New spaces constructed in the 1990s using implicitly defined norms led to the solution of many of the most outstanding problems in the history of Banach space theory. (See [2] and [4] for a survey.)

The question is whether every Banach space with an explicitly defined norm will always contain one of the classical sequence spaces. An obstacle that Banach space theorists have encountered is the need to formalize the intuitively clear notions of explicit and implicit definability. (See [2], [3].)

We will present a progress report for the asymptotic case of this problem.

[1] B. S. Tsirelson, *It is impossible to imbed l_p of c_0 into an arbitrary Banach space*, ***Funkcional. Anal. i Priložen.***, vol. 8 (1974), no. 2, pp. 57–60.

[2] W. T. Gowers, *Recent results in the theory of infinite-dimensional Banach spaces*, in ***Proceedings of the International Congress of Mathematicians, vol. 1, 2 (Zürich, 1994)***, Birkhäuser, Basel, 1995, pp. 933–942.

[3] E. Odell, *On subspaces, asymtotic structure, and distortion of Banach spaces*: *Connections with logic*, to appear in ***Analysis and Logic***, Cambridge University Press.

[4] E. Odell and Th. Schlumprecht, *Distortion and stabilized structure in Banach spaces*; *new geometric phenomena for Banach and Hilbert spaces*, in ***Proceedings of the International Congress of Mathematicians, vol. 1, 2 (Zürich, 1994)***, Birkhäuser, Basel, 1995, pp. 955–965.

► ALEX M. MCALLISTER, *The Turing order of ω-models defined via completions of arithmetic*.

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We examine the partial ordering induced by Turing reducibility on various families of sets that are definable via completions of Peano arithmetic (PA). We consider the families of sets that are representable in completions of PA; that is, the families of sets that are computable in bounded pieces of a completion. We also consider the Turing closure of the families of sets that are computably enumerable in bounded pieces of a completion. The families of representable sets are ω-models of WKL_0 and the Turing closures of the families of weakly representable sets are ω-models of an extension of WKL_0 that we call $\mathsf{WKL}_0 + \Delta^0_2(\mathcal{S})$. We examine the partial ordering of these families of sets including minimal and maximal elements, infinite ascending and descending chains, and incomparable elements and antichains. There are many similarities between these partial orders, but also some striking differences. For ω-models of $\mathsf{WKL}_0 + \Delta^0_2(\mathcal{S})$, we are able to obtain a complete answer to the following question that remains open for ω-models of WKL_0: if $\mathcal{S}$ is an ω-model of a subsystem of second-order arithmetic and $X \in \mathcal{S}$ is noncomputable, does there exist $Y \in \mathcal{S}$ such that X and Y are Turing incomparable?

► TIMOTHY H. MCNICHOLL, *Truth-table degrees and the Ershov hierarchy*.
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For all n, let $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ be the set of all truth-table degrees that contain an n-c. e. set; let $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}} = \mathcal{D}^{\mathrm{tt}}_{1\text{-c. e.}}$. We show that for all n, any proper final segment of $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ can be embedded into any proper final segment of $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ in such a way that zeros and ones are preserved. We also show that for all n and any $d \in \mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ there is an embedding of $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ into $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ that preserves zeros and ones and that hits d. We show that for all n there is an epimorphism of $\mathcal{D}^{\mathrm{tt}}_{\text{c. e.}}$ onto $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$. We answer a question of Downey by showing that for all n, $\mathcal{D}^{\mathrm{tt}}_{n\text{-c. e.}}$ is co-embeddable with the c. e. *wtt*-degrees in a way that preserves zero and ones. These results are demonstrated to hold throughout the Ershov Hierarchy.

► RUSSELL MILLER, *The computable dimension of trees of height ω*.
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The *computable dimension* of a computable structure $\mathcal{A}$ is the number of equivalence classes in

$$\{\, \mathcal{B} \mid \mathcal{B} \text{ is computable and } \mathcal{B} \cong \mathcal{A} \,\}$$

under computable isomorphism. If $\mathcal{A}$ has computable dimension 1, we say that $\mathcal{A}$ is *computably categorical*. This implies that every computable structure isomorphic to $\mathcal{A}$ is isomorphic to it via some computable map.

We prove that no computable tree of height ω is computably categorical, and indeed that all such trees have computable dimension ω. The necessary construction requires that we prove several corollaries of Kruskal's Theorem, which states that if $\{\, T_i \mid i \in \omega \,\}$ is an infinite set of finite trees, then some T_i must embed into some T_j with $j > i$. Recently we have extended this result, proving that all computable trees of infinite height have computable dimension ω.

► CARLO TOFFALORI, *Comparing theories of modules over group rings*.
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This is a joint work with Saverio Cittadini [1]. Several wildness/undecidability results for modules over rings $D[G]$, where D is a Dedekind domain and G is a finite group, are based on the interpretation of the word problem for groups inside the class of D-torsion $D[G]$-modules. Hence it is natural to check what happens among D-torsionfree objects. Preliminarily we compare the (first order) theory T of all these modules and the theory T_0 of the finitely generated ones (so of $D[G]$-lattices). One realizes that they are equal if and only if D is a field. The obstruction is the existence of D-divisible D-torsionfree $D[G]$-modules. Accordingly we consider D-reduced D-torsionfree $D[G]$-modules for a local D. We show that the key conditions ensuring that their theory equals T_0 are:

(1) $D[G]$-lattices have a finite representation type;

(2) each $\widehat{D}[G]$-lattice is isomorphic to $\widehat{M}$ for some $D[G]$-lattice M

($\widehat{}$ = completion with respect to the maximal ideal of D). However $T \neq T_0$ in most cases. Then we compare the behavior of these theories with respect to decidability. It should be noticed that, in principle, T is r. e. and T_0 is co-r. e. in a natural way, so the starting situations are opposite. A helpful tool for T is the analysis of D-torsionfree part of the Ziegler spectrum of $D[G]$ given in [2]. We use those results to prove that T is decidable when (1) holds (the decidablility of T_0 under the same assumption is trivial).

[1] S. Cittadini and C. Toffalori, *Comparing first order theories of modules over group rings*, preprint.

[2] A. Marcja, M. Prest, and C. Toffalori, *The torsionfree part of the Ziegler spectrum of RG when R is a Dedekind domain and G is a finite group*, preprint.

► AMY K. C. S. VANDERBILT, *A partial order for nonmonotonic rule systems*.
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Nonmonotonic Rule Systems have been extensively studied in papers by Marek, Nerode, and Remmel [1, 2]. We introduce a partial order for nonmonotonic rule systems $\mathcal{U} = \langle \mathcal{U}, \mathcal{N} \rangle$. Further, we explore the induced structure and some consequences.

[1] W. Marek, A. Nerode, and J. B. Remmel, *Nonmonotonic rule systems I*, ***Annals of Mathematics and Artificial Intelligence***, vol. 1 (1990), pp. 241–273.

[2] ———, *Nonmonotonic rule systems I*, ***Annals of Mathematics and Artificial Intelligence***, vol. 5 (1992), pp. 229–263.