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2000-2001 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The New York Hilton and Towers, New York, NY
December 28–29, 2000

A Winter Meeting of the Association for Symbolic Logic was held December 28–29, 2000, in New York, New York, at the New York Hilton and Towers in conjunction with the meeting of the Eastern Division of the American Philosophical Association. The Program Committee consisted of Paul Benacerraf and Kit Fine. The program included three invited talks and a symposium on the foundations of mathematics.

The following two-hour symposium was presented:

Foundations of mathematics: *past, present and future*,
Chair: Alasdair Urquhart (Toronto University),
Speaker: Harvey Friedman (Ohio State University),
Commentators: Harold Hodes (Cornell University) and Warren D. Goldfarb (Harvard University),

The following one-hour invited talks were presented:

Dana S. Scott (Carnegie Mellon University), *Reconsidering the axioms for geometry.* (Chair: Haim Gaifman, Columbia University).
Raymond M. Smullyan, *Logical legerdemain.* (Chair: Melvin C. Fitting, Lehman College, CUNY).

Abstracts of invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
KIT FINE

Abstracts of invited talks

► HARVEY FRIEDMAN, *Foundations of mathematics: Past, present, and future*.
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Foundations of Mathematics originated to serve a great perceived need of the mathematics community. This original development was followed by a period of great developments in the 1930s that startled the mathematics community. However, continued strong interest in foundations of mathematics in the mathematica community is now dependent on the subject focusing on more normal mathematical situations. We discuss such new investigations in foundations of mathematics that promise to rekindle interest in the mathematics community.

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The main thrusts of foundations of mathematics are readily accessible to the philosophical community. We also discuss the main developments as well as the new investigations in philosophical terms.

► DANA S. SCOTT, *Reconsidering the axioms for geometry*.
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Euclid's Elements were considered a paragon of the axiomatic method for centuries, but during the 19th century problems became apparent. These had nothing to do the Parallel Postulate but rather concerned questions of order of points on a line and intersections of lines and curves with each other. Around the turn of the 20th century Hilbert produced a rigorous axioms system that was very influential. In the later 1920s Tarski developed a lean, first-order axiom system based on the primitives of betweeness and equidistance. This work led to a famous completeness proof, and later investigators substantially reduced the number of necessary axioms.

It has to be said, however, that the deduction of standard theorems from Hilbert's or Tarski's axioms can be quite tiresome and logically tricky—and not suitable for high-school students. Could there be another approach using "self-evident" primitives and properties that would make life easier? A report on some preliminary efforts will be given using points and rays (i.e., half-lines) as primitive objects. There are several very simple operations on these geometric entities—particularly involving order. Combining statements in this style with some basic properties of congruence of compound figures might produce a more transparent basis for axiomatic presentations. (This is joint work with John Mumma, a graduate student in Philosophy at Carnegie Mellon.)

► RAYMOND M. SMULLYAN, *Logical legerdemain*.
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A partly humorous, partly serious combination of logic, magic, and paradoxes, including a version of Newcombe's paradox of the perfect predictor, only with the predictor entirely removed.

Abstracts of contributed talks

► G. ALDO ANTONELLI AND RICHMOND H. THOMASON, *Representability in second-order propositional multi-modal S5*.
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A propositional system of modal logic is *second-order* if it contains quantifiers $\forall p$ and $\exists p$, which, in the standard interpretation, are construed as ranging over sets of possible worlds (propositions). Most second-order systems of modal logic are highly intractable; for instance, when augmented with propositional quantifiers, K, B, T, K4 and S4 all become effectively equivalent to full second-order logic. An exception is S5, which, being interpretable into monadic second-order logic, is decidable.

In this paper we generalize this framework by allowing multiple modalities. While this does not affect the undecidability of K, B, T, K4 and S4, multi-modal second-order S5 is dramatically more expressive than its mono-modal counterpart. As an example, we establish the definability of the transitive closure of finitely many modal operators. We also take up the decidability issue, and, using a novel encoding of sets of unordered pairs by partitions of the leaves of certain trees, we show that the second-order propositional logic of two S5 modalities is also equivalent to full second-order logic.

► DENNIS F. CUDIA, *Existential universal judgments in Aristotelian logic*.
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Let $G' = (S', P', V'_N, V_T)$ be the context sensitive grammar defined in [1]. Let $\bar{S}$, $\bar{P}$, $\bar{M}$, respectively, denote $\sim S$, $\sim P$, $\sim M$, respectively, and define $V_X = \{S^*, P^*, M^*, \bar{S}^*, \bar{P}^*, \bar{M}^*\}$ where Y^* means Y is non-empty.

Definition. An *extended mood* is a quadruple whose first and third components are universal propositions and whose second component is an element of V_X, and whose fourth component is a particular proposition. An *existential universal analytic mood* is an extended mood such that there exists a figure such that the figure-extended mood combination is an analytic judgment.

Let $G'' = (S', P'', V'_N, V''_T)$ be the context sensitive grammar with $V''_T = V_T \cup V_X$, whose context sensitive productions are the same as those of G', and whose context free productions are:

$S' \to \langle\text{figure}\rangle\langle\text{mood}\rangle$

$\langle\text{mood}\rangle \to \langle S \text{ and } P \text{ or neither distributed}\rangle \mid \langle S \text{ or } P \text{ distributed}\rangle$

$\langle S \text{ and } P \text{ or neither distributed}\rangle \to \langle\text{one figure only}\rangle \mid \langle\text{all figs}\rangle$

$\langle\text{one figure only}\rangle \to \langle\text{fig. 1}\rangle \mid \langle\text{fig. 2}\rangle \mid \langle\text{fig. 3}\rangle \mid \langle\text{fig. 4}\rangle$

$\langle\text{fig. 1}\rangle \to AS^*AI \mid GP^*GI \mid AM^*GI \mid G\bar{S}^*GZ \mid A\bar{P}^*AZ \mid G\bar{M}^*AZ$

$\langle\text{fig. 2}\rangle \to GS^*AI \mid AP^*GI \mid GM^*GI \mid A\bar{S}^*GZ \mid G\bar{P}^*AZ \mid A\bar{M}^*AZ$

$\langle\text{fig. 3}\rangle \to AS^*GI \mid GP^*AI \mid AM^*AI \mid G\bar{S}^*AZ \mid A\bar{P}^*GZ \mid G\bar{M}^*GZ$

$\langle\text{fig. 4}\rangle \to GS^*GI \mid AP^*AI \mid GM^*AI \mid A\bar{S}^*AZ \mid G\bar{P}^*GZ \mid A\bar{M}^*GZ$

$\langle\text{all figs}\rangle \to FS^*EI \mid EP^*FI \mid EM^*EZ \mid E\bar{S}^*FZ \mid F\bar{P}^*EZ \mid F\bar{M}^*FI$

$\langle S \text{ or } P \text{ distributed}\rangle \to \langle\text{fig. 1, 2}\rangle \mid \langle\text{fig. 3, 4}\rangle \mid \langle\text{fig. 2, 4}\rangle \mid \langle\text{fig. 1, 3}\rangle$

$\langle\text{fig. 1, 2}\rangle \to ES^*AO \mid EP^*AU \mid EM^*GO \mid F\bar{S}^*GU \mid F\bar{P}^*GO \mid F\bar{M}^*AU$

$\langle\text{fig. 3, 4}\rangle \to ES^*GO \mid EP^*GU \mid EM^*AO \mid F\bar{S}^*AU \mid F\bar{P}^*AO \mid F\bar{M}^*GU$

$\langle\text{fig. 2, 4}\rangle \to AS^*EO \mid AP^*EU \mid GM^*EU \mid G\bar{S}^*FU \mid G\bar{P}^*FO \mid A\bar{M}^*FO$

$\langle\text{fig. 1, 3}\rangle \to GS^*EO \mid GP^*EU \mid AM^*EU \mid A\bar{S}^*FU \mid A\bar{P}^*FO \mid G\bar{M}^*FO$

The context free productions of G'' generate exactly the set of 54 existential universal analytic moods and $L(G'')$ is the set of 96 analytic judgments that includes the 9 singular analytic categorical syllogisms. Using [2] it is shown that neither G' nor G'' violates the second law of thermodynamics.

[1] D. Cudia, *Information and entropy in Aristotelian logic*, ***Bulletin of Symbolic Logic***, vol. 4 (1998), p. 221.

[2] ———, *The information in the genetic code*, ***Journal of Symbolic Logic***, vol. 53 (1988), pp. 1291–1292.

► MICHAEL GLANZBERG, *A contextual-hierarchical approach to truth and the liar paradox*.
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One sort of *Liar Sentence* is a sentence l that 'says' l does not express a true proposition. A well-known idea in the study of truth and the Liar Paradox is that such sentences exhibit a kind of context-dependence. In a null context, l does not express a proposition; yet in the course of an inference reaching this conclusion, a context shift occurs. In the shifted context, l does express a proposition, indeed a true one.

This paper provides a formal model of this behavior, using the techniques of model theory in admissible sets. A context is argued to fix a domain of truth conditions that corresponds to the structures in an admissible $HYP_{\mathfrak{M}}$ for appropriate ground structure $\mathfrak{M}$. An interpretation for the relation 'sentence s is true in structure $\mathfrak{A}$' is provided for such a domain of structures. Using this interpretation, it is shown that in a null context the Liar Sentence does not express a proposition. It is then argued that the problematic context shift induces a shift to an expanded ground structure $\langle \mathfrak{M}, P \rangle$. The domain of truth conditions for the shifted context corresponds to the structures in $HYP_{\langle \mathfrak{M}, P \rangle}$. It is shown that this $HYP_{\langle \mathfrak{M}, P \rangle}$ contains structures differing in $\mathcal{L}_{HYP_{\mathfrak{M}}}$-theory from all structures in $HYP_{\mathfrak{M}}$. Hence, the domain of truth conditions genuinely *expands*. Relative to this expanded domain, it is shown how the Liar Sentence can express a proposition.

► STEPHEN HARNISH, *Non-well-founded set theories with a universal set*.
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Many set theories with a universal set have been proposed throughout the 20th Century [2]. Numerous set theories with *other* non-well-founded sets have also been studied during this period. Following the publication of Aczel's book *Non-well-founded Sets* [1], much research has focused on the properties and applications of the non-well-founded set theories of Aczel, Scott, and Finsler. Each of these set theories can be characterized as an axiomatic system of the form $(\mathrm{ZFC}^- + \mathrm{AFA}^\sim)$. Here we assume that $\sim$ is a regular bisimulation, ZFC^- is (ZFC without the foundation axioms), and that the axiom $\mathrm{AFA}^\sim$ states "an accessible pointed graph (apg) is an exact picture if and only if it is $\sim$-extensional." It is known that these systems are consistent relative to ZFC.

This paper introduces two set theories whose universes not only contain the non-well-founded sets of Finsler and Scott, respectively, but that also contain a universal set. This is accomplished by changing several axioms in $(\mathrm{ZFC}^- + \mathrm{AFA}^\sim)$. The basic change from traditional non-well-founded set theories is that the canonical pictures of sets can have two kinds of nodes: those whose children are exactly the *elements* of the node, and those whose children are the *non-elements* of the node. Hence, the canonical picture of the universal set has no edges, but only one node of the second kind. These new set theories are consistence relative to ZFC. This paper presents the basic properties of these two theories, and contrasts them to $(\mathrm{ZFC}^- + \text{Finsler's AFA})$, $(\mathrm{ZFC}^- + \text{Scott's AFA})$, and Quine's New Foundations.

[1] P. Aczel, ***Non-well-founded sets***, Lecture Notes, no. 14, Center for the Study of Language and Information, Stanford, 1988.

[2] T. E. Forster, ***Set theory with a universal set***, Oxford University Press, New York, 1995.

► KEVIN C. KLEMENT, *A paradox in Frege's theory of meaning*.
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The logical system of Frege's 1893 *Grundgesetze der Arithmetik* is widely known for its inconsistency due to Russell's paradox. The inconsistency was traced by Frege himself to its naïve class theory enshrined in its "Basic Law V". Indeed, if one removes Basic Law V from the extant system, what remains is consistent. However, the logical system does not fully reflect the entirety of Frege's philosophical commitments. Frege's theory of meaning commits him to a realm of senses and thoughts not reflected in his logical system. I show that even the consistent core of Frege's system—the system *without* Basic Law V—becomes inconsistent when his commitment to senses is axiomatized therein. In particular, it is possible to generate a Cantorian antinomy involving thoughts and concepts. For each concept it is possible to generate a different universal thought, e.g., for the concept of being self-identical, we get the thought that *everything is self-identical*, for the concept of being human, we have the thought that *everything is human*, etc. Some such thoughts themselves fall under the concepts they generalize, some do not. The thought that everything is self-identical is self-identical, but the thought that everything is human is not itself human. Consider now the concept k, of *being a universal thought that does not fall under the concept of which it states the generalization*. Then consider the thought that everything is k. Does it itself fall under the concept of k? It does just in case it does not. A logical system based upon the *Grundgesetze*, but expanded in line with Frege's own views about meaning, is developed but shown to be inconsistent, even without Basic Law V.

► BILLY JOE LUCAS, *Time and necessity*.
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Let L^m be a modal language that includes $\sim$, $\supset$, and a necessity operator L, and L^t be a temporal language that includes at least $\sim$, $\supset$, a "heretofore" operator P, and a "hereafter" operator F. Define the omnitemporal translation function $^\circ$ from L^m to L^t: $(\sim \alpha)^\circ = \sim (\alpha^\circ)$, $(\alpha \supset \beta)^\circ = (\alpha^\circ) \supset (\beta^\circ)$, $(L\alpha)^\circ = P(\alpha^\circ) \cdot (\alpha^\circ) \cdot F(\alpha^\circ)$.

THEOREM. *$(L\alpha \supset LL\alpha)^\circ$ and $(\sim L\alpha \supset L \sim L\alpha)^\circ$ are each (separately) valid on a class of temporal frames $\langle W, R\rangle$ if and only if both*

(a) *R contains no forks and*
(b) *R is CT connected (if $\exists z\ (x\ R\ z \cdot z\ R\ y)$ then $x\ R\ y$ or $y\ R\ x$).*

Now consider three ideas about time and necessity. Idea One: Time is such that it is logically possible that there be a possible time relative to which there are at least two alternative possible futures, one of which does not come about. Or, somewhat less precisely put: there could be ways things might have turned out, but didn't. Idea Two: There is no time at which a necessary truth is false. Idea Three: The modal status of modal propositions is necessary. The dominant trend in recent analytic metaphysics of taking the concept of necessity to coincide with the concept of truth in all possible worlds is little more than a new version of the MSN thesis. Ideas One, Two, and Three taken together are inconsistent with the Theorem above. If in fact all three of these ideas are as reasonable as they seem, we have here a genuine philosophical paradox.

► FRED SEYMOUR MICHAEL, *Entailment and bivalence*.
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In classical two valued logic, we cannot infer a false statement from one that is not false, any more than we can infer a statement that is not true from one that is. In classical logic in fact preserving truth and preserving non-falsity are one and the same thing. They are not the same in non-bivalent logics however and I will argue that the classical notion of entailment that preserves only truth is not strong enough for such a logic. I will show that it we retain the classical notion of entailment in a logic with three values, true, false and a third value in between, an inconsistency can be derived that can be resolved only by measures that seriously disable the logic. I will show this for a logic designed to allow for semantic presuppositions, then I will show that we get the same result in any three valued logic that has the same value ordering. I will finally suggest how the notion of entailment should be generalized so that this problem may be avoided. In the opinion of Dana Scott, no really workable three valued logic has yet been developed. It is hard to disagree with this. A workable three valued logic could perhaps be developed, however, if we had a notion of entailment suitable to non-bivalent logics.

► GRIGORI MINTS, *Russell's anticipation of intuitionistic logic*.
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In Section 18 (Chapter II) of [1] B. Russell anticipated intuitionistic logic by clearly distinguishing propositional principles implying the law of the excluded middle from remaining valid principles. This is important in connection with the method used later by A. Heyting (see [2], Section 3.1) for finding his axiomatization of intuitionistic logic. Russell lists ten propositional axioms. First nine principles are intuitionistically valid: $(p \to q) \to (p \to q)$; $(p \to q) \to (p \to p)$; $(p \to q) \to (q \to q)$; $p \to (q \to p)$; $pq \to p$; $(p \to q)(q \to r) \to (p \to r)$; $(p \to (q \to r)) \to (pq \to r)$; $(pq \to r) \to (p \to (q \to r))$; $(p \to q)(p \to r) \to (p \to qr)$.

The next principle, called *reduction* is essentially what was later called Pierce law: $((p \to q) \to p) \to p$.

Russell claimed the following principles follow from the first nine principles:

$$\neg(p(\neg p)); p \to \neg\neg p; (p \to \neg q) \iff (q \to \neg p)$$
$$(p \to \neg q) \iff \neg(pq); (p \to q) \to (\neg q \to \neg p); p \to (\neg p \to q)$$
$$\neg p \iff (p \to \neg p); (p \to \neg q) \iff (\neg\neg p \to \neg q).$$

All these principles are provable in intuitionistic and even in minimal logic.

Next we list the principles Russell claims cannot be proved without the reduction principle. None of these principles is provable intuitionistically, since eacn of them has an instance which intuitionistically implies $p \vee \neg p$.

$$p \vee \neg p; \exists q\ (p \iff \neg q); \neg\neg p \to p; (\neg q \to \neg p) \to (p \to q);$$
$$(\neg p \to p) \to p; (p \to q) \to (q \vee \neg p).$$

So Russell was right both in positive and in negative part. This is especially interesting in view of the evidence [3] that Brouwer was studying Russell's work at the time when the principles of intuitionism were first developed.

[1] B. RUSSELL, ***The Principles of Mathematics***, Allen and Unwin, London, 1903.

[2] A. TROELSTRA, *On early history of intuitionistic logic*, in ***Mathematical logic*** (P. Petkov, editor), Plenum Press, New York and London, 1990, pp. 3–18.

[3] D. VAN DALEN, ***Mystic, geometer, and intuitionist: the life of L. E. J. Brouwer***, Clarendon Press, Oxford, 1999.

► MICHAEL SCANLAN, *C. H. Langford and the origins of model theory*.
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Robert Vaught traces the origins of Tarski's work in model theory to a seminar he held in 1927–29 in Warsaw [3]. The seminar was devoted to the study of the method of elimination of quantifiers to prove the completeness and decidability of theories. The two examples of the method that were available for study were by Skolem [2] and by C. H. Langford [1]. It is probably not surprising to find the work of Skolem influencing Tarski in such an important period, but the name of Langford might create surprise. This talk will outline how Langford's work might very well have influenced Tarski's recognition of the need for precise semantic concepts such as truth or definability.

Langford's paper demonstrates semantic completeness for a number of theories of linear order. The proofs are best understood retrospectively in terms of satisfaction of formulas and truth of sentences in interpretations. The argument for the influence of Langford on Tarski is that in studying Langford's work he realized that he could not make sense of it without using such notions. The talk will illustrate this with an example of how such notions of truth and satisfaction occur implicitly at crucial points in Langford's completeness/decidability proofs.

[1] C. H. LANGFORD, *Some theorems on deducibility*, ***Annals of Mathematics***, vol. 28 (1926), pp. 16–40.

[2] T. SKOLEM, ***Untersuchungen über die Axiome des Klassenkalküls und über Produktations- und Summationsprobleme, welche gewisse Klassen von Aussagen betreffen***, Skrifter, Videnskabakademiet i Kristiana, no. 3, 1919.

[3] R. L. VAUGHT, *Alfred Tarski's work in model theory*, ***Journal of Symbolic Logic***, vol. 51 (1986), pp. 869–882.

► PAUL STRAUSS, *A semantic paradox*.
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Consider the following sentence:

"I love people that love people, and I hate people that hate people."

Let John be a person who loves people. Then I love myself since I love John. Also, let Adolph be a person who hates people. Then I hate myself since I hate Adolph. So I love myself and hate myself at the same time. A contradiction.

Another resolution of the same paradox is that suppose there's a person, say Bob, who loves some people but hates others, then I love Bob and I hate Bob at the same time. A contradiction.