THE BULLETIN OF SYMBOLIC LOGIC
Volume 7, Number 1, March 2001

2000 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM 2000

La Sorbonne, Paris, France, July 23–31, 2000

Logic Colloquium 2000, the 2000 European Meeting of the Association for Symbolic Logic, took place in La Sorbonne, Paris, France, on July 23–31, 2000, on the site of Hilbert's presentation of his famous list of problems in the summer of 1900. LC2000 was part of World Mathematical Year 2000 and was organized to have a special impact on the future of logic, in recognition of the new millennium. The main themes of the meeting were: Proof theory and logical foundations of computer science, Set theory, Model theory, Computability and complexity theory, History of 20th century logic, Philosophy and applications of logic to cognitive sciences. The tutorials, philosophy symposium, historical talks, and certain other lectures also comprised the European Logic Summer School 2000, emphasizing the importance of the training of young researchers in logic.

The Program Committee consisted of D. Andler, C. Berline, B. Cooper, D. van Dalen, A. Kanamori, C. Parsons, A. Razborov, H. Schwichtenberg, J. Steel, S. Todorčević, A. Wilkie, and C. Wood (Chair). The Local Organizing Committee included C. Berline, Z. Chatzidakis, R. Cori (Chair), M. Dickmann, J. Dubucs, J.-B. Joinet, D. Lascar, Y. Legrandgérard, J. Mosconi, M.-H. Mourgues, C. Muhlrad-Grief, L. Pacholski, J.-P. Ressayre, B. Veličković, and F. Ville. The program consisted of 4 tutorials and 24 invited plenary talks. There were also over 200 contributed papers, most of which were presented during 11 blocks of time, each block having 7 parallel sessions. A few papers were presented by title only.

This Logic Colloquium is conjectured to be the largest ever, certainly the largest in recent memory, with 426 registered participants present from 52 countries, plus several others in attendance for shorter times. Through grants from the ASL, the European Community, and the French and US governments, financial support was provided to 98 participants. July 2000 was the coldest in record in Paris, with rain throughout the first three weeks. As Lance Armstrong entered Paris in triumph in the Tour de France, LC2000 participants arrived and the skies turned vivid blue, interrupted only by an occasional summer shower. The conference setting was very congenial for guests, thanks to the non-trivial efforts of the organizing committee and to the beauty of the setting. In addition to the pleasure of visiting Paris, participants enjoyed plenary talks in the magnificent and historical Amphitheatre Richelieu, and found the large central courtyard ideal for informal discussions. The organizing committee arranged a variety of social and cultural events in the evenings and the free afternoons, and hosted a banquet on July 27. The banquet was held outside the city at Château de Ferrières, where it was shown to be possible (at least in France) to serve a gourmet meal to over 400 guests.

The following tutorials were given:

Lenore Blum and Stephen Smale (Carnegie Mellon University, Pittsburgh), *Complexity and real computation*.

Elisabeth Bouscaren (Université Paris VII), *Model theory and geometry*.

1079-8986/01/0701-0004/$9.20

John Longley (University of Edinburgh), *Realizability and higher-type computability*.
W. Hugh Woodin (University of California, Berkeley), *The continuum hypothesis*.

The following invited plenary talks were presented:

Peter Aczel (University of Manchester, U.K.), *Can ZF classical mathematics be understood from a constructive standpoint?*.
Sam Buss (University of California at San Diego), *Definability and complexity in bounded arithmetics*.
Martin Davis (New York University), *Computability theory in the twentieth century*.
Ilijas Farah (Rutgers University), *Continuous liftings*.
Leo Harrington (University of California, Berkeley), *Automorphisms of the computably enumerable sets*.
Wilfrid Hodges (Queen Mary and Westfield College, England), *Model theory in its first century*.
Martin Hofmann (University of Edinburgh), *On the decision problem for typed lambda calculus with coproducts*.
Gerhard Jäger (Universität Bern), *Metapredicative and explicit Mahlo: a proof-theoretic perspective*.
Jean-Louis Krivine (Université Paris VII), *Realizability and generalized forcing in set theory*.
Yves Lafont (CNRS), *Proof theory and computer science: from typographical engineering towards geometry*.
Richard Laver (University of Colorado), *Large cardinal results and problems*.
Angus Macintyre (University of Edinburgh), *Model theory of Frobenius on Witt vectors*.
David Marker (University of Illinois at Chicago), *Differential equations and logarithmic-exponential series*.
Donald Martin (University of California at Los Angeles), *Set theory in the 20th century*.
Daniel Osherson (Rice University, Texas), *A psychologist looks hopefully to logic*.
Peter Shor (AT&T Labs, USA), *Quantum computing*.
Theodore A. Slaman (University of California, Berkeley), *Aspects of the Turing jump*.
Otmar Spinas (Christian-Albrechts-Universität zu Kiel, Germany), *Canonical behaviour of Borel functions on Polish planes*.
S. Starchenko (University of Notre Dame, USA), *On the Zil'ber principle for structures definable in o-minimal models*.
Martin Zeman (Universität Wien, Austria), *Combinatorial principles in inner models*.
Boris Zil'ber (University of Oxford, U.K.), *Analytic and pseudo-analytic structures*.

Three of the invited plenary talks were organized into a Philosophy Symposium entitled *The Foundation of Mathematics around 1900*, chaired by Charles Parsons:

Philippe de Rouilhan (Université Paris I), *Russell's logics*.
William Ewald (University of Pennsylvania), *The philosophical significance of Hilbert's problems*.
Richard Heck (Harvard University), *Frege's contributions to foundational studies*.

In addition, a session on *The history of logic in France* (*Petite histoire de la logique en France*) was held in late afternoon on July 28, with participants M. Demazure, S. Grigorieff, M. Guillaume, B. Jaulin, J.-L. Krivine, D. Lacombe, K. McAloon, and Ph. de Rouilhan.

Abstracts of tutorials and invited plenary talks, and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
CAROL WOOD

Abstracts of invited tutorials

► LENORE BLUM AND STEPHEN SMALE, *Complexity and real computation*.
Computer Science Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA.
E-mail: lblum@icsi.berkeley.edu.
Mathematics Department, City University of Hong Kong, Tat Chee Ave., Kowloon, Hong Kong, People's Republic of China.
E-mail: masmale@Math.cityu.edu.hk.

The tutorial will to a great extent be based on the material in the book of the same title written jointly by L. Blum, F. Cucker, M. Shub and S. Smale, and published by Springer 1998. Thus, NP-completeness over a general field will be important. With the field of 2 elements this is the work of Cook-Karp. With the field of real numbers one comes close to the algorithms of Newton and Gauss and eventually modern numerical analysis. With the field of complex numbers one obtains a uniform version of classical algebaic complexity theory.

A natural extension of the problem "is $P = NP$?" to the complex numbers is developed. Now the universal problem is Hilbert's Nullstellensatz considered as a decision problem. Relations of the extended problem to the original Cook-Karp problem will be discussed.

► ELISABETH BOUSCAREN, *Model theory and geometry*.
UFR de Mathématiques, Université Paris VII, 2 Pl. Jussieu, 75251 Paris Cedex 05, France.
E-mail: elibou@logique.jussieu.fr.

In this tutorial we will present some aspects of the recent developments of Geometric Model Theory, illustrated by examples from the Model Theory of Fields.

In particular we will use different theories of fields (algebraically closed fields, differentially closed fields, separably closed fields and algebraically closed fields with an automorphism) to introduce the following topics:

- independence, stability and simplicity,
- definable groups,
- local modularity.

We hope to be able to finish by presenting a simple case of one of the applications to algebraic geometry.

► JOHN LONGLEY, *Realizability and higher-type computability*.
Division of Informatics, University of Edinburgh, JCMB, The King's Buildings, Mayfield Road, Edinburgh EH9 3JZ, United Kingdom.
E-mail: jrl@dcs.ed.ac.uk.

It is well known what is meant by a computable function on the natural numbers, but it is far from obvious what should be meant by a "computable" operation of higher type—say, of type $((N \to N) \to N) \to N$. Indeed, it is not even clear *a priori* whether there are any really sensible notions of higher type computability. Nevertheless, the study of computability at higher types turns out to be mathematically fruitful, and yields a small handful of different notions of computability that appear to be mathematically natural and robust. Besides its intrinsic conceptual interest, the subject has connections with:

- Constructive logic and metamathematics
- Programming language semantics and logic
- Complexity theory
- Real number computability
- Computability in physics.

Since the 1950s, the subject has evolved in a somewhat chaotic fashion, different strands being scattered widely across the literature in recursion theory, constructive logic and computer science. However, it now seems possible, with hindsight, to give a coherent account of much of this material within a uniform framework.

In the first lecture I will give a brief historical survey of the subject and the contributions of Kleene, Kreisel, Platek, Scott and many others. I will then describe some recent approaches to imposing a sense of coherence on this landscape. I will explore, within a simple abstract framework, the various known notions of higher type computable *functional* and how they are related. I will then show how ideas from realizability offer a framework for discussing even more general (e.g., non-extensional) notions of "computable operation".

Throughout the course I will emphasize the close interplay between computability and realizability, and the potential applications to computer science.

► W. HUGH WOODIN, *The continuum hypothesis*.
Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA.
E-mail: woodin@math.berkeley.edu.

I shall survey in these three lectures some recent mathematical developments which support the claim that the Continuum Hypothesis is false. The approach is to investigate abstractly the possibilities for "good" theories for the structure of the set of all subsets of ω_1, the least uncountable ordinal. This structure is simply the structure $H(\omega_2)$ of sets of hereditary cardinality at most $\aleph_1$.

Informally a theory is "good" if it is finitely axiomatizable modulo consistency statements. More formally, a theory is good if it is finitely axiomatizable, over ZFC, in Ω-logic. Ω-logic is a specific strong logic which is immune to forcing. The Ω-Conjecture asserts that Ω-logic is the strongest such logic. The validities of Ω-logic define abstractly a consistency hierarchy which the Ω-Conjecture implies is cofinal among all large cardinal axioms.

There are good theories for $H(\omega_2)$, but any good theory must yield that CH is false. At the core of these results are the combinatorics associated to the stationary, co-stationary, subsets of ω_1. Indeed these sets play the pivotal role.

Abstracts of invited plenary talks

► PETER ACZEL, *Can ZF classical mathematics be understood from a constructive standpoint?*.
Departments of Mathematics and Computer Science, University of Manchester, Manchester M13 9PL, United Kingdom.
E-mail: petera@cs.man.ac.uk.

Hilbert's Program (HP) was intended to lead to a positive answer to something like this question when 'constructive' is replaced by 'finitist'. When HP was first conceived there was not yet a clear distinction between the two standpoints. Today we understand the finitist standpoint to be a restricted one, where the abstractions of a constructive standpoint are not available. It is now generally agreed that Godel's incompleteness results showed that the original HP is not achievable. Instead variants of HP have been formulated and a great variety of results have been obtained, particularly by proof theorists, reducing fragments of classical mathematics to systems which are arguably acceptable from the constructive standpoint or even to systems that are finitistically acceptable. Proof theoretic technics have been refined to such an extent in more recent years, particularly through the work of Michael Rathjen, that one might now hope that some of the tools might eventually be applicable even to full second order arithmetic (PA_2). But, because of the extreme complexity of this work and doubts about the constructive character of the systems that PA_2 might be reduced to, one can wonder if this line of work will really lead to a constructive 'understanding' of classical mathematics. Here I will follow another direction.

Of course the problem is the full impredicativity of PA_2 and stronger systems such as ZF. In my talk I will discuss the status of full impredicativity in the context of my prefered approach to the foundations of constructive mathematics, Martin-Lof's Constructive Type Theory (CTT). I will review my earlier work on the type theoretic interpretation of the constructive set theory, CZF, and some extensions, and I will discuss ways of strengthening CTT so as to get systems strong enough to interprete ZF. The first crudest approach is to simply add the law of excluded middle to the Curry-Howard treatment of logic in CTT. A more subtle second approach is to add a version of Russell's Axiom of Reducibility to CTT and use Friedman's 'double-negation' reduction of ZF to IZF. Yet another possible approach is to add axioms for an impredicative type Prop, as in the Huet-Coquand Calculus of Constructions. But it remains an open problem whether this last possible extension is strong enough to interprete IZF and so ZF.

I will conclude by arguing that the second approach gives a more coherent approach to understanding ZF than the standard 'classical iterative conception'. Although the approach is not completely constructive I believe that it may be the appropriate approach to a constructive 'understanding' of classical mathematics.

► SAM BUSS, *Definability and complexity in bounded arithmetics*.
Department of Mathematics, University of California San Diego, San Diego, California 92093-0112, USA.
E-mail: sbuss@ucsd.edu.

This talk is a survey of bounded arithmetic especially on its connections to the P vs. NP problem and related problems in computational complexity. We present the definitions of bounded arithmetic and the basic witnessing theorems. We then discuss relativized and conditional separations of the fragments S^i_2, T^i_2, S^{i+1}_2 of bounded arithmetic. We discuss the strong pseudo-random number generator (SPRNG) conjecture and Razborov's proof that S^2_2 cannot prove superpolynomial lower bounds on circuit size of polynomial hierarchy predicates unless the SPRNG conjecture fails. We conclude that there are reasons for optimism about the prospects of logic to make progress on the P versus NP question.

► MARTIN DAVIS, *Computability theory in the twentieth century*.
Visiting Scholar UC Berkeley—Professor Emeritus, New York University, USA.
E-mail: martin@eipye.com.

Some of the highlights in the development of computability/recursion theory will be surveyed. The importance of recursive definitions became evident in some of the foundational work of Dedekind and Peano around the turn of the century. In this same period, Axel Thue studied operations on strings. Moving to the 1920s Skolem proposed to use recursive definitions to found arithmetic in a free variable calculus, and Hilbert wanted to use them in order to prove the Continuum Hypothesis. In 1931 Gödel used the class of (what are now called) primitive recursive functions in his classic paper proving the existence of undecidable arithmetic propositions in formal systems. In lectures in 1934, Gödel suggested that if the notion of recursive definition were appropriately extended, it might lead to a class of functions coextensive with those algorithmically calculable. Very soon thereafter, Church concluded that the class of functions expressible in his lambda-calculus indeed had this very property and his student Kleene proved that this lambda-definability was equivalent to Gödel's recursiveness. Meanwhile in England, Turing proposed his notion of computability by an abstract machine, and provided an analysis of the process of computation, concluding that his machines were sufficient to encompass any computation. It soon became clear that Turing's notion was equivalent to the other two. Post had found his own (also equivalent to the others) path in the 1920s but had never managed to produce a convincing development.

After Church's work had appeared in print, but before he knew about Turing's work, Post produced his own variant of Turing's machines.

The year 1936, in which the ***Journal of Symbolic Logic*** was launched, saw the publication of the proofs of the equivalence of the various notions, and the first example of an important problem that could be proved unsolvable using the new techniques, namely the proof by Turing and by Church that Hilbert's Entscheidungsproblem is algorithmically unsolvable. Computability theory was now launched as a coherent subject that could be developed in a number of directions, only highlights of which can be dealt with in this talk.

In a seminal paper in 1938 Kleene introduced the notion of partial recursive function, proved the S-m-n theorem and the two recursion theorems, and applied this apparatus to the constructive ordinals. In a 1943 paper he studied the arithmetic hierarchy. Turing in his 1939 dissertation had introduced the seminal notion of computability relative to an oracle. Post in a highly influential address to the American Mathematical Society in 1944 launched the study of recursively enumerable (r. e.) sets and their degrees and posed the question ("Post's problem") of the existence of an non-computable r. e. set of lower degree than that of his creative set K. Post also showed how the levels of the arithmetic hierarchy were related to the "jump" from a set to sets r. e. in that set. Davis and Mostowski (1950) studied extensions of the arithmetic hierarchy (the hyperarithmetic hierarchy) later studied by Kleene and Spector.

Post and Markov each proved the unsolvability of the word problem for semi-groups, and Novikov and Boone later extended this to the unsolvability of the word problem for groups. In 1950 Davis moved towards a proof of the unsolvability of Hilbert's 10th problem by obtaining an arithmetic normal form for r. e. sets. At the same time Julia Robinson approached the problem from the opposite direction studying the problem of defining exponentiation by a Diophantine equation. Joined by Hilary Putnam the two approaches met in the middle in their 1960 proof of the unsolvability of the decision problem for exponential Diophantine equations. To complete the proof of the unsolvability of Hilbert's 10th problem, a Diophantine equation whose solution grow exponentially was needed and such an equation was supplied in Matiyasevich's (1970) ingenious construction.

The key to the solution of Post's problem was the "priority method" introduced by Friedberg and Muchnik (1957): this method enabled infinitely many apparently conflicting requirements to all be ultimately satisfied using a limit process of which the result was an r. e. set. This initiated a continuing development in which great virtuosity was applied to obtain a number of surprising theorems. In this talk two examples due to Sacks will be explained.

► ILIJAS FARAH, *Continuous liftings*.
Department of Mathematics, CUNY, College of Staten Island, 2800 Victory Blvd., Staten Island, New York 10314, USA.
E-mail: ilijas@math.rutgers.edu.

CONJECTURE. *Assuming a sufficiently strong forcing axiom* (*e.g., PFA*), *for every pair I, J of projective ideals on N, every isomorphism between their quotient algebras has a continuous lifting.* (*An ideal is projective if it corresponds to a projective subset of the Cantor space.*)

An affirmative answer to the above would imply that two quotient algebras are isomorphic if and only if there is a definable isomorphism. This was confirmed for P-ideals in [1], using an axiom much weaker than PFA. I will survey recent progress on the question, and explain connections with the structure of gap-spectra in quotient algebras.

The above should not be confused with a complementary conjecture of Todorcevic, that every homomorphism between quotients over analytic P-ideals with a continuous lifting has a completely additive lifting (see [1]). While the latter is concerned with the structure of definable homomorphisms, the former is concerned with the interplay between definable and arbitrary sets of reals. It belongs to a program of investigating when are the connecting maps between definable objects necessarily definable.

An affirmative answer to both conjectures implies a strong rigidity phenomenon: If two ideals are not isomorphic in the Rudin-Keisler sense, then their quotients are not isomorphic (assuming PFA). Indeed, this is true for all analytic P-ideals (see [1]).

[1] I. Farah, *Analytic quotients: theory of liftings for analytic ideals over the integers*, ***Memoirs of the American Mathematical Society***, vol. 148, no. 702, November 2000, 200 pp., to appear.

► LEO HARRINGTON, *Automorphisms of the computably enumerable sets*.
Department of Mathematics, University of California, Berkeley, California 94720-3840, USA.
E-mail: leo@math.berkeley.edu.

There is a technique for building Δ^0_3 automorphisms of the lattice $\mathcal{E}$ of computably enumerable sets. A recent result with Peter Cholak shows that there are certain parts of $\mathcal{E}$ (actually these are parts of $\mathcal{E}^{\text{eq}}$) such that any automorphism restricted to such parts must be Δ^0_3. This talk will be about this result and some of its consequences.

► WILFRID HODGES, *Model theory in its first century*.
School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, England.
E-mail: w.hodges@qmw.ac.uk.

Model theory is a set of theorems, and of people. But more fundamentally it is a collection of ways of looking at mathematical structures. It gained its name in a paper of Alfred Tarski in 1954, which followed on from addresses given by Tarski and Abraham Robinson to the 1950 International Congress of Mathematicians. Several strands of work from the first half of the century came together in those addresses, and in the second half of the century many lines of research traced back to them.

► MARTIN HOFMANN, *On the decision problem for typed lambda calculus with coproducts*.
LFCS Edinburgh, Mayfield Rd, Edinburgh EH9 3JZ, United Kingdom.
E-mail: mxh@dcs.ed.ac.uk.

We consider the decision problem for simply-typed lambda calculus with coproduct (disjoint union) types generated over one base type. Thus one adds to the simply typed lambda calculus over one base type, a type former $A + B$, and the following terms:

$$\begin{aligned} \text{inl}(t) : A + B, &\qquad \text{when } t : A \\ \text{inr}(t) : A + B, &\qquad \text{when } t : B \\ \text{case}(t_1, t_2, t_3) : C, &\qquad \text{when } t_1 : A + B,\, t_2 : A \to C,\, t_3 : B \to C. \end{aligned}$$

Two terms are semantically equivalent if they have the same denotation under the obvious set-theoretic interpretation which interprets the base type as an infinite set and coproduct types as disjoint union of sets.

For example, we have the equivalence:

$$\text{case}(x, \lambda x.y, \lambda x.y) = y$$

and more generally, if $t_1 = t_2$ then

$$\text{case}(x, \lambda x.t_1, \lambda x.t_2) = t_1 (= t_2)$$

provided that t_1, t_2 do not contain x.

Dougherty and Subrahmanyam (1995) have shown that semantic equivalence is captured by the natural equational theory (BiCCC) arising from the theory of bicartesian closed categories.

A long-standing problem is to give an effective decision procedure for this theory BiCCC.

We report here on a solution based on the technique of normalisation by evaluation (NBE) introduced by Schwichtenberg and Berger (1991) and further refined by the authors using category-theoretic methods.

We use NBE to associate normal forms with terms.

The category-theoretic refinement of NBE (using sheaf theory and Grothendieck topologies) is used to establish completeness of the approach with respect to BiCCC. We remark that the relationship between BiCCC and Grothendieck topologies has been used by Fiore and Simpson (1997) to give a syntax-free characterisation of definability of set-theoretic functionals in the calculus above.

While a decision procedure for BiCCC has been announced by Ghani (1994) our method is quite different and we believe conceptually simpler. Our method also encompasses product types and an initial object.

This reports on joint work with T. Altenkirch (University of Munich, Germany), P. Dybjer (Chalmers University, Sweden), and P. Scott, (University of Ottawa, Canada).

► GERHARD JÄGER, *Metapredicative and explicit Mahlo: a proof-theoretic perspective*.
Institut für Informatik und angewandte Mathematik, Universität Bern, Neubrueckstrasse 10, CH-3012 Bern, Switzerland.
E-mail: jaeger@iam.unibe.ch.

Mahlo cardinals and their recursive analogues have played an important role in several areas of traditional logic, most prominently in set theory and recursion theory. The least recursive Mahlo ordinal, for example, is the least ordinal not recursive in Gandy's superjump, it is the closure ordinal of several interesting nonmonotone inductive definitions, and the inital segment of the constuctible hierarchy up to this ordinal provides for the least standard model of Π_2 reflection on admissibles.

The proof-theoretic treatment of Mahloness is more recent. First a theory for admissible sets formalizing a Mahlo universe and a corresponding subsystem of analysis were analyzed by Rathjen. Later Mahlo axioms were brought into the context of Martin-Löf type theory by Setzer. Research about Mahloness in the framework of Feferman's explicit mathematics is based on the work by Jäger, Strahm and Studer.

In this talk we discuss several new aspects of Mahloness, in particular from the point of view of explicit mathematics. However, related principles in subsystems of second order artihmetic will also be mentioned. In doing so we will also shed some new light on *metapredicativity*, an interesting area of formal systems couched between what is predicative and properly impredicative.

► JEAN-LOUIS KRIVINE, *Realizability and generalized forcing in set theory*.
UFR de Mathématiques, Université Paris VII, 2 Pl. Jussieu, 75251 Paris Cedex 05, France.
E-mail: krivine@logique.jussieu.fr.

The well known Curry-Howard correspondence between proofs and programs was originally discovered for a very restricted logic: the *system of simple types*, which uses the intuitionistic propositional calculus, with the only connective $\rightarrow$. It was extended to second order intuitionistic logic, in 1970, by J.-Y. Girard, under the name of *system F*. The relation with programming languages was made by J. Reynolds.

In 1990, this correspondence was extended to classical logic, following Felleisen and Griffin who found that the *law of Peirce* corresponds to *control instructions* in functional programming languages. Since then, several type systems have been given, which extend the Curry-Howard correspondence to second order classical logic. It was left the problem of extending this correspondence to the whole of classical Zermelo-Frænkel set theory, and this is the subject of the present talk.

In order to do this, we inductively build realizability models, like in the well known construction of forcing. We show that each axiom of ZF (except the axiom of choice) is then realized; we obtain in this way a type system in which set-theoretic proofs are formalizable and give rise to programs, which are terms of the λ-calculus extended with control instructions. In this system, the normalization property is "essentially true" in the sense that we get correct computations on data types. Of course, not every typable term is normalizable since, for example, the fixed point operator Y has the type of the foundation axiom.

This construction is indeed a generalization of forcing, the realizability models being the analog of Boolean valued models in usual forcing. Since we work in classical logic, it is now easy to turn them into ordinary models of ZF. Models obtained in this way are rather fascinating objects, which differ deeply from forcing models and pose several interesting problems. First of all, they always have non standard integers (except in a "degenerate case", which is usual forcing), and therefore, they are not end extensions of the original model of ZFC. Although I did not yet prove this, it is very likely that they never satisfy the axiom of choice.

[1] J.-L. KRIVINE, *Typed lambda-calculus in classical Zermelo-Frænkel set theory*, ***Archive for Mathematical Logic***, to appear, preprint at `http://www.logique.jussieu.fr/www.krivine`.

► YVES LAFONT, *Proof theory and computer science: from typographical engineering towards geometry*.
CNRS, Institut de Mathematiques de Luminy, 163 avenue de Luminy, Case 907, 13288 Marseille Cedex 9, France.
E-mail: `lafont@iml.univ-mrs.fr`.

What is a proof? Is it a number (Gödel), a text (Hilbert), or a tree (Gentzen)?

- *Numbers* are old and respectable mathematical objects, but Gödel's numbering is arbitrary. It is just a tool for the proof of the incompleteness theorem.
- *Texts* are closer to the material reality of mathematical proofs, but they are more *typographical* objects than mathematical ones. Furthermore, there are many arbitrary choices in the textual representation.
- The structure of proofs is better shown by *trees*, and this representation is used indeed by systems for interactive proof search, but trees are still not the good objects!

Linear logic suggests a more geometrical viewpoint, with the notion of *proof net* and its avatars, *interaction nets* and *geometry of interaction*. In a related area, namely *symbolic computation* and *rewriting*, geometrical notions also appear, such as *Penrose diagrams* and *homological invariants*. A symbiosis between logic and geometry could be as fruitful as the one which already exists between algebra and geometry.

► RICHARD LAVER, *Large cardinal results and problems*.
Department of Mathematics, University of Colorado, Boulder, Colorado, 80309, USA.
E-mail: `laver@euclid.colorado.edu`.

We will survey some results on very large cardinals, including facts about the axioms themselves and about some algebraic and other low-level statements which arise in the study of this topic.

► ANGUS MACINTYRE, *Model theory of Frobenius on Witt vectors*.
E-mail: `angus@maths.ed.ac.uk`.

Joint work with L. Belair and T. Scanlon.

I will describe an approach, basically along the lines of Ax-Ershov-Kochen, to the model theory of the rings $W[k]$, where k is a perfect field of finite characteristic, augmented with the important lifting of Frobenius. Under some natural restrictions on k (restrictions which are irrelevant in the original purely field-theoretic work) one can obtain exact analogues of the classical transfer theorems, quantifier eliminations, and decidability results. But for this one needs to develop a much more refined theory of pseudoconvergence, and henselization in the category of valued difference fields. Our results in the above setting involve the formalism of p-derivations. But we develop also an extensive theory of general valued difference fields without any analogue of p-derivations, and obtain in particular an analogue of the classical work relating p-adic fields and power series over finite fields.

We pay attention, too, to uniformity in p.

► DAVID MARKER, *Differential equations and logarithmic-exponential series*.
Department of Mathematics, Statistics and Computer Science, Univesity of Illinois at Chicago, 851 S. Morgan St., Chicago, Illinois 60607-7045, USA.
E-mail: marker@math.uic.edu.

Van den Dries, Macintyre and I introduced the field of LE-series as a natural nonstandard model of the theory of the real field with exponentiation. The LE-series have a natural derivation and there seems to be a close relationship between solving differential equations in the LE-series and finding solutions in Hardy fields.

► DONALD MARTIN, *Set theory in the 20th century*.
Department of Mathematics, University of California at Los Angeles, Los Angeles, California 90095-1555, USA.
E-mail: dam@math.ucla.edu.

It is the first third of the century that I will discuss in the most detail. This was the period that witnessed the formal axiomatization of set theory. It also witnessed the discovery of the iterative concept of set, a concept that the ZFC axioms may be thought of as axiomatizing. There are still several misconceptions about this early period, particularly misconceptions having to do with credit. The contributions of Cantor and Mirimanoff, for example, are not adequately appreciated. Zermelo, on the other hand, is deservedly much appreciated, but his real contributions are not quite the same as the contributions for which he is given credit.

The relative consistency theorems of Gödel and Cohen revitalized set theory, but they also raised fundamental questions about the future of the subject. The various kinds of set-theoretic research after Cohen might be thought of as resulting from different responses to these questions. Fortunately, there are many interconnections between the different kinds of research. What is perhaps unique about set theory at the end of the century is that work in it might shed light on basic questions about the nature of mathematics. How can we come to know the truth of fundamental mathematical axioms? Do all mathematical questions have definite answers? Is such a set as the set of all real numbers a determinate totality or something ultimately vague?

► DANIEL OSHERSON, *A psychologist looks hopefully to logic*.
Rice University, Psychology Department, MS-25, P. O. Box 1892, Houston, Texas 77251-1892, USA.
E-mail: osherson@rice.edu.

The development of science is among the most distinctive accomplishments of the human species. To help clarify how science is achieved, psychologists would like to understand the kind of reasoning that underlies empirical discovery. A natural hypothesis is that such reasoning is based on elements of deductive and inductive logic, in something like the form these disciplines take today. However, if "inductive logic" is interpreted in the sense of (Bayesian) probability, the hypothesis runs into a problem: most people appear to lack even the most elementary ability to distribute probabilities over logically related statements.

This mental lacuna calls for elucidation in its own right. A potential explanation is suggested by recent psychological data. If correct, the explanation makes it implausible that basic scientific competence rests on probabilistic inductive logic.

As a starting point for understanding scientific discovery, therefore, the psychologist needs an alternative perspective on inductive logic. One alternative starts from the all-or-none concept of acceptance as opposed to graded belief. The inductive logic of acceptance is governed by principles of hypothesis selection and revision rather than probability. This theory of inductive logic has already seen development, but many questions remain.

► PETER SHOR, *Quantum computing*.
AT&T Labs, Florham Park, USA.
E-mail: `shor@research.att.com`.

Quantum computers are hypothetical devices which use the principles of quantum mechanics to perform computations. For some difficult computational problems, including the cryptographically important problems of prime factorization and finding discrete logarithms, the best algorithms known for classical computers are exponentially slower than the algorithms known for quantum computers. Although they have not yet been built, quantum computers do not appear to violate any fundamental principles of physics. I will explain how quantum mechanics provides this extra computational power, and outline the factoring algorithm. One of the main difficulties in building quantum computers is in manipulating coherent quantum states without introducing errors or losing coherence. This problem can be alleviated by the use of quantum error correcting codes; if a quantum computer can be built with only moderately reliable hardware, software can be used to make it extremely reliable. I will briefly discuss these issues.

► THEODORE A. SLAMAN, *Aspects of the Turing jump*.
Department of Mathematics, University of California, Berkeley, California 94720-3840, USA.
E-mail: `slaman@math.berkeley.edu`.

The *Turing jump* is the function which maps X, a set of natural numbers, to X', the halting problem relative to X. X' is the canonical example of a set which is arithmetic in X but not recursive in X.

We will discuss two aspects of this remarkable function and its iterations. First, we will examine the thesis that the only robust definable way to produce a set from X which is strictly more complicated than X is to produce a set which is as least as complicated as X'. Second, we will discuss how this canonical aspect of the jump and of the double-jump leads to Shore and Slaman's proof that the jump is definable in the partial order of the Turing degrees.

► OTMAR SPINAS, *Canonical behaviour of Borel functions on Polish planes*.
Mathematisches Seminar, Christian-Albrechts-Universität zu Kiel, Ludewig-Meyn-Strasse 4, 24098 Kiel, Germany.
E-mail: `spinas@math.uni-kiel.de`.

A subset of the Baire space ${}^{\omega}\omega$ is called *superperfect* if it is closed and homeomorphic to ${}^{\omega}\omega$. A rectangle in $({}^{\omega}\omega)^2$ is called *superperfect* if both sides are superperfect. I will present the canonization of all Borel measurable functions from $({}^{\omega}\omega)^2$ to $\mathbb{R}$ modulo restriction to superperfect rectangles. By Hurewicz' Theorem this yields, for every Polish space X which is not σ-compact, the canonization of all Borel functions $f\colon X^2 \to \mathbb{R}$ modulo restriction to rectangles with both sides closed and not σ-compact. It is known that an analogous result is impossible in dimension 3 and higher. Below I explain how the canonization looks like. This is a continuation of the work in [1] and is presented in [2].

DEFINITION 1. We say that $(x, y) \in ({}^{\omega}\omega)^2$ *oscillates infinitely often* if there exists a strictly increasing sequence $\langle k_i : i < \omega \rangle \in {}^{\omega}\omega$ such that $k_0 = 0$ and the following hold for all $n < \omega$:

1. $k_{2n} = \min\{ i < \omega : y(i) > \max\{ k_{2n+1}, x(j) : j < k_{2n+1} \} \}$,
2. $k_{2n+1} = \min\{ i < \omega : x(i) > \max\{ k_{2n+2}, y(j) : j < k_{2n+2} \} \}$.

As $\langle k_i : i < \omega \rangle$ is uniquely determined, we let $\mu_0(x, y) = \emptyset$, $\mu_{n+1}(x, y) = x \restriction k_{2n+1}$, $\nu_n(x, y) = y \restriction k_{2n}$, for all $n < \omega$. Let IO denote the set of all $(x, y) \in ({}^{\omega}\omega)^2$ which oscillate infinitely often. Let ETP (the set of *even type-pairs*) denote the set of all pairs of the form $(\mu_n(x, y), \nu_n(x, y))$, for some $(x, y) \in \mathrm{IO}$ and $n < \omega$. Similarly, OTP denotes the set of all *odd type-pairs*, i.e., those of the form $(\mu_{n+1}(x, y), \nu_n(x, y))$, for some $(x, y) \in \mathrm{IO}$ and $n < \omega$.

DEFINITION 2. Let $c : \mathrm{ETP} \to 2$, $d : \mathrm{OTP} \to 2$ be arbitrary. Define functions f_c and f_d with domain IO as follows: Fix $(x, y) \in \mathrm{IO}$ and let $(\mu_n, \nu_n) = (\mu_n(x, y), \nu_n(x, y))$. If $\langle (\mu_{n_k}, \nu_{n_k}) : k < N \leq \omega \rangle$ is the subsequence of those pairs (μ_n, ν_n) with $c(\mu_n, \nu_n) = 1$, then let $f_c(x, y) = \langle \mu_{n_k+1} : k < N \rangle$. Similarly, if $\langle (\mu_{n_k+1}, \nu_{n_k}) : k < N \leq \omega \rangle$ is the subsequence of those (μ_{n+1}, ν_n) with $d(\mu_{n+1}, \nu_n) = 1$, let $f_d(x, y) = \langle \nu_{n_k+1} : k < N \rangle$.

THEOREM. *Suppose that $f : ({}^{\omega}\omega)^2 \to \mathbb{R}$ is Borel measurable. There exists a superperfect rectangle $S_0 \times S_1 \subseteq \mathrm{IO}$ such that precisely one of the following alternatives holds*:

1. $\forall (x, y), (x', y') \in S_0 \times S_1\ f(x, y) = f(x', y')$.
2. $\forall (x, y), (x', y') \in S_0 \times S_1\ f(x, y) = f(x', y') \iff x = x'$.
3. $\forall (x, y), (x', y') \in S_0 \times S_1\ f(x, y) = f(x', y') \iff y = y'$.
4. *There exists $c : \mathrm{ETP} \to 2$ such that for all $(x, y), (x', y') \in S_0 \times S_1$ we have $c(\mu_n(x, y), \nu_n(x, y)) = 1$ for infinitely many n and $f(x, y) = f(x', y') \iff f_c(x, y) = f_c(x', y')$.*
5. *There exists $d : \mathrm{OTP} \to 2$ such that for all $(x, y), (x', y') \in S_0 \times S_1$ we have $d(\mu_{n+1}(x, y), \nu_n(x, y)) = 1$ for infinitely many n and $f(x, y) = f(x', y') \iff f_d(x, y) = f_d(x', y')$.*
6. $\forall (x, y), (x', y') \in S_0 \times S_1\ f(x, y) = f(x', y') \iff (x, y) = (x', y')$.

[1] O. SPINAS, *Ramsey and freeness properties of Polish planes*, ***Proceedings of the London Mathematical Society***, to appear.
[2] ———, *Canonical behaviour of Borel functions on superperfect rectangles*, preprint.

► S. STARCHENKO, *On the Zil'ber principle for structures definable in o-minimal models*.
Department of Mathematics, University of Notre Dame, USA.
E-mail: starchenko.1@nd.edu.

Let $\mathcal{M}$ be a fixed o-minimal model, and $\mathcal{N}$ be a structure definable in $\mathcal{M}$ (i.e., the universe of $\mathcal{N}$ is a definable subset of $\mathcal{M}$ and all basic relations and functions of $\mathcal{N}$ are definable in $\mathcal{M}$).

A general question one can ask is whether the Zil'ber principle holds for such $\mathcal{N}$.

In this talk we will discuss mainly the following part of the Zil'ber principle (so called "purity"): If $\mathcal{M}$ is an expansion of a field then every $\mathcal{M}$-definable set is definable in the pure field language.

This part of the Zil'ber principle, does not hold in general. For example, $\mathbb{R}_{\mathrm{an}}$, the expansion of the field of reals by restricted analytic functions, is a proper expansion of the field of reals.

However we will show that "purity" holds under extra assumption that $\mathcal{M}$ is stable.

From this one can derive another proof of Chow's theorem and also the "purity" principle for the category of compact analytic complex manifolds.

► MARTIN ZEMAN, *Combinatorial principles in inner models*.
Institut für Formale Logik, Universität Wien, Währinger Str. 25, 1090 Wien, Austria.
E-mail: zeman@logic.univie.ac.at.

We shall present the Friedman-Jensen approach to the theory of fine structural inner models $\mathbf{L}[E]$ for overlapping extender sequences and analyse several combinatorial principles in these models. Among these principles are $\diamondsuit_\kappa(A)$, $\diamondsuit_\kappa^+$, $\square_\kappa$, global $\square$, weak $\square$ principles like Jensen's $\square_\kappa^*$, Schimmerling's $\square_{\kappa,\lambda}$ and $\square_{\kappa,<\kappa}$, existence of κ^+-Aronszajn and Souslin trees. Unlike in $\mathbf{L}$, most of these principles need not hold in $\mathbf{L}[E]$, but we give a complete characterisation of them, that is, a condition equivalent to the failure of these principles in $\mathbf{L}[E]$. This condition is in $\mathbf{L}[E]$ equivalent to Jensen's weak extendibility axiom and its consistency strenth ranges at the level of superstrong cardinals.

► BORIS ZIL'BER, *Analytic and pseudo-analytic structures*.
Mathematical Institute, University of Oxford, 24-29 St. Giles, Oxford, OX1 3LB, United Kingdom.
E-mail: `zilber@maths.ox.ac.uk`.

As is well-known Hrushovski constructed a series of strongly minimal structures that have no interpretation in algebraic geometry. We discuss his construction and give arguments towards the understanding that the structures that the construction provides are basically similar to classical ones such as complex numbers with extra analytic transcendental functions.

We are going to discuss how, pursuing systematically this approach we can

(i) develop a model-theoretic point of view on transcendental number theory;
(ii) find a generalisation of analytic theory for abstract algebraically closed fields with applications to algebraic geometry.

Abstracts of invited plenary talks in the Philosophy Symposium entitled *The Foundation of Mathematics around 1900*

► PHILIPPE DE ROUILHAN, *Russell's logics*.
Université Paris 1, Panthéon Sorbonne, France.
E-mail: `rouilhan@ext.jussieu.fr`.

In 1903, in Appendix B of the Principles of Mathematics, Russell described a paradox he had discovered late in 1902. This paradox is more difficult than the better known paradox of 1901, which depends upon the notion of set. The later paradox concerns what we should call today the hyperintensional notion of proposition, understood as the meaning of a possible sentence, and it reveals that this notion is subject to difficulties analogous to those that the earlier paradox uncovered in the concept of set. Russell must have forgotten or ignored the 1903 paradox, for he seems never to have mentioned it again. John Myhill rediscovered it, in a slightly different form, in 1958, and consequent to this it attracted the attention of logicians, notably Church, in the seventies.

The hyperintensional paradox, and others of the same sort, have been misunderstood and neglected. When they and their implications are taken into account, the development of Russell's thought, and the history of logic in general, from the Principles to Principia appears in a new light. I argue that in the rational reconstruction and evaluation of the logics that Russell successively conceived, following the way that Russell himself understood them, too much weight has been attributed to the semantical paradoxes and not enough to the hyperintensional paradox. In this talk, I propose to correct this.

The great Russellian logical enterprise was subject to powerful constraints quite exogenous to Russell's thought and mind. Russell may not have been aware of them, but their effects were in no way diminished by his ignorance, and these same forces apply today to any attempt to construct a hyperintensional logic.

► WILLIAM EWALD, *The philosophical significance of Hilbert's problems*.
University of Pennsylvania Law School, 3400 Chestnut St., Philadelphia, Pennsylvania 19104, USA.
E-mail: `wewald@law.upenn.edu`.

Hilbert is often regarded, in Otto Blumenthal's phrase, as 'the man of problems'—as a brilliant technician who moved from problem to problem, picking up new problems and discarding the old as his interests shifted. This aspect of Hilbert's practice has created the impression that the various aspects of his mathematical work—in number theory, in geometry, in mathematical physics, in foundations—were not related in any systematic manner. In particular his search for a consistency proof tends to be viewed as one problem among many, and specifically as an attempt to overcome the difficulties occasioned by the discovery of the paradoxes. It is argued here that Hilbert's search for consistency proofs was not motivated by the discovery of the paradoxes but had independent grounds, and that a recognition of this fact makes it possible to view Hilbert as engaged in a more systematic enterprise than is commonly recognized.

► RICHARD HECK, *Frege's contributions to foundational studies*.
208 Emerson Hall, Harvard University, Cambridge, Massachusetts 02138, USA.
E-mail: heck@fas.harvard.edu.

Gottlob Frege is well-known as the inventor of modern logic, in particular, of quantification theory, and as a defender of the view that arithmetic is "a branch of logic". The details of his views, and of his formal work, are less well-known, however. I intend to provide an overview of these matters and then to discuss, with reference to some recent work by Jamie Tappenden and Patricia Blanchette, aspects of Frege's famous dispute with Hilbert regarding proofs of the independence of geometrical axioms.

Abstracts of contributed talks

► TAREK SAYED AHMED, *On neat reducts of algebras of logics*.
Department of Mathematics, Faculty of Science, Cairo University, Egypt.
E-mail: sahmed@math-inst.hu.

SC_α, CA_α, QA_α and QEA_α stand for the classes of Pinter's substituton algebras, Tarski's cylindric algebras, Halmos' quasipolyadic algebras, and quasipolyadic equality algebras of dimension α, respectively. Generalizing a result of Nemeti on cylindric algebra, we show that for $K \in \{SC, CA, QA, QEA\}$, and ordinals $\alpha < \beta$, the class $Nr_\alpha K_\beta$, of α-dimensional neat reducts of β-dimensional K-algebras, though closed under taking homomorphic images and products, is not closed under forming subalgebras (i.e., is not a variety), if and only if $\alpha > 1$.

From this it easily follows that for $1 < \alpha < \beta$, the operation of forming α-neat reducts of algebras in K_β does not commute with forming subalgebras, a notion to be made precise.

We give a contrasting result concerning Halmos' polyadic algebras (with and without equality). For such algebras, we show that the class of infinite dimensional neat reducts forms a variety.

We comment on the status of the property of neat reducts commuting with forming subalgebras for various reducts of polyadic algebras that are also expansions of cylindric like algebras. We try to draw a borderline between reducts that have this property and reducts that do not.

Following research initiated by Pigozzi, we also emphasize the strong tie that links the (apparently non-related) property of neat reducts commuting with forming subalgebras, to proving amalgamation results in CA-like algebras of relations. We show that (like amalgamation) neat reducts commuting with forming subalgebras, is another algebraic expression of definability, and hence is also strongly related to the well known metalogical properties of Craig, Beth and Robinson in the corresponding logics.

► YUUKI ANDOU, *Strong normalization of classical natural deduction*.
Faculty of Liberal Arts (I), Hosei University, Tokyo 102-8160, Japan.
E-mail: norakuro@i.hosei.ac.jp.

We show a proof of strong normalization theorem (SN) for a system of natural deduction for first order classical logic. Our system has full logical connectives primitively including those for disjunction and for existential quantifier. In [1] we have proved Church-Rosser property directly for our system and also made a comment about SN of it. Our proof of SN is obtained by using a method of de Groote. In [2] he proved SN of intuitionistic natural deduction system which includes disjunction, and in his proof essential reductions and structural reductions are treated separately. Applying his method to our system on the base of Parigot's proof in [3] of SN for classical natural deduction, we obtain our proof which has the same separation on the treatment of two sorts of reductions.

[1] Y. ANDOU, *Church-Rosser property of a simple reduction for full first order classical natural deduction*, submitted.

[2] P. DE GROOTE, *On the strong normalization of natural deduction with premutation-conversions*, in ***Rewriting techniques and applications***, Lecture Notes in Computer Science, vol. 1631, Springer-Verlag, Berlin, 1999, pp. 45–59.

[3] M. PARIGOT, *Proofs of strong normalization for second order classical natural deduction*, ***Journal of Symbolic Logic***, vol. 62 (1997), pp. 1461–1479.

► MOHAMMAD ARDESHIR, *A Gentzen-style axiomatization for basic predicate logic*.

Department of Mathematics, Sharif University of Technology, P. O. Box 11365-9415, and Institute for Studies in Theoretical Physics and Mathematics, P. O. Box 19395-5746, Tehran, Iran.
E-mail: ardeshir@math.sharif.ac.ir, ardeshir@karun.ipm.ac.ir.

This is joint work with M. Aghaei.

W. Ruitenburg introduced [1] Basic Predicate Logic, BQC as the first order extension of Basic Propositional Logic, BPL invented by A. Visser [2]. Basic Logic is a proper subsystem of Intuitionistic Logic, IL in which *Modus Ponens* is weakened and is axiomatized in the same language of IL. Basic Logic is characterized as the logic of Kripke models with transitive frames.

Visser's axiomatization of BPL is in natural deduction style and Ruitenburg's BQC is in sequent notation (a sequent is an expression of the form $A \Longrightarrow B$, where A and B are formulas in the language of BQC).

We introduce a cut free Gentzen style axiomatization for BQC and its propositional fragment BPL. Our new system has the subformula property. Among other things, we are able to give a syntactic proof for decidability of BPL.

[1] W. RUITENBURG, *Basic predicate calculus*, ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), pp. 18–46.

[2] A. VISSER, *A propositional logic with explicit fixed points*, ***Studia Logica***, vol. 40 (1981), pp. 155–175.

► MARAT ARSLANOV, *The local enumeration degree structures*.

Kazan State University, Department of Mathematics, Kremlevskaja 18, Kazan 420008, Russia.

In this paper, we study first-order theories of the partial orderings of the Σ_2, Δ_2, total and n-c. e. enumeration degrees for different $n > 1$. We describe some recent results related to these structures. They consist of two parts. In the first part, we consider sentences in an expanded language which distinguish n-c. e. degrees for different $n > 1$ and study the complexity of these sentences. In the second part, we study algebraic properties of these structures (diamond embedding, cupping and capping properties, splitting of a given degree above another degree, etc.).

This is joint work with S. B. Cooper and I. Sh. Kalimullin.

► JEREMY AVIGAD, *Cut elimination revisited.*
Department of Philosophy, Carnegie Mellon University, Pittsburgh, Pennsylvania, 15213, USA.
E-mail: avigad@cmu.edu.

Algebraic proofs of the cut-elimination theorems for classical and intuitionistic logic are presented. A variation of the double-negation translation is also discussed: if φ is provable classically, then $\neg(\neg\varphi)^{\mathrm{nf}}$ is provable in minimal logic, where θ^{nf} denotes the negation-normal form of θ. The translation is used to show that the cut-elimination theorem for classical logic can be viewed as a special case of the cut-elimination theorem for intuitionistic logic.

► ARNON AVRON, *Safety of formulas in query languages, in set theory and in metamathematics*.
Computer Science Department, Tel Aviv University, Tel Aviv, Israel.
E-mail: aa@math.tau.ac.il.

We show that there have been common ideas concerning safety properties of formulas which were independently developed in the research on query languages and in the mathematical disciplines of set theory and of metamathematics (of elementary Arithmetics). We introduce a uniform notion of safety of a formula relative to a subset of its set of free variables as a common generalization of the various notions which have been employed in these areas, and show that it is useful for all of them. In the database case the main interest so far has in fact been in safety of a formula relative to the whole set of its free variables. The generalized relative notion provides, however, a firm theoretical foundation for using in queries complex terms, infinite relations, and (most important:) relations that are only partially accessible through links (like in the world wide web). In set theory our notion provides a new understanding and formulation of the axioms of ZF. We show that the various comprehension axioms of this theory (pairing, union, separation axioms, replacement axioms and powerset) can be replaced by the following principle: accept as an axiom any instance of $\exists Z\ (\forall x.\ x \in Z \iff A)$ in which A is *safe* relative to $\{x\}$. On the other hand safety relative to the empty set of variables is equivalent to absoluteness. In Arithmetics our notion provides a new understanding of $\sum_0$ formulas and of p. r. formulas. In this case the main interest is again in safety relative to the empty set of variables. However, the more general notion not only leads to a neat new definition of these classes, but also to a refined classification of them according to the choice of basic language and the safety properties of its atomic formulas.

► JOAN BAGARIA, *Determinacy and weakly-Ramsey sets in Banach spaces*.
Departament de Lògica, Història i Filosofia de la Ciència, Universitat de Barcelona, Spain.
E-mail: bagaria@trivium.gh.ub.es.

We give a sufficient condition for a set of block subspaces in an infinite-dimensional Banach space to be weakly-Ramsey ([2]). Using this condition we prove that in the Levy-collapse of a Mahlo cardinal over L, every projective set is weakly-Ramsey. This, together with a construction of W. H. Woodin ([4]), is used to show that the Axiom of Projective Determinacy implies that every projective set is weakly-Ramsey. For Hereditarily Indecomposable spaces we show that the Axiom of Determinacy plus the Axiom of Dependent Choices imply that every set is weakly-Ramsey. These results are the generalizations to the class of projective sets of some theorems from W. T. Gowers [2], [3], and our paper [1].

[1] J. Bagaria and J. López-Abad, *Weakly-Ramsey sets in Banach spaces*, submitted.
[2] W. T. Gowers, *Analytic sets and games in Banach spaces*, preprint IHES M/94/42.
[3] ———, *An infinite Ramsey theorem and some Banach-space dichotomies*, preprint.
[4] W. H. Woodin, *On the consistency strength of projective uniformization*, in ***Proceedings of Herbrand Symposium, Logic Colloquium '81***, North-Holland, 1982.

► ALEXANDRU BALTAG, *Universal set theory and strong modal reflection principles*.
CWI, P. O. Box 94079, 1090 GB Amsterdam, The Netherlands.
E-mail: abaltag@cwi.nl.

I consider infinitary modal logic $L^{\diamondsuit}_{\infty}$ as a foundational language for set theory. The Barwise-Moss semantics for modal logic (see [4]) is given by considering sets and classes as Kripke structures, with converse-membership as the accessibility relation. I use this semantics to *axiomatize and model* a non-classical, non-wellfounded universe of sets, containing a "universal" set U and other "large" sets. This universe can be seen as the largest extension of Peter Aczel's non-wellfounded universe which still satisfies Aczel's Strong Extensionality principle. My central axiom $SAFA$ ("Super-Antifoundation Axiom") is a strengthening of Aczel's AFA, stated as a *strong Reflection Principle* $L^{\diamondsuit}_{\infty}$. My consistency proof for this set theory uses a generalization of the standard "canonical model" construction for modal logic. The resulting model is isomorphic to one of the models constructed by researchers on "topological set theory" ([5], [6], etc.). I use this set theory to give non-classical solutions to classical set-theoretical paradoxes, to prove fixed-point theorems relating recursion and corecursion, to formalize a "self-referential" model theory and a "reflexive" category theory, to provide natural solutions for domain equations, and to justify some well-known "large cardinal" axioms.

[1] A. BALTAG, *STS: A structural theory of sets*, ***Ph. D. Dissertation***, Indiana University, 1998.

[2] ———, *A structural theory of sets*, in ***Proceeedings of "Advances in modal logic" (AiML '98)***, Uppsala, Sweden, 1998, pp. 32–41.

[3] ———, *STS: A structural theory of sets*, ***Logic Journal of the IGPL***, vol. 7 (1999), no. 4, pp. 481–515.

[4] J. BARWISE and L. S. MOSS, ***Vicious circles***, CSLI Lecture Notes, vol. 60, Stanford, 1996.

[5] M. FORTI and F. HONSELL, *The consistency problem for positive comprehension principles*, ***Journal of Symbolic Logic***, vol. 54 (1989), no. 4, pp. 1401–1417.

[6] ———, *A general construction of hyperuniverses*, ***Quadermi dell' Istituto di Matematiche Applicate "U. Dini"***, no. 9, 1992.

► STEFANO BARATELLA AND SIU-AH NG, *Applications of quantifier elimination in Banach spaces*.
Dipartimento di Matematica, via Sommarive 14, 38050 Povo TN, Italia.
E-mail: baratell@science.unitn.it.
University of Natal, Pietermaritzburg, 3209 South Africa.
E-mail: ngs@math.unp.ac.za.

Following an approach to quantifier elimination pursued in an abstract setting by Keisler [3], in [1] we introduced a notion of quantifier elimination (briefly: QE) for structures based on Banach spaces. Such a notion can be proved equivalent to Henson's quantifier elimination in sufficiently saturated nonstandard hulls. QE refers to a class of infinitary expressions, called by Keisler *neocompact formulas*, extending the class of *positive bounded formulas* introduced in [2].

First we present some properties equivalent to QE, then we discuss consequences of QE (or weaker properties like *density* or *back-and-forth*) on the geometry of a Banach space.

Eventually, we point out relationship between the abovementioned properties and fixed point theorems in reflexive nonstandard hulls of Banach spaces.

[1] S. BARATELLA and S.-A. NG, *Neocompact quantifier elimination in structures based on Banach spaces*, to appear in ***Annals of Pure and Applied Logic***.

[2] C. W. HENSON, *Nonstandard hulls of Banach spaces*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 108–144.

[3] H. J. Keisler, *Quantifier elimination for neocompact sets*, ***Journal of Symbolic Logic***, vol. 63 (1998), no. 4, pp. 1442–1472.

► RUSSELL BARKER, *A sketch of the proof that there is no recursive link between the k-size of a model and its cardinality*.

Mathematical Institute, University of Oxford, 24-29 St Giles, Oxford, OX1 3LB, UK.

In this talk I shall introduce the notion of L^k, the fragment of first order logic which contains variables $x_1, \ldots, x_k$, k-size and k-equivalence. Then I shall go on to use these tools to present two ideas conjectured by Anuj Dawar [1] which give finitary analogies to the Löwenheim-Skolem theorems. After this I will introduce the notions of diophantine and unary-exponential diophantine equations and give some results about these. Later a sketch refutation of Dawar's first conjecture, which is due to Martin Grohe [2], will be presented. This refutation involves building a translation from systems of diophantine equations into specific kinds of L^k theories called scales and then using a theorem of Matijasevic [3] to show that the desired result is impossible. Finally, I shall extend Grohe's proof, using the notions of unary exponential diophantine equations and exponentiation scales, and then use another of Matijasevic's theorems [4] to show that Dawar's second conjecture is also false.

[1] A. Dawar, *Type and indiscernables in finite models*, in ***Logic Colloquium '95***, 1998.

[2] M. Grohe, *Large finite models with small lk-types*, in ***Proceedings of the 12th IEEE symposium on logic in computer sciences***, 1997.

[3] J. V. Matiyasevic, *Enumerable set are diophantic*, ***Soviet Mathematics Doklady***, 1970.

[4] ———, *The existance of non-effectizable estimates in the theory of exponential diophantine equations*, ***Journal of Soviet Mathematics***, 1974.

► ARNOLD BECKMANN, *Model-theoretic characterizations of the separation problem of bounded arithmetic*.

Department of Mathematics, UCSD, 9500 Gilman Drive, La Jolla, California 92093-0112, USA.

E-mail: abeckman@math.ucsd.edu.

Bounded arithmetic is designed to characterize low complexity computability, i.e., the polynomial hierarchy. The main open problem for bounded arithmetic is the question if bounded arithmetic is finitely axiomatizable, which is equivalent to asking if the levels of bounded arithmetic S_2^i, T_2^i, etc., collapse. Furthermore, this question is also connected to the open problem whether the polynomial hierarchy collapses. The precise connection is that bounded arithmetic is finitely axiomatized if and only if bounded arithmetic can prove that the polynomial hierarchy collapses (cf. [2]).

In this talk we will give model-theoretic characterizations of the separation problem of bounded arithmetic. The main tool will be restricted consistency statements, whose complexity is captured by a bounded analog of Π_1. Our characterizations will be as the following one:

> Two levels of bounded arithmetic—say S_2^i vs. T_2^j for $j \geq i$—are separated by a bounded Π_1 sentence if and only if there exists a model of the smaller theory (i.e., S_2^i) without proper bounded-elementary extensions to a model of the stronger theory (i.e., T_2^j).

Remark for experts: By "bounded Π_1" we mean the universal closure of Π_1^b, and "bounded-elementary extensions" is the restriction of elementary extensions to sharply bounded formulas.

[1] A. Beckmann, *Model-theoretic characterizations of $\forall\Pi_1^b$-separations in bounded arithmetic*, preprint, 2000, available at http://wwwmath.uni-muenster.de/u/beckmann/publ/artikel9.html.

[2] S. R. BUSS, *Relating the bounded arithmetic and the polynomial time hierarchies*, ***Annals of Pure and Applied Logic***, vol. 75 (1995), pp. 67–77.

► EDWARD BELAGA, *Post-Hilbertian programme and its post-Gödelian stumbling block, II: Logical, phenomenological, and philosophical limits of the set-theoretical quest for mathematical infinity*.
IRMA, Université Louis Pasteur, 7, rue René Descartes, 67084 Strasbourg Cedex, France.
E-mail: belaga@math.u-strasbg.fr.

David Hilbert's foundational expectations and their brutal refutal by Kurt Gödel created the most disheartening paradigm of the formal scientific thinking: the stronger are the axiomatic foundations (in other words, expressive power) of a formal theory, the more inevitably stumbles one upon very natural and formally admissible in this theory claims, which cannot be neither proved no refuted without some nontrivial extension of the said axiomatic foundations. And in most interesting cases, the extension in question has to be either the claim itself, or, rather arbitrary and with equal plausibility, its logic negation!

There is no other modern domain of formal studies where this post-modern paradigm is so pronounced as in set theory and its *ZF* axiomatics.

The trouble is hidden exactly where our *ZF* pride resides: in the powerful built-in iterative mechanism of set generation. Thus, *ZF* has gained in its creative power on the expense of its descriptive power, becoming an interactive programming language which supplies an advanced "user" with sophisticated options of creation of, and manipulation with, artificial infinite totalities (similar to computer graphic images): "*Only the first few levels of the cumulative hierarchy bear any resemblance to external reality. The rest are a huge extrapolation based on a crude model of abstract thought processes. Gödel himself comes close to admitting as much.*" [3], p. 362.

[1] EDWARD G. BELAGA, *Post-Hilbertian programme and its post-Gödelian stumbling-block*, ***Bulletin of Symbolic Logic***, vol. 4 (1998), pp. 449–450, abstract.

[2] ———, *Mathematical infinity, its inventors, discoverers, detractors, defenders, masters, victims, users, and spectators*, Preprint 00/27, Université Louis Pasteur, Strasbourg, http://irmasrv1.u-strasbg.fr:80/irma/publications/2000/00027.shtml.

[3] STEPHEN G. SIMPSON, *Partial realizations of Hilbert's program*, ***Journal of Symbolic Logic***, vol. 53 (1988), pp. 349–363.

► LUC BÉLAIR, *Delta-rings and axioms for Witt vectors with Frobenius*.
UQAM, C. P. 8888, Succursale Centre-ville, Montréal, Québec, H3C 3P8, Canada.
E-mail: belair.luc@uqam.ca.

We discuss the role of the delta-ring structure in the model theory of Witt vectors with the Frobenius automorphism.

► GARY D. BELL, *On evidence concerning the continuum hypothesis*.
University of California, Irvine, California, USA.
E-mail: gdbell@uci.edu.

From results of Gödel and Cohen it follows that the continuum hypothesis (CH) is consistent with, but not provable from, ZFC, the commonly accepted axioms of set theory. Gödel had proposed that adoption of stronger axioms of infinity might settle the CH. But a result of Solovay and Levy shows that large cardinal axioms do not settle the question of CH directly. Nonetheless, W. H. Woodin has claimed that results concerning the $P_{\max}$ model, some forcing axioms, and a notion of consequence in Ω-logic provide, assuming the existence of large cardinals, evidence that the CH is false. I will present results on my attempt to extract the philosophical and methodological principles lying behind this claim and to philosophically evaluate these principles. How do these results constitute evidence that CH is false? For example, Woodin's approach to the CH seems not to be merely the usual sort of strategy of proposing a new axiom of which not-CH is a consequence and then saying why the axiom should be accepted as an axiom of set theory. What justifications might be given for the large cardinal assumptions, the forcing axioms, and other assumptions used to make Woodin's claim? For example, the notion of consequence in Ω-logic involves a restriction to a certain class of models. What justifies this restriction?

► ANATOLY P. BELTIUKOV, *Smullyan rudimentary predicates and Skolem elementary functions on trees*.
Mathematics Department, Udmurt State University, Izhevsk, 426034, Russia.
E-mail: belt@uni.udm.ru.

Analogs of Smullyan rudimentary predicates and Skolem elementary functions on binary trees are constructed. Tree analogs of Grzegorczyk classes based on bounded recursion are also considered. Differences between the properties of these classes and those of ordinary Smullyan, Skolem and Grzegorczyk classes are studied.

► JEAN-YVES BÉZIAU, *Universal logic: Towards a general theory of logics*.
Stanford University, Centre for the Study of Language and Information, Ventura Hall, Stanford, California, 94305-4115, USA.
E-mail: jyb@csli.stanford.edu.

By universal logic, we mean here a general theory of logics considered as fundamental mathematical structures in the sense of Bourbaki (cf. [4]). The aim of this paper is to explain the philosophical and historical background of such a theory and to give some examples of technical problems that are dealt with.

We use the expression *universal logic* by analogy with the expression *universal algebra*: universal algebra was developed as a very general theory of algebras considered as abstract structures. As Birkhoff explains (cf. [3]), he was led to the now standard notion of abstract algebra (a set with a family of operators) by the need of systematization of heterogeneous structures.

During the XXth century, numerous logics were developed (intuitionistic logic, many-valued logic, relevant logic, paraconsistent logic, quantum logic, modal logic, polar logic, non monotonic logic, linear logic, etc.). The basic idea of universal logic is to consider logics as structures and study general properties of these structures and their relationships (cf. [1] and [2]).

[1] J.-Y. Béziau, *Universal logic*, in ***Logica '94—Proceedings of the 8th International Symposium*** (T. Childers and O. Majer, editors), Czech Academy of Science, Prague, 1994, pp. 73–93.

[2] ———, *Recherches sur la logique universelle*, ***Ph. D. thesis***, Department of Mathematics, University of Paris 7, 1995.

[3] G. Birkhoff, *Universal algebra*, in ***Selected papers on algebra and topology by Garret Birkhoff*** (G.-C. Rota and J. S. Oliveira, editors), Birkhäuser, Basel, 1987, pp. 111–115.

[4] N. Bourbaki, *The architecture of mathematics*, ***American Mathematical Monthly***, vol. 57 (1950), pp. 221–232.

► D. A. BREDIKHIN, *N-variables positive logic and algebras of relations*.
Saratov State Technical University, 410002 Lermontova 7-22, Saratov, Russia.
E-mail: bredikhin@mail.ru.

The theory of algebras of relations plays an important role in modern algebraic logic. In the investigation of algebras of relations, one of the most important problems is to study their laws that can be expressed by equations or conditional equations. For any set Ω of operations on binary relations, denote by $R\{\Omega\}$ the class of algebras whose elements are binary relations and whose operations are members of Ω. Let L_∞ be a positive first-order logic [2] and L_n by a restriction of L_∞ on formulas with n individual variables. Denote by $\mathrm{Eq}_n\{\Omega\}$ $(\mathrm{Qeq}_n\{\Omega\})$ the set of all equations (conditional equations) of algebras from $R\{\Omega\}$ that can be derivate in L_n, and let $V_n\{\Omega\}$ $(Q_n\{\Omega\})$ be the corresponding variety (quasivariety). We shall consider the operations of relation product $\circ$, relation inverse $^{-1}$, intersection $\cap$, union $\cup$, and identity relations Δ. It follows from [3] that equations and conditional equations of algebras with these operations can be expressed in L_3. Note that the case $n = 3$ is more or less trivial and we suppose that $n \geq 4$. The class $R\{\circ, ^{-1}, \cap, \Delta\}$ was introduced and studied in [1].

THEOREM 1. *$Q_4\{\circ, ^{-1}, \cap, \Delta\} = V_4\{\circ, ^{-1}, \cap, \Delta\}$; $V_4\{\circ, ^{-1}, \cap, \Delta\}$ is finitely based; an algebra $(A, \cdot, ^{-1}, \wedge, e)$ belongs to $V_4\{\circ, ^{-1}, \cap, \Delta\}$ if and only if it satisfies the following equations: $(xy)z = x(yz)$, $(x^{-1})^{-1} = x$, $(xy)^{-1} = y^{-1}x^{-1}$, $xe = ex = x$, $x \wedge x = x$, $x \wedge y = y \wedge x$, $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \wedge y)^{-1} = x^{-1} \wedge y^{-1}$, $xy \wedge x(y \wedge z) = x(y \wedge z)$, $x \wedge yz = x \wedge y(z \wedge x^{-1}y)$. $Q_5\{\circ, ^{-1}, \cap, \Delta\}$ does not contain in $V_\infty\{\circ, ^{-1}, \cap, \Delta\}$, in particular, $Q_5\{\circ, ^{-1}, \cap, \Delta\} \neq V_5\{\circ, ^{-1}, \cap, \Delta\}$.*

[1] B. JÓNSSON, *Representation of modular lattices and of relation algebras*, ***Transactions of the American Mathematical Society***, vol. 92 (1959), pp. 449–464.

[2] H. RASIOWA and R. SIKORSKI, ***The mathematics of metamathematics***, Panstwowe Wydawnictwo Naukowe, Warszawa, 1963.

[3] A. TARSKI, *On the calculus of relations*, ***Journal of Symbolic Logic***, vol. 6 (1941), pp. 73–89.

► BERND BULDT, *An epistemological difference between G1 and G2*.
Department of Philosophy, University of Konstanz, D-78457 Konstanz, Germany.
E-mail: bernd.buldt@uni-konstanz.de.

The first and the second Gödel incompleteness theorem ("G1" and "G2" for short), though they are intimately connected, differ considerably. While G1 remains stable under substitution of provability predicates, G2 shows a certain sensibility for the provability predicate employed. Using suitable provability predicates is it easy to formally prove consistency within the system itself. This fact, the intensionality of G2, is still yet not widely acknowledged nor adequately understood.

Continuing earlier work on this problem (see [1] and [2]), I will argue for the thesis, that the G1/G2-difference, i.e., the extensional/intensional-difference, goes hand in hand with an epistemological difference. The argument rests on an anlysis of the conditions each G2-proof has to satisfy.

[1] B. BULDT, *Re-extensionalizing Gödel's second theorem*, in ***Philosophy of mathematics*** (J. Czermak, editor), Vienna, 1993, pp. 221–230.

[2] ———, *Anmerkungen zum Beweis des Zweiten Gödelschen Satzes*, in ***Analyomen 2*** (G. Meggle, editor), Berlin, 1997, pp. 31–42.

► V. K. BULITKO, *Post problem for restricted reducibilities*.
Odessa State University, Fontanskaya Doroga 67-40, Odessa 65062, Ukraine.
E-mail: booly@te.net.ua.

We consider the well known [1] problem by E. Post on construction of such sets that with respect to a given reducibility have degrees intermediate between the degrees of recursive sets and the degrees of complete sets.

Post considered reducibilities $\leq_m$, $\leq_{btt}$, $\leq_{tt}$, $\leq_T$ and solved the problem for all of them except $\leq_T$. For the latter Post constructed classes of simple and hyper-simple sets.

In [2] the following result was presented: Post's approach is applicable to any "characteristically bounded" reducibility $\leq_{TM}$ that does not coincide with $\leq_T$ on the class of recursively enumerable sets (r. e. s.). Furthermore, even the class of hyper-simple sets suffices to show that.

In this work another complexity measure for Turing reductions is considered. We call it *segment complexity* [3]. Let $A[n] = (A(i))_{i<n}$ denote the initial segment of set A. So $A(i)$ denotes the i-th element of A. The number n is the length of the segment $A[n]$. We can now say $A \leq_{MT} B$ if and only if there exist an index $x \in \mathbb{N}$ and a total function $h\colon \mathbb{N} \to \mathbb{N}$ such that the reduction $A \leq_T^x B$ takes place, $h \leq_T M$, and the initial segment of oracle $B[h(n)]$ is sufficient to compute $A[n]$ via the reduction almost for any n. For any set M relation $\leq_{MT}$ is also a reducibility with respect to decidability.

Therefore, the problem of interest is to describe such T-degrees $d_T(M)$ that Post's ideas are applicable to reducibilities $\leq_{MT}$.

As a part of our solution, we introduce *radius of recursiveness* $R_B\colon \mathbb{N} \to \mathbb{N}$ of any set B defined as a function that on the input n yields the length of the maximal initial segment of B computable by a program with the length no greater than n. M-radially-h-simple set is such a r. e. s. A that the radius of recursiveness $R_{\bar{A}}$ of its complement can not be majorated by any increasing total recursive in M function.

THEOREM. *For any set M of low Turing degree there exist M-radially-h-simple non-simple sets. All such sets are not $\leq_{MT}$-complete.*

In his talk at the Colloquium the author presented this amplification of the theorem: for any set M such that $\emptyset' \not\leq_T M$ there exist M-radially-h-simply sets what are not $\leq_{MT}$-complete.

[1] H. ROGERS,JR., ***Theory of recursive functions and effective computability***, McGraw-Hill, 1967.

[2] V. K. BULITKO, *On Post's problem for reducibilities with information bounds*, ***11th International Congress of LMPS***, Cracow, Poland, 1999, volume of abstracts, p. 76.

[3] ———, *About segment complexity of Turing reductions*, ***Mathematical Logic Quarterly***, vol. 45 (1999), no. 4, pp. 561–571.

► XAVIER CAICEDO, *The Ehrenfeucht-Mostowski theorem in abstract logics*.
Departamento de Matemáticas, Universidad de los Andes, A. A. 4976, Bogotá, Colombia.
E-mail: `xcaicedo@uniandes.edu.co`.

Compact logics satisfying the interpolation theorem and the Feferman-Vaught (uniform reduction) property for finite disjoint sums of structures satisfies the Ehrenfeucht-Mostowski theorem. That is, any theory with infinite models must have models containing any given linearly ordered set so that any automorphism of the order extends to an automorphism of the model. Without compactness, and under various weakenings of the definability and uniform reduction properties, we still get non rigid models. For example, any sentence having infinite models in a logic satisfying Beth's definability theorem and an infinitary version of uniform reduction for finite disjoint sums must have models with infinitely many automorphisms.

► WILLIAM C. CALHOUN, *Embedding finite lattices into the lattice of ideals of computably enumerable Turing degrees*.
Department of Mathematics and Computer Science, Bloomsburg University of Pennsylvania, Bloomsburg, Pennsylvania 17815, USA.
E-mail: `wcalhoun@bloomu.edu`.

This talk will be a survey of results concerning lattice embeddings into the lattice of ideals of computably enumerable Turing degrees, $\mathcal{I}$. The problem of embedding lattices into $\mathcal{I}$ is related to the well-known problem of embedding lattices into the computably enumerable Turing degrees, $\mathcal{R}$.

I will discuss recent joint work with Lerman in which we show that the lattice L_{20} provides an example of a finite lattice that is not embeddable into $\mathcal{I}$. The proof uses a reorganized and adapted version of Lempp and Lerman's proof that the lattice L_{20} is not embeddable into $\mathcal{R}$.

We also obtain a Π_2 necessary and sufficient condition for embedding lattices into $\mathcal{I}$. The condition may be obtained by reduction to Lerman's condition for embedding ranked finite partial lattices into $\mathcal{R}$.

[1] W. C. CALHOUN, *The lattice of ideals of recursively enumerable degrees*, ***Ph. D. Thesis***, University of California, Berkeley, California, 1990.

[2] S. LEMPP and M. LERMAN, *A finite lattice that cannot be embedded into the enumerable Turing degrees*, ***Annals of Pure and Applied Logic***, vol. 87 (1997), pp. 167–185.

[3] M. LERMAN, *A necessary and sufficient condition for embedding ranked finite partial lattices into the computably enumerable degrees*, ***Annals of Pure and Applied Logic***, vol. 94 (1998), pp. 143–180.

► G. CAMPERO-ARENA AND J. K. TRUSS, *Homogeneous and* 1*-transitive coloured, countable linear orderings*.
Department of Pure Mathematics, University of Leeds, Leeds LS2 9JT, England.
E-mail: gabriela@amsta.leeds.ac.uk.
E-mail: j.k.truss@leeds.ac.uk.

In this paper we tackle a classification problem for coloured linear orderings, that is, linear orderings in which each point is assigned a unique member of a set C, thought of as "colours". We restrict here to the countable case (so that also C may be taken to have cardinality at most $\aleph_0$), and, so that we have some chance of classification, impose a suitable homogeneity assumption. Now, if we require full homogeneity (sometimes called"ultrahomogeneity", [2]), meaning that any isomorphism between finite substructures extends to an automorphism, then the classification consists of just the so-called "C-coloured rationals" Q_C or structures built up from these and isolated points by concatenation, as we shall see in the last section. In the first sections rather we adapt Morel's classification [1] of the countable 1-transitive linear orders to the coloured case, where now "1-transitive" means that the automorphism group acts singly transitively on each set of points coloured by a fixed colour.

[1] A. C. MOREL, *A class of relation types isomorphic to the ordinals*, ***Michigan Mathematics Journal***, vol. 12 (1965), pp. 317–321.

[2] A. H. LACHLAN and R. E. WOODROW, *Countable ultrahomogeneous undirected graphs*, ***Transactions of the American Mathematical Society***, vol. 262 (1980), pp. 51–94.

► WALTER A. CARNIELLI AND MARCELO E. CONIGLIO, *Translations and model theory*.
Group in Theoretical and Applied Logic, CLE/IFCH—State University of Campinas-UNICAMP, 13083-970 Campinas, SP, Brazil.
Group in Theoretical and Applied Logic, CLE/IFCH—State University of Campinas-UNICAMP, 13083-970 Campinas, SP, Brazil.
E-mail: coniglio@cle.unicamp.br.

We propose a formalism for expressing general properties of abstract logics (conceived as two-sorted first-order structures in the sense of [1]) and translations between them (where translations are conceived as homomorphisms between such structures). The concept of translation between logics (see for example [5] and [3]) is a powerful tool for relating abstract logical systems and for conveying meaning to heterodox logics through the notion of possible-translations semantics (cf. [2]).

However, a general theory able to express general properties of abstract logics and explain the meaning of translations between them is still missing. This paper contributes to this question, introducing a formal framework in which abstract logics and their translations can be treated and understood. We show (details given in [4]) that interesting results from the point of view of combination of abstract logics can also be obtained. In particular, it is possible to prove that the category of abstract logics is closed under ultraproducts and that the process of fibring (cf. [6]) is also expressable in the proposed formalism.

We stress how classical results of model theory can be used to establish several properties of translations between abstract logics, helping to assign a philosophical meaning to this concept.

[1] D. J. BROWN and R. SUSZKO, *Abstract logics*, ***Dissertationes Mathematicae***, vol. 102 (1973), pp. 9–41.

[2] W. A. CARNIELLI, *Possible-translations semantics for paraconsistent logics*, in ***Frontiers of paraconsistent logic*** (D. Batens, C. Mortensen, G. Priest, and J. P. Van Bendegem, editors), Logic and Computation Series, 2000, to appear.

[3] W. A. CARNIELLI and I. M. L. D'OTTAVIANO, *Translations between logical systems: a manifesto*, ***Logique et Analyse***, vol. 157 (1997), pp. 67–81.

[4] M. E. CONIGLIO and W. A. CARNIELLI, *A model theoretic approach to translations between logics*, submitted to WoLLIC '2000.

[5] J. J. DA SILVA, I. M. L. D'OTTAVIANO, and A. M. SETTE, *Translations between logics*, in ***Models, algebras and proofs*** (X. Caicedo and C. H. Montenegro, editors), Marcel Dekker, 1999, pp. 435–448.

[6] A. ZANARDO, A. SERNADAS, and C. SERNADAS, *Fibring: Completeness preservation*, ***Journal of Symbolic Logic***, to appear.

► ZOÉ CHATZIDAKIS AND EHUD HRUSHOVSKI, *Some asymptotic results on fields*.
Université Paris 7, UFR de mathématiques, Case 7012, 2, place Jussieu, 75251 Paris Cedex 05, France.
E-mail: zoe@logique.jussieu.fr.
Institute of Mathematics, The Hebrew University, Givat Ram, Jerusalem 91904, Israel.
E-mail: ehud@math.huji.ac.il.

First set of results. Let Q be the set of all prime powers, and for $q = p^n \in Q$, p a prime, let K_q be a separably closed, non algebraically closed field of characteristic $p > 0$, and consider the difference field (K_q, σ_q), where $\sigma_q : x \mapsto x^q$. We consider the theory T consisting of all sentences true in almost all K_q, and give an axiomatisation of T. In the language obtained by adding to $\{+, \cdot, 0, 1, \sigma\}$ functions (which will be interpreted as the λ-functions of K if the characteristic is $p > 0$) and predicates for $\sigma(K)$-linear independence, we show that the expansion by definition of T is model-complete. All completions of T are simple, and various results valid in models of ACFA extend.

Second set of results. For each prime p, let $F(p)$ be the differential field $(\mathbb{F}_p(t)^s, +, \cdot, D_p)$, where t is transcendental over $\mathbb{F}_p$ and D_p is the unique derivation on $\mathbb{F}_p(t)^s$ satisfying $D_p t = 1$. We let T_D be the set of all sentences true in almost all $F(p)$. We show that T_D is bi-interpretable with a certain second-order theory of pseudo-finite fields. We also show that $T_{D\forall}$ and $T_{D\exists}$ are Σ_2^0-complete.

► JOHN CORCORAN, *What are logical relations?*.
Department of Philosophy, University of Buffalo, SUNY, Buffalo, New York 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Logical consequence and the converse relation of implication, contradiction and the complementary relation of compatibility, independence and the opposite relation of logical equivalence are all logical relations between propositions. Discussion stemming from Tarski's definition of "logical notions" makes it evident that the parallel question of the definition of "logical relation" has not been addressed. This paper locates the concept of logical relation in the literature of twentieth-century logic by colleagues of American Postulate Theorists often in connection with relations between members of postulate sets and between members of "squares of opposition". Relations considered to be logical are all found to share two properties: “formality” and “contentuality”. A formal relation is one that relates every pair that is in the same logical form as a pair that it relates. Formal relations “hold in virtue of form”. A contentual relation is one that relates any pair whose members are respectively logically equivalent to the corresponding members of a pair that it relates. Contentual relations “hold in virtue of content”. Relations considered non-logical in the relevant sense are found to lack one or the other of these properties. The relation that relates a disjunction to either of its disjuncts is non-logical; it is formal but not contentual. Material implication is non-logical; it is not formal but it is contentual. Some non-logical relations relevant to logical considerations are neither formal nor contentual; e.g., the relation of an axiom to a theorem of a particular axiomatic theory. The paper concludes with the proposal that in order for a relation to be logical (in the relevant sense) it is necessary and sufficient that it be both formal and contentual.

► JAIRO JOSÉ DA SILVA, *The many senses of completeness*.
University of the State of São Paulo (UNESP), Rio Claro, S. P., Brazil.
E-mail: `jairomat@linkway.com.br`.

A century ago, as well as today, the notion of completeness had many different and not always equivalent senses. Both Edmund Husserl and David Hilbert introduced more than one notion of completeness in the last years of the 19th Century. These concepts, albeit differently motivated, have nonetheless certain similarities. Husserl himself thought that the notion of completeness related to Hilbert's axiom of completeness derived from the concepts of completeness that he introduced in order to deal with the problem of imaginary entities in mathematics.

In this paper, which I conceive as a sequel to my “Husserl's two notions of completeness” (to appear in *Synthese*), I want to consider this question in more details than has been done before in the literature. Beginning with a careful analysis of the notions of completeness defined in paragraph 72 of *Ideen I*, I try to put Husserl's ideas concerning formal systems and their properties into the broader context of his philosophy. In particular his ideas concerning formal ontology as a province of formal logic.

The fact that the study of formal systems belonged, according to Husserl, to formal ontology and not, as Hilbert thought, to a purely combinatorial metamathematics, explains, I believe, why Husserl thought that the property of deductive completeness, which Hilbert considered as purely syntactic, is related to the property of semantic completeness implicit in the axiom of completeness that Hilbert introduced in the axiomatisation of geometry and the theory of real numbers.

► P. DELLUNDE, F. DELON, AND F. POINT, *Separably closed fields considered as modules*.
Àrea de Lògica, Universitat Autònoma de Barcelona 08193-Bellaterra, Barcelona, Spain.
E-mail: `Pilar.Dellunde@uab.es`.
Équipe de Logique Mathématique, UFR de Mathématiques, Case 7012, Université Paris 7-Denis Diderot, 2, Place Jussieu, 75251 Paris, France.
E-mail: `delon@logique.jussieu.fr`.
Institut de Mathématique et Informatique, Université de Mons-Hainaut, Le Pentagone, 6, avenue du Champ de Mars, B-7000 Mons, Belgium.
E-mail: `point@logique.jussieu.fr`.

We present some results on the theory of modules of separably closed fields ([2]). Let $SCF_{p,e}$ be the first-order theory of separably closed fields of characteristic p and imperfection degree $e \in \omega \cup \{\infty\}$ in the language of fields ([3]). We consider the models of $SCF_{p,e}$ in a weaker language. We fix a model K of $SCF_{p,e}$ and a p-basis B of K. We may consider K as a module over the skew polynomial ring $R = \mathbb{F}_p(B)[t;\alpha]$, where α is the Frobenius map (see, for instance, [1]). We extend the ordinary language of modules with the analog of the p-component functions and we prove that the theory of separably closed fields considered as R-modules, is complete and admits quantifier elimination.

We prove also that our theory (though stable) is not superstable, we give a description of the types and we study the Ziegler space of indecomposables ([4]).

[1] P. M. COHN, *Skew fields*, in ***Encyclopedia of mathematics and its applications*** (G.-C. Rota, editor), vol. 57, Cambridge University Press, 1995.

[2] P. DELLUNDE, F. DELON, and F. POINT, *Theory of modules of separably closed fields*, preprint, 1999.

[3] Y. ERSHOV, *Fields with a solvable theory*, ***Doklady***, vol. 174 (1967), pp. 19–20; English translation in ***Soviet Mathematics***, vol. 8 (1967), pp. 575–576.

[4] M. ZIEGLER, *Model theory of modules*, ***Annals of Pure and Applied Logic***, vol. 26 (1984), pp. 149–213.

► STÉPHANE DEMRI AND RAJEEV GORÉ, *Theoremhood preserving maps characterising cut elimination for modal provability logics*.
Laboratoire Leibniz C. N. R. S., 38000 Grenoble, France.
E-mail: demri@imag.fr.
Automated Reasoning Group and Department of Computer Science, Australian National University, Canberra 0200, Australia.
E-mail: rpg@arp.anu.edu.au.

Propositional modal provability logics like $\mathtt{G}$ and $\mathtt{Grz}$ have arithmetical interpretations where $\Box\varphi$ can be read as "formula φ is provable in Peano Arithmetic". These logics are decidable but are characterised by classes of Kripke frames which are not first-order definable. By abstracting the aspects common to their characteristic axioms we define the notion of a formula generation map $\mathtt{F(p)}$ in one propositional variable. We then define a class of (provability) logics where each logic $\mathtt{LF}$ is obtained from a first-order definable (Sahlqvist) modal logic $\mathtt{L}$ by the addition of an axiom based upon a formula generation map $\mathtt{F(p)}$ so that $\mathtt{LF} = \mathtt{L} + \mathtt{F(p)}$. The class of such logics includes $\mathtt{G}$ and $\mathtt{Grz}$. By appropriately modifying the right introduction rules for $\Box$, we give (not necessarily cut-free) display calculi for every such logic having a conservative tense extension. We define the pseudo-displayable subset of these logics as those whose display calculi enjoy cut-elimination for sequents of the form $\top \vdash \varphi$ for any formula φ. We then show that for any provability logic $\mathtt{LF}$ there is a map f on formulae such that $\mathtt{LF}$ is pseudo-displayable if and only if f maps theorems of $\mathtt{LF}$ to theorems of the underlying logic $\mathtt{L}$ and vice-versa. By using a standard renaming technique we can guarantee that there is a polynomial-time translation from $\mathtt{LF}$ into $\mathtt{L}$. All proofs are purely syntactic and show the versatility of display calculi since similar results using traditional Gentzen calculi are not possible for as broad a range of logics and require further conditions. Our maps generalise previously known maps from $\mathtt{G}$ into $\mathtt{K4}$. An application of our results gives an $\mathcal{O}((n.logn)^3)$ translation from the ("second order") provability logic $\mathtt{Grz}$ into a decidable subset of first-order logic. Since each of our logics $\mathtt{L}$ is a Sahlqvist logic, it is first-order definable, and hence each $\mathtt{L}$ has a translation into first-order logic. Our results therefore show that all pseudo-displayable logics $\mathtt{LF}$ are "essentially first-order" even though their characteristic axiom may not be first-order definable.

Authors supported by an Australian Research Council International Researcher Exchange Award and Queen Elizabeth II Award respectively.

► CARLOS A. DI PRISCO, JIMENA LLOPIS, AND STEVO TODORCEVIC, *Parametrized partitions*.
Instituto Venezolano de Investigaciones Cientificas, Apartado 21827, Caracas 1020-A, Venezuela.

E-mail: cdiprisc@ivic.ve.
Universidad Simón Bolivar, Caracas, Venezuela.
E-mail: jllopis@usb.ve.
CNRS-Université Paris VII, 2 Place Jussieu, 75251 Paris Cedex 05, France.
E-mail: stevo@logique.jussieu.fr.

A. Miller [1] showed that for every Borel $f : \omega^\omega \times [\omega]^\omega \to 2$, there is a perfect set $P \subseteq \omega^\omega$ and an infinite $H \subseteq \omega$ such that f is constant on the product $P \times [H]^\omega$. This can be seen as a Galvin-Prikry parametrization of the Berstein property. Parametrized partition properties of this type have interesting applications.

Infinite products of finite sets of natural numbers constitute particularly nice perfect subsets of the Baire space, and it is interesting to consider the possibility of parametrizing the corresponding strengthening of the Bernstein property. We show that such a parametrization is possible, even for analytic partitions. Moreover, we show that for every sequence $\{m_i\}$ of natural numbers there is an algebra $\mathbb{PCL}$ of subsets of $\omega^\omega \times [\omega]^\omega$ such that for every $\mathbb{PCL}$-measurable function $f : \omega^\omega \times [\omega]^\omega \to 2$, there exists a sequence $\{ H_i : i \in \omega \}$ with $H_i \subseteq \omega$ and $|H_i| = m_i$ for every $i \in \omega$, and a set $H \in [\omega]^\omega$, such that f is constant on $\prod_{i\in\omega} H_i \times [H]^\omega$. The algebra $\mathbb{PCL}$ contains the Borel subsets of $\omega^\omega \times [\omega]^\omega$ and it is closed under Suslin's operation. Moreover, it contains all subsets of $\omega^\omega \times [\omega]^\omega$ in $L(\mathbb{R})$ when $L(\mathbb{R})$ is a Solovay model. This can be used to answer a question of [2] regarding the relationship between a strong infinite dimensional polarized partition relation and $\omega \to (\omega)^\omega$.

[1] A. W. Miller, *Infinite combinatorics and definability*, ***Annals of Pure and Applied Logic***, vol. 41 (1989), pp. 179–203.

[2] C. A. Di Prisco and J. Henle, *Partitions of products*, ***Journal of Symbolic Logic***, vol. 58 (1993), pp. 860–871.

► MARKO DJORDJEVIC, *Recursive upper bounds for a class of L^n-theories*.

Department of Mathematics, Uppsala University, Box 480, 751 06 Uppsala, Sweden.
E-mail: marko@math.uu.se.

M. Grohe has shown (see [2]) that if $n \geq 3$ and the vocabulary of L contains at least one $(n-1)$-ary relation symbol then there is *no* recursive function f such that for all L^n-theories T with finite models, $\min\{ |M| : M \models T \} \leq f(|S^n_n(T)|)$, where $S^n_n(T)$ is the set of L^n-types in n free variables with respect to T. In [1], for every $n \geq 2$, a well behaved class of L^n-theories (where the vocabulary of L may contain relation symbols of all arities $\leq n$), call it $\mathcal{T}_n$, is studied. There it is proved that every theory in $\mathcal{T}_n$ has finite models, and in this talk we show that there exists a recursive function $f : \omega^2 \to \omega$ such that for every $n \geq 2$ and every $T \in \mathcal{T}_n$, $\min\{ |M| : M \models T \} \leq f(n, |S^n_n(T)|)$. This result depends heavily on the theory of stable homogeneous structures, a survey of which can be found in [3].

[1] M. Djordjević, *Finite variable logic, stability and finite models*, ***Journal of Symbolic Logic***, to appear.

[2] M. Grohe, *Large finite structures with few L^k-types*, in ***Proceedings of the 23rd International Symposium on mathematical foundations of computer science***, Lecture Notes in Computer Science, vol. 1450, Springer-Verlag, 1998.

[3] A. H. Lachlan, *Stable finitely homogeneous structures, a survey*, in ***Algebraic model theory*** (B. T. Hart et al., editors), Kluwer Academic Publishers, 1997, pp. 145–159.

► ROD DOWNEY, DENIS R. HIRSCHFELDT, AND BAKHADYR KHOUSSAINOV, *Uniformity in computable structure theory*.

School of Mathematical and Computing Sciences, Victoria University of Wellington, New Zealand.
E-mail: rod.downey@vuw.ac.nz.
School of Mathematical and Computing Sciences, Victoria University of Wellington, New

Zealand.
Department of Computer Science, University of Auckland, New Zealand.

We investigate the effects of adding uniformity requirements to concepts in computable structure theory such as computable categoricity (of a structure) and intrinsic computability (of a relation on a computable structure). In computability theory, uniformity is usually defined in terms of indices. From a model-theoretic viewpoint, however, it seems more natural to consider effective procedures that take structures, rather than indices for structures, as input. We consider and compare these two different notions of uniformity, and discuss connections with the relative computable structure theory of Ash, Knight, Manasse, and Slaman [1] and Chisholm [3] and with previous work of Ash, Knight, and Slaman [2] on uniformity in a general computable structure-theoretical setting.

[1] C. Ash, J. Knight, M. Manasse, and T. Slaman, *Generic copies of countable structures*, ***Annals of Pure and Applied Logic***, vol. 42 (1989), pp. 195–205.

[2] C. Ash, J. Knight, and T. Slaman, *Relatively recursive expansions II*, ***Fundamenta Mathematicae***, vol. 142 (1993), pp. 147–161.

[3] J. Chisholm, *Effective model theory vs. recursive model theory*, ***Journal of Symbolic Logic***, vol. 55 (1990), pp. 1168–1191.

► MIRNA DŽAMONJA, *D-spaces*.

School of Mathematics, University of East Anglia, Norwich, NR4 7TJ, United Kingdom.
E-mail: `h020@uea.ac.uk`.

Let X be a topological space, and τ its topology. An open neigbourhood assignment ONA on X is a function assigning to each x in X a member of τ that contains x. A topological space is called a D-space if for every ONA U of X there is a closed discrete subspace D of X such that $\bigcup\{\, U(x) : x \in D \,\} = X$. We show that it is consistent that every locally countable space X of size $\leq \aleph_2$ in which every open subspace of size $\leq \aleph_1$ is strongly D, is D itself.

► MÁRIO J. EDMUNDO, *O-minimal cohomology and definably compact definable groups*.

Mathematical Institute, University of Oxford, 24-29 St Giles, OX1 3LB Oxford, United Kingdom.
E-mail: `edmundo@maths.ox.ac.uk`.

Let N an o-minimal expansion of a real closed field. We develop the theory of covers for N-spaces (when N is $\aleph_1$-saturated) and the cohomology theory for the category of closed and bounded N-definable sets with continuous N-definable maps and use this to solve the problem of existence of torsion points on N-definably compact N-definable abelian groups. Namely we prove the following result: Let G be an N-definably compact N-definably connected N-definable group. Then the o-minimal Euler characteristic of G is zero. Moreover, if G is abelian and $\dim G = n$, then $\pi_1(G) = \mathbb{Z}^n$ and for all $k > 1$, $\operatorname{card}(\{\, x \in G : kx = 0 \,\}) = k^n$. We also compute the cohomology rings of N-definably compact N-definable groups.

► PHILIP EHRLICH, *Real and surreal arithmetic continua*.

Department of Philosophy, Ohio University, Athens, Ohio, 45701, USA.
E-mail: `ehrlich@oak.cats.ohiou.edu`.

In his monograph ***On numbers and games*** [1], J. H. Conway introduced an ordered field No which contains (in a suitable sense that can be made precise) "all numbers great and small." However, in addition to its distinguished structure as an ordered field No has a rich algebraico-tree-theoretic structure, or simplicity hierarchy, that emerges from the recursive clauses in terms of which it is defined. In a recent paper [2], we introduced a class of ordered fields with simplicity hierarchies whose properties generalize those of No and drew attention to some of the important relations that exist between No and this more general class of s-hierarchical ordered fields as we call them. Among a host of other results, we showed that an ordered field together with a tree structure is an s-hierarchical ordered field if and only if it is isomorphic to an ordered field that is an initial subtree of No (a property that is satisfied by every real-closed ordered field). In the present paper, we examine the relation between No and its unique complete s-hierarchical ordered subfield of reals and establish a variety of theorems which naturally suggest that whereas the s-hierarchical ordered field of reals should merely be regarded as constituting an Archimedean s-hierarchical arithmetic continuum, No may be regarded as a sort of absolute s-hierarchical arithmetic continuum (modulo von Neumann-Bernays-Gödel set theory). For proofs of these and related results, see [3].

[1] J. H. Conway, ***On numbers and games***, Academic Press, 1976.

[2] P. Ehrlich, *Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers*, ***Bulletin of Symbolic Logic***, vol. 4 (1998), no. 2, p. 212.

[3] ———, *Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers*, ***Journal of Symbolic Logic***, forthcoming.

► ALI ENAYAT, Σ_0^1-*definitions of infinity*.
Department of Mathematics and Statistics, American University, Washington, D. C. 20016-8050, USA.
E-mail: enayat@american.edu.

Given a model M in a language L, a logic $\mathbf{L}$ (such as first order logic $L_{\omega,\omega}$ or monadic second order logic $L_{\omega,\omega}^{\mathrm{Mon}}$) and an infinite cardinal κ, let us say that $\mathbf{L}$ *defines* κ in M if there is an $\mathbf{L}$-formula $\varphi(X)$ in the language $L \cup \{X\}$, where X is a new unary predicate, such that for all subsets A of M, $(M, A) \models \varphi(A)$ if and only if $|A| \geq \kappa$. For example, $L_{\omega,\omega}$ does not define $\aleph_0$ in the linear order $(\mathbb{Q}, <)$ of the rationals but $L_{\omega,\omega}^{\mathrm{Mon}}$ defines $\aleph_0$ in any linear order. However, $L_{\omega,\omega}$ defines $\aleph_0$ in $(\mathbb{R}, <)$, and $L_{\omega,\omega}^{\mathrm{Mon}}$ can define $\aleph_1$ in $(\mathbb{R}, <)$, at least in the presence of CH (by an old result of Gurevic). In this talk we survey known results dealing with such definability/undefinability results, and prove some new theorems.

► YU. L. ERSHOV, *On wonderful extensions of the field $\mathbb{Q}$ of rational numbers*.
Institute of Discrete Mathematics and Informatics, Prospect ac. Koptuga,4, 630090, Novosibirsk, Russia.
E-mail: ershov@nsu.ru.

A field F of characteristic 0 is called a wonderful extension of the field $\mathbb{Q}$ if there are the embeddings $(l(r); l(p), p \in P)$ of the field F into the field $\mathbb{R}$ of real numbers and the fields $\mathbb{Q}_p$ of p-adic numbers for p in P (here P is the set of all prime numbers) such that the following three conditions are satisfied:

1. Condition *NB*: For any element a in F, for almost all p in P, $l(p)(a)$ lies in the ring of p-adic integers.
2. Condition M: There is no proper algebraic extension of the field F having embeddings into all fields $\mathbb{Q}_p$.
3. Hasse's principle for smooth rational points of algebraic varieties defined over F relative to embeddings $l(r); l(p), p \in P$.

Theorem 1 (Existence). *Any countable field F of characteristic 0 with embeddings $l(r)$; $l(p)$, $p \in P$, satisfing condition NB, has a countable wonderful extension.*

Theorem 2 ('Uniqueness').

(a) *Any two wonderful extensions are elementary equivalent.*
(b) *Any extension of wonderful extensions is elementary.*

Theorem 3 (Decidability). *The elementary theory of the wonderful extensions is decidable.*

The proofs of Theorems 1 and 2 are based on the model theory of multi-valued fields with near boolean (*NB*) families of valuation rings developed in [1]. For the proof of Theorem 3 one needs the details of the elementary theory of prime finite fields from [2].

[1] YU. L. ERSHOV, ***Multi valued fields***, Scientific Books, Novosibirsk, 2000.
[2] M. FRIED and M. JARDEN, ***Field arithmetic***, Springer-Verlag, 1986.

► SOLOMON FEFERMAN, *On the proof-theoretical strength of some systems with the numerical omniscience scheme*.
Department of Mathematics, Stanford University, Stanford, California 94305, USA.
E-mail: `sf@csli.stanford.edu`.

The Numerical Omniscience Scheme (NOS) asserts for each formula $A(n)$ (parameters allowed) of a given language that if $A(n)$ is decided for each natural number n then either for all n, $A(n)$, or there exists n with not-$A(n)$. The scheme is considered in systems based on intuitionistic logic. A special case of NOS is the statement of the Limited Principle of Omniscience (LPO), which asserts that for all functions f of natural numbers, either f is identically zero or f has some non-zero value. Errett Bishop pointed out that all the theorems of classical analysis for which he found constructive substitutes follow from those substitutes together with LPO. NOS is here considered in the system of Heyting Arithmetic extended to finite types together with the Axiom of Choice AC in all types. It is shown that the Gödel functional ("Dialectica") interpretation of this system together with NOS follows from its interpretation with LPO. The interpretation validates full AC. Thus the proof-theoretic strength of the interpreted system and a number of its variants has been determined by the author as reported in [1], sections 8 and 9. In particular, if axioms for countable tree ordinals are added, the resulting system has strength measured by the Howard ordinal. This then establishes the strength of the system W introduced in [2], which uses only part of AC.

[1] J. AVIGAD and S. FEFERMAN, *Gödel's functional ("Dialectica") interpretation*, in ***Handbook of proof theory*** (S. R. Buss, editor), Elsevier, Amsterdam, 1998, pp. 337–405.
[2] T. COQUAND and E. PALMGREN, *Intuitionistic choice and classical logic*, ***Archive for Mathematical Logic***, vol. 39 (2000), pp. 53–74.

► SOLOMON FEFERMAN AND THOMAS STRAHM, *Unfolding finitist arithmetic*.
Department of Mathematics, Stanford University, Stanford, California 94305, USA.
E-mail: `sf@csli.stanford.edu`.
Institut für Informatik und angewandte Mathematik, University of Bern, Neubrückstr. 10, CH-3012 Bern, Switzerland.
E-mail: `strahm@iam.unibe.ch`.

The notion of *unfolding a schematic formal system* was introduced in Feferman [1] in order to answer the following question:

> Given a schematic system S, which operations and predicates, and which principles concerning them, ought to be accepted if one has accepted S?

A paradigmatic example of a schematic system S is the basic system NFA of non-finitist arithmetic. In Feferman and Strahm [2], three unfolding systems for NFA of increasing strength have been analyzed and characterized in more familiar proof-theoretic terms; in particular, it was shown that the full unfolding of NFA, $\mathcal{U}(\mathsf{NFA})$, is proof-theoretically equivalent to predicative analysis.

The aim of the present contribution is to elucidate the concept of unfolding in the context of the schematic system for *finitist arithmetic* FA in order to answer the crucial question of what principles are implicit in the actual finitist conception of the set of natural numbers. The main results state that the operational unfolding $\mathcal{U}_0(\mathsf{FA})$ and even the full (predicate) unfolding $\mathcal{U}(\mathsf{FA})$ of finitist arithmetic FA conservatively extend Skolem's system of primitive recursive arithmetic PRA. These results support Tait's informal analysis of finitism in [3] according to which finitist mathematics does not go beyond PRA.

[1] S. FEFERMAN, *Gödel's program for new axioms: Why, where, how and what?*, in ***Gödel '96*** (P. Hájek, editor), Lecture Notes in Logic, vol. 6, Springer-Verlag, Berlin, 1996, pp. 3–22.

[2] S. FEFERMAN and T. STRAHM, *The unfolding of non-finitist arithmetic*, ***Annals of Pure and Applied Logic***, vol. 104 (2000), pp. 75–96.

[3] W. TAIT, *Finitism*, ***Journal of Philosophy***, vol. 78 (1981), pp. 524–546.

► MARCO FORTI, *Strongly extensional vs. complementable collections*.
Dip. Matematica Applicata "U. Dini", Universita' di Pisa, V. Bonanno 25B, 56100 Pisa, Italy.
E-mail: forti@dma.unipi.it.

The general notion of *collection*, as "any aggregate of distinct objects concieved as a *single* object", originates from the fusion of two different naive concepts: the "extension of an arbitrary property" and the "content of a list". The first (*Fregean*) notion makes implicit reference to a *universal collection* V, from which objects are *separated* (possibly by means of some *definite* property). It suggests *Weak Extensionality* up to urelements, and claims for suitable *Comprehension Principles* (but the graph of *membership* $E \notin V$). The second (say *Barwisean*) notion, has to specify when two lists have the *same* content. According to [1], it yields *Strong Extensionality* up to atoms, thus *Singleton* and *Complement* are incompatible.

The usual axiomatizations identify "sethood" and "elementhood", a *Limitation of Size Principle* being the basic criterion for both. Large collections, being *uncollectable*, are a mere "façon de parler". We consider all collections, large and small, as *first class objects*, and we introduce the collections *Coll* of all collections and *Ins* of all sets (= "small manageable collections"). Adopting a "Barwisean" attitude, one can assume the very strong *Generalized Positive Comprehension Principle GPK* (see [2]). Then $E \in$ *Coll*, and one obtains a good approximation of the "most wanted" theory *SEC* asked for in [1].

Following instead a "Fregean" attitude, we considered in [3] a few very strong *Comprehension Principles* in the form of "Gödel Operations" acting on *Coll*, including *complement* (hence $E \notin$ *Coll*). The *simple and controlled internal structure* of sets is rendered by putting into *Coll* the graph of *membership between objects and sets*. We should add unions and intersections of *arbitrary collections*, but what we got is only that unions and intersections of *sets of collections* and of *collections of sets* are collections, and so is the *cartesian product of a set of collections*.

[1] J. BARWISE and L. MOSS, ***Vicious circles***, CSLI Lecture Notes, vol. 60, Stanford, 1996.

[2] M. FORTI and F. HONSELL, *Choice principles in hyperuniverses*, ***Annals of Pure and Applied Logic***, vol. 77 (1996), pp. 35–52. Addendum and Corrigendum, *Ibid.*, vol. 92 (1998), pp. 211–214.

[3] M. FORTI, F. HONSELL, and M. LENISA, *Operations, collections and sets within a general axiomatic framework*, in ***Logic and foundations of mathematics*** (A. Cantini et al., editors), Synthese Library, vol. 280, Kluwer, Dordrecht, 1999, pp. 1–24.

► MICHELE FRIEND, *Is 'analytic truth in mathematics' co-extensional with 'theorem'?*.
London School of Economics, The Old Vicarage, Pytchley, Nr. Kettering, Northants, Angleterre NN14 1EP.
E-mail: michele@philosophers.net.

Here, a theorem is a sentence which is yielded by an effective proof procedure, 'analytically true' can mean (a) true in virtue of meaning, (b) tautologous, or (c) follows from axioms and definitions alone.

An argument for the thesis that 'analytically true', when applied to the mathematical context, is co-extensional with 'theorem' is as follows, 'Analytic' is an epistemological term. There is a positive and a negative characterisation, both of which can be found in Frege. The positive characterisation is (c) above, The negative characterisation is: a statement is analytically true just in case to know that it is true, we neither have to appeal to sense perception, nor do we need to appeal to Kantian spatial or temporal intuition. Looking at the notation of the Begrifsschrift and the Grundgesetze together with the notion of a gapless proof, we have a good idea of how to interpret 'follows from' in (c). In hindsight, we can replace 'follows from' with 'yielded by an effective, proof procedure'. Under the (c) understanding of analyticity in mathematics and the 'effective proof' understanding of 'follows from' we have a position which holds that 'analytically true' in mathematics is co-extensional with 'theorem'.

In contrast, an argument against the thesis runs as follows. Under definitions (a) and (b), 'analytic' is a semantic notion. Apprised of the Gödel incompleteness theorem we know that truth in a sufficiently sophisticated system is not co-extensional with theorems of that system. We can now jump one of two ways: (1) we can seek to preserve the thesis and then conclude that 'analytic truth' in mathematics is restricted to complete systems, or significant proportions of incomplete system; or (2) we can drop the thesis. (1) is not an interesting option, for it makes the thesis empty or trivial.

To argue for (2) we consider the Gödel sentences, argue that they are analytically true, and then draw out the philosophical significance of the qualification of 'analytic' to a sentence in mathematics.

► J.-S. GAKWAYA AND CH. MICHAUX, *Graded universal function for BSS-Grzegorczyk hierarchy and separation of complexity classes*.
Université de Mons-Hainaut, Institut de Mathématique et d'Informatique, Le Pentagone, 6 avenue du Champ de Mars, 7000 Mons, Belgique.
Université de Mons-Hainaut, Institut de Mathématique et d'Informatique, Le Pentagone, 6 avenue du Champ de Mars, 7000 Mons, Belgique.
E-mail: `christian.michaux@umh.ac.be`.

In this communication we will assume some basic knowledge of Blum-Shub and Smale model of computation (see [1]). We will first define an extension of Grzegorczyk hierarchy (see [3] for the definition of this classical hierarchy) to the framework of BSS-model of computation (extending ideas from [2]). We show that this BSS-Grzegorczyk hierarchy is strict. As in the classical case a hierarchy of classes of relations is associated to BSS-Grzegorczyk hierarchy. By building a graded universal function for this hierarchy we prove that it is a strict hierarchy from the third level. The method of proof (which has some reminiscence from [4]) gives us a unified proof of the classical result. But it has no consequence on the main open problem in the subject: separation of the three first levels of this hierarchy of relations.

[1] L. Blum, M. Shub, and S. Smale, *On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines*, ***Bulletin of the American Mathematical Society***, vol. 21 (1989), pp. 1–46.

[2] J.-S. Gakwaya, *Extended Grzegorczyk hierarchy in the BSS model of computability*, in ***Foundations of computational mathematics*** (Cucker and Shub, editors), Springer-Verlag, 1997, pp. 127–151.

[3] A. Grzegorczyk, *Some classes of recursive functions*, ***Instytut Matematyczny Polskiej Akademii Nauk, Warszawa*** (1953), pp. 1–44.

[4] R. W. Ritchie, *Classes of predictably computable functions*, ***Transactions of the American Mathematical Society***, vol. 106 (1963), pp. 139–179.

► MATTHEW B. GIORGI, *Minimal degrees below high degrees*.
Department of Pure Mathematics, University of Leeds, Leeds, LS2 9JT, England.
E-mail: `matt@amsta.leeds.ac.uk`.

A degree $\mathbf{a} \leq \mathbf{0}'$ is *high* if $\mathbf{a}' = \mathbf{0}''$, and a degree $\mathbf{a}$ is *generalized high* if $\mathbf{a}' = (\mathbf{a} \cup \mathbf{0}')'$.

We note that generalized high degrees and high degrees coincide on the degrees below $\mathbf{0}'$. A degree $\mathbf{m}$ is *minimal* if $\mathbf{m} > \mathbf{0}$ and there is no degree $\mathbf{d}$ such that $\mathbf{0} < \mathbf{d} < \mathbf{m}$. Minimal degrees do indeed exist, and it has been shown that there are $2^{\aleph_0}$ minimal degrees. Every high degree bounds a minimal degree, and no high degree is itself minimal. In fact every generalized high degree bounds a minimal degree. There are results of Cooper and Epstein, and of Li and Yang, showing that there are elementary differences in the way individual members of familiar classes of degrees known to bound minimal degrees do so. We investigate possible distributions of minimal degrees below high degrees.

► MARIAGNESE GIUSTO, *Free sets and Reverse Mathematics*.
University of Torino.
Postal address: Via Loreto Vecchia 9/10/A, 17100 Savona, Italy.
E-mail: giusto@dm.unito.it, mariagnese@savonaonline.it.

In this talk we present some results—obtained in collaboration with Prof. P. Cholak and Prof. J. L. Hirst—related to Ramsey's theorem and the free set theorem in the context of Reverse Mathematics. A posting of H. Friedman in the FOM email list sparked our interest in this topic.

The purpose of Reverse Mathematics is to study the role of set existence axioms, trying to establish the weakest subsystem of second order arithmetic in which a theorem of ordinary mathematics can be proved. The basic reference is Simpson's recent monograph.

The Free Set Theorem (FS) reads as follows: "Let $f : [\mathbb{N}]^k \to \mathbb{N}$. There exists a *free* set for f, i.e., an infinite $A \subseteq \mathbb{N}$ such that for all $x_1, \dots, x_k \in A$, if $f(x_1, \dots, x_k) \in A$ then $f(x_1, \dots, x_k) \in \{x_1, \dots, x_k\}$." For $k \geq 1$, we use $FS(k)$ to denote the statement FS restricted to a particular value of k.

We recall that Ramsey theorem (RT^k_n) reads as follows: "For all n-coloring of $[\mathbb{N}]^k$ there is an infinite homogeneous set".

The most relevant result we found is that Ramsey's theorem for pairs and two colors (RT^2_2) implies the free set theorem for pairs $(FS(2))$. This result provides some interesting corollaries: "$FS(2)$ is Π^1_1-conservative over $RCA_0 + I\Sigma_2$" and "every computable $f : [\mathbb{N}]^2 \to \mathbb{N}$ has a low_2 free set".

► BÉATRICE GODART-WENDLING AND PIERRE JORAY, *On Ajdukiewicz's categorial analysis of quantifiers*.
CNRS, Laboratoire d'histoire des théories linguistiques, Université Paris 7; 2, Place Jussieu, F-75005 Paris, France.
E-mail: godart@ccr.jussieu.fr.
Séminaire de Logique, Université de Neuchâtel, Louis-Agassiz 1, CH-2000 Neuchâtel, Switzerland.
E-mail: pierre.joray@lettres.unine.ch.

The first time that a solution was given to the categorial analysis of formal quantifiers was in the second part of K. Ajdukiewicz's famous paper of 1935: "*Die syntaktische Konnexität*". Inspired by S. Lesniewski's logical inquiries, this solution allows both to specify and to complete the theory of semantic categories which was still problematic on this point. Let us mention, as a reminder, that Lesniewski did not attach any categorial index to the quantifiers in his formal language. Such a characterization would have led Lesniewski to put his high-order quantification among the functors of his calculus and call into question the unity of quantification, imposing a different index on the quantifier each time it has to bind a variable of a different semantic category.

Ajdukiewicz's treatment consists then to introduce an essential distinction between what he called "functor" and "operator". This allows him to indicate accurately, in the categorial formalism, the notion of scope and the binding role of quantifiers. Adding to his categorial notation the use of a vertical dash, left to the quantifiers fractional indexes, Ajdukiewicz manages to represent in the formalism both the shared and the distinctive properties of functors and operators. Just like any functor, a quantifier can form a composite coherent whole with the argument to which it applies. However, as being an operator, a quantifier can never be one of the arguments of a functor of an higher order. The vertical dash takes this difference into account. It introduces a new condition into the calculus concerning the order of application of Ajdukiewicz's unique categorial rule. Extending, moreover, this analysis to Whitehead and Russell's circumflex operator '$\frown$', Ajdukiewicz manages then to show that every quantifier can be specifically described as a combination including a functor and a pure abstraction operator.

In this paper, we will show the way Ajdukiewicz's formalization constitutes a valuable generalization of Lesniewski's theory of semantic categories. This study will also lead us to specify how Ajdukiewicz's solution allows an homogeneous treatment of all the formal quantifiers.

[1] K. Ajdukiewicz, *Die syntaktische Konnexität*, ***Studia Philosophica***, vol. 1 (1935), pp. 1–27; English translation in S. McCall, ***Polish logic, 1920–1939***, Clarendon, Oxford, 1967, pp. 207–231.

[2] S. Lesniewski, ***Collected works*** (S. Surma, J. T. Srzednicki, and D. I. Barnett, editors), Dordrecht, Varsaw; Kluwer, Polish Scientific Publications, 1992.

[3] A. N. Whitehead and B. Russell, ***Principia mathematica***, second ed., Cambridge University Press, 1927.

► ROBERT GOLDBLATT, *Equational definability of some classes of coalgebras*.
School of Mathematical and Computing Sciences, Victoria University, P. O. Box 600, Wellington, New Zealand.
E-mail: `Rob.Goldblatt@vuw.ac.nz`.

Certain kinds of coalgebra have proven useful in the modelling of features of object-oriented programming languages [3, 2]. The kind we consider has a non-empty set X of *states* (which may be thought of as the possible realisations of some notion of object), and a set of *method* operations $m\colon X \times I^m \to X$ and *attribute* functions $a\colon X \times I^a \to O^a$, where I^m and I^a are sets of inputs and O^a a set of outputs or *observable values*.

A calculus of *terms* for these coalgebras is developed using variables and constants for members of input and output sets, symbols for methods and attributes, and a special state symbol σ which may be thought of as a parameter denoting the "current" state. An *observable equation* $t_1 \approx t_2$ has terms t_1 and t_2 taking observable values.

The *ultrafilter enlargement* of a coalgebra A is defined as a new coalgebra whose states are certain "rich" ultrafilters on the state set of A. These ultrafilters are required to contain special sets of states defined by observable equations. It is shown that

> a class of coalgebras is definable by Boolean combinations of observable equations if and only if it is closed under disjoint unions, ultrafilter enlargements, and domains and images of coalgebraic morphisms (or equivalently, images of bisimulation relations).

This may be viewed as a coalgebraic analogue of Birkhoff's characterisation of equationally definable classes of algebras by closure under homomorphic images, subalgebras, and direct products.

[1] Robert Goldblatt, *What is the coalgebraic analogue of Birkhoff's variety theorem*?, ***Research Report 99-34***, School of Mathematical and Computing Sciences, Victoria University of Wellington, December 1999.

[2] Bart Jacobs, *Objects and classes, coalgebraically*, in ***Object-orientation with parallelism***

and persistence (B. Freitag, C. B. Jones, C. Lengauer, and H.-J. Schek, editors), Kluwer Academic Publishers, 1996, pp. 83–103.

[3] HORST REICHEL, *An approach to object semantics based on terminal co-algebras*, ***Mathematical Structures in Computer Science***, vol. 5 (1995), pp. 129–152.

► VALENTIN GORANKO AND BRUCE KAPRON, *The modal logic of the countable random frame*.

Department of Mathematics, Rand Afrikaans University, P. O. Box 524, Auckland Park 2006, Johannesburg, South Africa.
E-mail: `vfg@na.rau.ac.za`.
Department of Computer Science, University of Victoria, Victoria, BC, V8W 3P6 Canada.
E-mail: `bmkapron@csr.uvic.ca`.

One of the major attractions in finite model theory is the study of asymptotic probabilities, 0-1 laws, and almost surely validity of properties of random finite structures, formalizable in various logical languages (see surveys [1], [2], and [3]).

An important problem in the area is to give a logical characterization of the almost sure validity in the finite, i.e., of those formulas which are valid with asymptotic probability 1 in a randomly chosen finite structure. However, this is not *a priori* a logically tractable concept, as it lacks explicit logical semantics in terms of particular models, but rather involves the class of all finite models as a whole. Usually such characterizations are associated with the countable random structure of the language and in most of the known cases when 0-1 laws hold, it is due to the property of that random structure to capture precisely the almost sure truth and falsity in the finite. In particular, Fagin proved that result for first-order logic, and Kolaitis and Vardi for the infinitary logic $L^{\omega}_{\infty,\omega}$ and for some prefix-defined fragments of monadic Σ^1_1.

We study the modal logic ML^r of the countable random frame which is contained in the modal logic of almost sure frame validity.

We give a sound and complete axiomatization of ML^r and show that it is not finitely axiomatizable. Then we describe the finite frames of that logic and show that it has the finite frame property and its satisfiability problem is in EXPSPACE. Finally, we show that there are modal formulas which are almost surely valid in the finite, yet fail in the countable random frame, and hence do not follow from the extension axioms.

[1] K. J. COMPTON, *0-1 laws in logic and combinatorics*, in ***NATO Advanced Study Institute on algorithms and order*** (I. Rival, editor), Reidel, 1988, pp. 353–383.

[2] Y. GUREVICH, *Zero-one laws*: *The logic in computer science column*, ***Bulletin of the European Association for Theoretical Computer Science*** (1992), pp. 90–106.

[3] PH. G. KOLAITIS and M. Y. VARDI, 0-1 *laws for fragments of second-order logic*: *an overview*, in ***Logic for computer science*** (Y. Moschovakis, editor), proceedings of workshop in 1989, pp. 265–286.

► PETR HÁJEK, *Properties of monadic fuzzy predicate logics*.

Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod vodarenskou vezi 2, 182 07 Prague, Czech Republic.
E-mail: `hajek@cs.cas.cz`.

Ł∀, G∀, Π∀ denote the three most important particular t-norm predicate logics: Łukasiewicz, Gödel and product logic. Their set of truth values is the real unit interval and the semantics is fully determined by the corresponding continuous t-norm as the truth function of conjunction. (See [1] for metamathematics of t-norm based logics.) Each of them has two monadic counterparts: mon-Ł∀ is Ł∀ with only unary predicates and mon_1-Ł∀ is Ł∀ with only unary predicates and only one object variable. Similarly for G∀ and Π∀. For each logic, TAUT is the set of all tautologies (formulas true, i.e., having truth value 1 in each interpretation), f-TAUT is the set of all finite tautologies (true in each finite interpretation) and similarly for SAT, f-SAT (true in at least one [finite] interpretation). Finite model properties: FMP1 is TAUT = f-TAUT, FMP2 is SAT = f-SAT. What is the arithmetical complexity of these sets? What finite model properties hold? We summarize the results (some of them are found in [1], [2], [3]).

THEOREM.

(1) *For* Ł∀, TAUT *is* Π_2*-complete*, f-TAUT *is* Π_1*-complete*, *both* FMP1 *and* FMP2*are false*. *For* mon-Ł∀, FMP1 *is true but* FMP2 *not*; TAUT = f-TAUT *is in* Π_1. *For* mon_1Ł∀, *both* FMP1 *and* FMP2 *are true*, TAUT = f-TAUT *is* Δ_1.
(2) *Both* FMP1 *and* FMP2 *are false for* mon_1-G∀ (*hence for* mon-G∀ *and* G∀). *For* G∀, TAUT *is* Σ_1*-complete and* f-TAUT *is* Π_1*-complete*.
(3) *For* Π∀, TAUT *is* Π_2*-hard and* f-TAUT *is* Π_1*-complete*. *Both* FMP1 *and* FMP2 *fails for* mon_1-Π∀.

[1] P. HÁJEK, ***Metamathematics of fuzzy logic***, Kluwer, 1998.

[2] ———, *Trakhtenbrot theorem and fuzzy logic*, in ***Computer science logic*** (Gottlob et al., editors), Springer-Verlag, 1999, pp. 1–8.

[3] M. RAGAZ, *Die Unentscheidbarkeit der einstelligen unendlichwertigen Prädikatenlogik*, ***Archiv für mathematische Logik***, vol. 23 (1983), pp. 129–139.

► JOEL DAVID HAMKINS, *A simple maximality principle*.
The City University of New York, CUNY Graduate Center, Mathematics Program, 365 Fifth Avenue, New York, New York 10016, USA.
E-mail: `jhamkins@gc.cuny.edu`.

The Maximality Principle I propose, following an idea of Christophe Chalons, asserts that any sentence that holds in some forcing extension V^P and all subsequent extensions V^{P*Q}, holds already in V and all forcing extensions of V. The principle is naturally expressed by the modal logic axiom '$\Diamond\Box\phi$ implies $\Box\phi$'. In this talk I will prove the relative consistency of the Maximality Principle and analyze its relative consistency strength, showing it to be closely related to the scheme asserting that V_δ is an elementary substructure of V.

► KAI HAUSER, *Objectivity over objects*.
Technische Universtät, Fachbreich 3, Sekr. MA 8-1, 10623 Berlin, Germany.
E-mail: `hauser@math.tu-berlin.de`.

Kurt Gödel's viewpoint in the philosophy of mathematics is usually characterized as a radical platonism whose underlying epistemology rests on mathematical intuition as a form of perception of abstract objects. However, in some places Gödel suggests a modified realism that can be described in terms of a primacy of objectivity over objects. In my talk I will sketch an account of the methodology of contemporary set theory (mainly in the context of the ongoing search for new axioms) based on an extrapolation of this idea.

The essence of objectivity is to view, from the outset, mathematical propositions as determinately true or false and to gather *evidence* for their truth or falsehood by rational and reliable methods. Causal interaction with abstract objects is not a part of this analysis, and this avoids the notorious difficulties plaguing naive realism. I will give a few examples to illustrate how one arrives at an adequate description of the process of theory formation in contemporary set theory, and (time permitting) discuss the reliability of the truth criteria used therein.

► MICHAEL HEATHER, *Formal representation of legal cognition*.
Law School, University of Northumbria at Newcastle, NE1 8ST, United Kingdom.
E-mail: `m.heather@unn.ac.uk`.

Applied symbolic logic attempts to develop rationally what works in practice and can provide the necessary universality on the one hand for the theory of legal reasoning and on the other for understanding the structure of distributed legal information systems now to be found in hypermedia and cyberspace. This turns out to be a very interesting example of the application of the Curry-Howard isomorphism between intuitionistic logic and computational type theory [2]. Geometric logic in category theory is essentially concerned with links between objects and is therefore particularly appropriate for modelling relationships both in legal reasoning and for legal information systems[1].

[1] M. A. HEATHER and B. N. ROSSITER, *Content self-awareness in distributed multimedia publishing: the need for a unifying theory*, in ***Third International Workshop on principles of document processing (PODP '96)*** (C. Nicholas and D. Wood, editors), Lecture Notes in Computer Science, vol. 1293, Springer-Verlag, 1997.

[2] MARK RYAN and MARTIN SADLER, *Valuation systems and consequence relations*, in ***Handbook of logic and computer science, I; Background: Mathematical structures*** (S. Abramsky, Dov. M. Gabbay, and T. S. E. Maibaum, editors), Clarendon Press, Oxford, 1992, pp. 2–78.

► VINCENT F. HENDRICKS AND STIG ANDUR PEDERSEN, *KK'ing diachronically*.

Department of Philosophy, University of Copenhagen, Njalsgade 80, DK-2300 Copenhagen S, Denmark.
E-mail: vincent@hum.ku.dk.
Departments of Philosophy and Mathematics, University of Roskilde, P. O. Box 260, 4000 Roskilde, Denmark.
E-mail: sap@ruc.dk.

In both epistemology and epistemic logic, a substantial amount of attention has been devoted to the famous *KK-thesis*. The characteristic axiom of *S4* interpreted epistemically amounts to the claim that a method δ knows that it knows p when it does know p: $K_\delta p \Longrightarrow K_\delta K_\delta p$. There are, and have been, various arguments for an against this thesis both philosophically and logically. From the *modal learning theoretical perspective* [1], [2], [3] (which is the perspective obtained by mixing modal, tense and epistemic logic with concepts drawn from formal learning theory) the *KK*-problem is somewhat more interesting to study because δ may be subject to various program commands like data consistency, data-retentiveness and data-entailment. It turns out that the *KK*-thesis is valid *diachronically* in the sense that δ has to converge to the antecedent claim first, then wait, and then converge to the consequent claim observing a special consistency command. For instance if δ obeys data-retentiveness and thus tries to converge to the consequent claim as soon as it converges to the antecedent claim, hence attempts a *synchronic* validation of *KK*, then the *KK*-thesis is impossible to learn. Thus, the *KK*-thesis requires the agent to behave quite *strategically* in order to succeed on *KK*.

[1] VINCENT F. HENDRICKS, *The convergence of scientific knowledge: a view from the limit*, 2000, in press: Studia Logica's Trends in Logic Series, Kluwer Academic Publishers.

[2] VINCENT F. HENDRICKS and STIG ANDUR PEDERSEN, *Operators in philosophy of science*, 2001, in preparation.

[3] ———, *Assessment and discovery in the limit of scientific inquiry*, to appear in ***Proceedings of the XI International Congress on logic, methodology and philosophy of science*** (LMPS '99), Kluwer Academic Publishers, 2000.

► ROBIN HIRSCH, IAN HODKINSON, AND AGNES KURUCZ, *Modal logics between* $\boldsymbol{K}^3$ *and* $\boldsymbol{S5}^3$.

University College London, Department of Computing, Imperial College, 180 Queen's Gate, London SW7 2BZ, United Kingdom.
E-mail: R.Hirsch@cs.ucl.ac.uk.

Imperial College London.
E-mail: imh@doc.ic.ac.uk.
Imperial College London.
E-mail: kuag@doc.ic.ac.uk.

Given Kripke frames (W_0, R_0), (W_1, R_1) and (W_2, R_2), their *product* is defined to be the relational structure $(W_0 \times W_1 \times W_2, \bar{R}_0, \bar{R}_1, \bar{R}_2)$, where

$$(x_0, x_1, x_2)\bar{R}_\ell(y_0, y_1, y_2) \quad \text{if and only if} \quad x_\ell R_\ell y_\ell \text{ and } x_j = y_j \text{ for } j \neq \ell.$$

$\mathbf{K}^3$ and $\mathbf{S5}^3$ are the 3-modal logics (i.e., sets of formulas of the language having the Booleans and three diamonds) determined by all (ternary) product frames and all products of **S5**-frames, respectively.

THEOREM. *Every* 3*-modal logic between* $\mathbf{K}^3$ *and* $\mathbf{S5}^3$ *is undecidable, non-finitely axiomatizable, and lacks the product finite model property. There is no algorithm deciding whether a finite frame validates the logic.*

This answers questions of Gabbay-Shehtman [1]. Thus, $\mathbf{K}^3$ is a natural example of an undecidable, recursively enumerable, Kripke complete logic which has the finite model property (for the latter, see [1]).

In the proofs we use the following result of Hirsch-Hodkinson [2]: It is undecidable whether a finite relation algebra is representable.

[1] D. M. GABBAY and V. B. SHEHTMAN, *Products of modal logics, part I*, ***Logic Journal of the IGPL***, vol. 6 (1998), pp. 73–146.

[2] R. HIRSCH and I. HODKINSON, *Representability is not decidable for finite relation algebras*, ***Transactions of the American Mathematical Society***, to appear.

► LEON HORSTEN AND HANNES LEITGEB, *Paradoxes for tense logic*.

Institute of Philosophy, University of Leuven, Kardinaal Mercierplein 2, B-3000 Leuven, Belgium.
E-mail: Leon.Horsten@hiw.kuleuven.ac.be.
Department of Philosophy, University of Salzburg, Franziskanergasse 1, A-5020 Salzburg, Austria.
E-mail: hannes.leitgeb@sbg.ac.at.

The difficulties with formalizing the intensional notions *necessity*, *knowability* and *omniscience*, and *rational belief* are well-known. If these notions are applied to (codes of) sentences, then from apparently weak and uncontroversial logical principles governing these notions outright contradictions can be derived.

Tense logic is one of the best understood and most extensively developed branches of intensional logic. In tense logic, the temporal notions past and future are formalized as sentential operators rather than as predicates. The question therefore arises whether the notions that are investigated in tense logic can be consistently formalized as predicates.

In this paper it is shown that the answer to this question is negative. Consider the language of arithmetic plus two additional primitive unary predicates P and F (past and future). Let K_t^* be the theory that is obtained by adding to Robinson Arithmetic the axioms and rules of the minimal tense logic K_t (except that past and future are now treated as predicates). Then an application of Gödel's diagonal lemma yield the following paradoxical result:

THEOREM 1. $K_t^* \vdash \neg P(\overline{\top}) \wedge P\neg(\overline{\top})$.

If, in addition, the principle $K_t^* + \neg P(\overline{\neg\varphi}) \rightarrow P(\overline{\varphi})$, which says that there exists a moment in the past, is assumed, an outright contradiction follows:

THEOREM 2. $K_t^* + \neg P(\overline{\neg\varphi}) \rightarrow P(\overline{\varphi}) \vdash \bot$.

These two paradoxes result from the *logical interplay* between the notions of past and future. In this sense, they are different from the paradoxes of Kaplan, Montague and

Thomason.

[1] D. Kaplan and R. Montague, *A paradox regained*, ***Notre Dame Journal of Formal Logic***, vol. 1 (1960), pp. 79–90.

[2] R. Montague, *Treatments of modality with corollaries on reflexion principles and finite axiomatizability*, ***Acta Philosophica Fennica***, vol. 16 (1963), pp. 153–167.

[3] R. Thomason, *Note on syntactical treatments of modality*, ***Synthese***, vol. 44 (1980), pp. 391–395.

► JOSÉ IOVINO, *A dichotomy for the asymptotic geometry of sequences*.
Division of Mathematics and Statistics, The University of Texas at San Antonio, San Antonio, Texas 78249-0664, USA.
E-mail: `iovino@math.utsa.edu`.

A central concept in Banach space geometry is that of spreading model. (See [2], [3] and [1].) Every basic sequence in a Banach space has a subsequence that generates a Banach space called a spreading model of the sequence, and the spreading models generated by a sequence are the standard to analyze in asymptotic geometry.

A central question is to identify under what conditions a spreading model contains either of the classical sequence spaces ℓ_p and c_0. To date, most of the results in this direction have been negative. (See, for example, [4] and [5]).

We will state the following dichotomy.

Theorem 1. *Every basic sequence in a Banach space X has a normalized block basis (x_n) which satisfies one and only one of the following conditions.*

(x_n) is cofinally isometric over X to either c_0 or ℓ_p, for some p with $1 \leq p < \infty$.
(x_n) has the largest possible number of spreading models.

The proof is based on constructions from model theory. Specifically, we use the fact that there is an intimate connection between whether a spreading model contains ℓ_p or c_0 and whether the spreading model has a type that is stable in the sense of model theory. The preceding theorem is a counterpart of Shelah's landmark result that an unstable theory has the maximum number of models.

[1] B. Beauzamy and J.-T. Lapresté, *Modèles étalés des espaces de Banach*, Travaux en Cours [Works in Progress], Hermann, Paris, 1984.

[2] A. Brunel, *Espaces associés à une suite bornée dans un espace de Banach*, 1974, p. 23.

[3] A. Brunel and L. Sucheston, *On B-convex Banach spaces*, ***Mathematical Systems Theory***, vol. 7 (1974), no. 4, pp. 294–299.

[4] E. Odell and Th. Schlumprecht, *On the richness of the set of p's in Krivine's theorem*, in ***Geometric aspects of functional analysis*** (Israel, 1992–1994), Operator Theory: Advances and Applications, vol. 77, Birkhäuser, Basel, 1995, pp. 177–198.

[5] ———, *A problem on spreading models*, ***Journal of Functional Analysis***, vol. 153 (1998), no. 2, pp. 249–261.

► ERIC JALIGOT, *"Small" groups of finite Morley rank*.
Institut Girard Desargues, UPRES-A 5028 Mathématiques, Université Claude Bernard Lyon-1, 43 blvd du 11 novembre 1918, 69622 Villeurbanne CEDEX, France.
E-mail: `jaligot@desargues.univ-lyon1.fr`.

It has been hoped for the last few years that infinite simple groups of finite Morley rank whose Sylow 2-subgroups are infinite of bounded exponent can be classified using ideas from the classification of finite simple groups. This is a large project mainly carried out by T. Altinel, A. Borovik, and G. Cherlin, with regards to an "ideal" conjecture according to which all such groups should be algebraic groups over algebraically closed fields.

The project has its origin in the *tame* context of groups of finite Morley rank which involve no *bad* groups and no *bad* fields, two pathologies for the ideal conjecture whose existences are not yet known by model-theorists. Nevertheless, it has been shown that this "tameness" assumption can be avoided in a fundamental recognition theorem of $\mathrm{PSL}_2(K)$, where K is of characteristic 2, in the context of groups of finite Morley rank and in an inductive setting for the ideal conjecture.

The aim of this talk is to give an overview of these crucial configurations of small simple groups of finite Morley rank which have to be studied, between bad groups and $\mathrm{PSL}_2(K)$: the *full Frobenius* groups (a notion which generalizes the notion of bad group), the lack of involutions and the need of an extension of the Feit-Thompson's theorem, and finally a recognition theorem of $\mathrm{PSL}_2(K)$ (where $\mathrm{char}(K) \neq 2$) for FT-groups of odd type.

► YING JIANG, *Positive types in system $\mathcal{F}$*.
Laboratory of Computer Science, Institute of Software, Chinese Academy of Sciences, P. O. Box 8718, Beijing 100080, China.
E-mail: `jy@ox.ios.ac.cn`.

We work on the class of the types with positive quantifiers of System $\mathcal{F}$.

We develop a nature method to decide whether or not a given type of this class is inhabited; and in the positive case, to construct effectively a base for the set of inhabitants of the type such that every inhabitant of the type is $\beta\eta$-equivalent to an element of the base. This method combines and extends methods in [1] and [2]

We present semantically a completeness result for the types of this class.

We prove that every closed inhabited type of the class is data type. Also the set of extended data types is closed under several type constructors.

[1] J.-R. Hindley, ***Basic simple type theory***, Cambridge Tracts in Theoretical Computer Science, vol. 42, Cambridge University Press.

[2] Y. Jiang, *Universal types in system $\mathcal{F}$*, preprint, 1998.

► REINHARD KAHLE, *Name induction in explicit mathematics*.
WSI, Universität Tübingen, Sand 13, D-72076 Tübingen, Germany.
E-mail: `kahle@informatik.uni-tuebingen.de`.

Systems of explicit mathematics were introduced by Feferman in the seventies [1, 2]. Using Jäger's naming relation [3], the theories can be presented as theories of types and names. Adding a natural induction principle for names to (meta-)predicative theories of types and names, we get, in a very uniform way, impredicate theories. From this point of view, name induction serves as an alternative for the principle of inductive generation, [4]. Moreover, it allows us to define a natural theory of explicit mathematics with the strength of the theory of inductive definitions ID_1, [5]. The presented work grows out of the close collaboration with Gerhard Jäger and Thomas Studer (IAM, Universität Bern).

[1] Solomon Feferman, *A language and axioms for explicit mathematics*, ***Algebra and logic*** (J. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer-Verlag, Berlin, 1975, pp. 87–139.

[2] ———, *Constructive theories of functions and classes*, ***Logic Colloquium '78*** (M. Boffa, D. van Dalen, and K. McAloon, editors), North-Holland, Amsterdam, 1979, pp. 159–224.

[3] Gerhard Jäger, *Induction in the elementary theory of types and names*, ***CSL '87*** (E. Börger et. al., editors), Lecture Notes in Computer Sciences, vol. 329, Springer-Verlag, Berlin, 1988, pp. 118–128.

[4] Gerhard Jäger, Reinhard Kahle, and Thomas Studer, *Universes in explicit mathematics*, submitted.

[5] Reinhard Kahle and Thomas Studer, *A theory of explicit mathematics equivalent to ID_1*, in ***CSL 2000*** (P. Clote and H. Schwichtenberg, editors), Lecture Notes in Computer Science, vol. 1862, Springer-Verlag, Berlin, 2000.

► REINHARD KAHLE AND VOLKER PECKHAUS, *Hilbert's paradox*.
WSI, Universität Tübingen, Sand 13, D-72076 Tübingen, Germany.
E-mail: kahle@informatik.uni-tuebingen.de.
Institut für Philosophie der Universität Erlangen-Nürnberg, Bismarckstr. 1, D-91054 Erlangen, Germany.
E-mail: vrpeckha@phil.uni-erlangen.de.

In 1903 Gottlob Frege sent a complimentary copy of the second volume of the *Grundgesetze der Arithmetik* to the Göttingen mathematician David Hilbert, containing in the postscript the description of Russell's Paradox and Frege's admission that this paradox can be formulated in the system of the Grundgesetze. In his response Hilbert declared that the paradox described had been known in Göttingen for a long time. He himself had found other, even more convincing, examples four to five years previously and Zermelo found the one, mentioned by Frege, three to four years ago.

It is well known that Zermelo indeed discovered a set-theoretical paradox in Cantor's theory, independently of Russell. But what were these contradictions Hilbert claimed to have found around 1898/1899? Hilbert never published a contradiction.

One of the authors was able to discover a paradox which is most likely the one Hilbert referred to in his letter to Frege. It is presented in an unpublished lecture course of Hilbert delivered in the summer term of 1905 on *Logische Principien des mathematischen Denkens*. There Hilbert discusses the paradoxes of set theory mentioning Zermelo's paradox and a contradiction of "purely mathematical nature" which was never published, as Hilbert stressed, but known to set-theorists, especially to Georg Cantor.

In the historical part of the lecture the story of Hilbert's Paradox will be told and it will be shown that Hilbert's axiomatic program was indeed affected by the paradoxes, contrary to the standard view. In the systematical part of the lecture Hilbert's Paradox will be described and reconstructed using modern tools. It will be shown that it is closer to Cantor's Paradox than to Russell's.

► YUZURU KAKUDA AND MAKOTO KIKUCHI, *An abstract scheme of design*.
Department of Computer and Systems Engineering, Kobe University, Kobe 657-8501, Japan.
E-mail: kakuda@kobe-u.ac.jp.
Division of Information and Media Science, Kobe University, Kobe 657-8501, Japan.
E-mail: kikuchi@pascal.seg.kobe-u.ac.jp.

Design can be taken as human-being's activity for realizing desire in the objective world. In engineering design, desire is mostly expressed as functions of a target entity, and a function of an entity is specified by behavior of the entity. Since physical attributes of an entity determine its behavior, design is possible if we know the relation between functions and attributes of entities. The concept of functions belongs to our knowledge world, and attributes are inhabitants of the real physical world. Thus, in design, a flow of information takes its rise from our knowledge world to the real world. This is a general scheme of design, and it is also applicable to many sort of human-being's activities of problem-solving. Based on the theory of Information Flow by Barwise and Seligman [1], we give a mathematical model of the design scheme in order to abstract common features of problem-solving phenomena and formulate them into a mathematical theory [2].

This research is partially supported by the project "Methodology of Emergent Synthesis" in the Research for the Future Program of the Japan Society for the Promotion of Science.

[1] J. BARWISE and J. SELIGMAN, ***Information flow—The logic of distributed systems***, Cambridge Tracts in Theoretical Computer Science, vol. 44, Cambridge University Press, 1997.

[2] Y. KAKUDA and M. KIKUCHI, *Abstract design theory*, 2000, in preparation.

► V. KANOVEI, *On Borel non-reducibility of P-ideals*.
MCCME, Bol. Vlasevski Per. 11, Moscow 121002, Russia.
E-mail: kanovei@math.uni-wuppertal.de.

Let $\mathcal{E}$ be the least class of equivalence relations on Polish spaces which contains all equalities and is closed under the operations of *countable power* E^∞ and *Fubini product* $\mathrm{Fin} \bigotimes_n \mathsf{E}_n$ over the ideal Fin of all finite sets.

THEOREM 1. *Any equivalence relation generated by a Borel P-ideal $\mathcal{I}$ other than* Fin *or* $0 \times \mathrm{Fin}$ *is not Borel reducible to an equivalence relation in $\mathcal{E}$.*

(Equivalence relations induced by Fin and $0 \times \mathrm{Fin}$ clearly belong to $\mathcal{E}$.)

The subclass $\mathcal{T} \subseteq \mathcal{E}$ obtained by the closure under countable powers only, includes only equivalence relations induced by continuous actions of S_∞, the symmetric group, so that for $\mathcal{T}$ instead of $\mathcal{E}$ Theorem 1 follows from Hjorth's theory of turbulence. (A particular case of $\mathcal{I}$ = the density-0 ideal here, was considered by H. Friedman.) Note that $\mathcal{E}$ contains many remarkable relations which do not belong to $\mathcal{T}$, for instance, *Frechet ideals* Fin_α, obtained from Fin by iteration of Fubini products.

Our technical approach is based on the notion of generic ergodicity, due to Hjorth. An equivalence relation E is *generically* F-*ergodic*, if any Borel map ϑ with $x \mathsf{E} y \Longrightarrow \vartheta(x) \mathsf{F} \vartheta(y)$ is F-constant on a comeager set. We prove that a certain stronger version of the ergodicity is preserved under the same transformations of F as above. Let a Polish group G act continuously on a Polish space X. Say that the induced equivalence E_X^G is *hereditarily generically* (hg, for brevity) F-ergodic if the local relation $\sim_Y^U$ is F-ergodic whenever $U \subseteq G$ is open, symmetric, and contains 1_G, and $\emptyset \neq Y \subseteq X$ is open, and the local orbit $\mathcal{O}(Y, U, x)$ is dense in Y for comeager-many $x \in Y$.

THEOREM 2. *Suppose that the action is generically turbulent. Let* F *be a family of analytic equivalence relations such that* E_X^G *is hg* F*-ergodic for each* F *in* $\mathcal{F}$. *Let* $\mathcal{F}'$ *be the closure of* $\mathcal{F}$ *under the operations of countable power and Fubini product over* Fin. *Then* E_X^G *is hg* F'*-ergodic for each* F' *in* $\mathcal{F}'$.

► JULIETTE KENNEDY AND SAHARON SHELAH, *On universality properties of reduced products*.
Department of Mathematics, University of Helsinki, Helsinki, Finland.
Institute of Mathematics, Hebrew University, Jerusalem, Israel.

In the early 1970's S. Tennenbaum proved that all countable models of $PA^- + \forall_1 - \mathrm{Th}(\mathbb{N})$ are embeddable into the reduced product $\mathbb{N}^\omega/\mathcal{F}$, where $\mathcal{F}$ is the cofinite filter.

THEOREM 1. *If M is a model of $PA^- + \forall_1 - \mathrm{Th}(\mathbb{N})$, and $|M| = \aleph_1$, then M is embeddable into $\mathbb{N}^\omega/\mathcal{F}$.*

THEOREM 2. *If M is a model of $PA^- + \forall_1 - \mathrm{Th}(\mathbb{N})$, $|M| = \lambda^+$, $(\aleph_1, \aleph_0) \to (\lambda^+, \lambda)$ and $\mathcal{F}$ is a regular filter on λ, then M is embeddable into $\mathbb{N}^\lambda/\mathcal{F}$.*

THEOREM 3. *If $\mathcal{F}$ is a regular ultrafilter on λ, $(\aleph_1, \aleph_0) \to (\lambda^+, \lambda)$ and M is infinite, then $M^\lambda/\mathcal{F}$ is λ^{++}-universal.*

(This answers Open Question 19 of Chang and Keisler, ***Model Theory***, North-Holland.)

► ASYLKHAN KHISAMIEV, *Relation between T-degrees and Σ-degrees of a model*.
Sobolev Institute of Mathematics of Siberian Branch of Russian Academy of Sciences, Apt. 154, Tereshkova street, 12, Novosibirsk, 630090, Russia.
E-mail: hisamiev@math.nsc.ru.

Consider those models that consist of a countable universe and a finite number of predicates and functions. Yu. L. Ershov has introduced in [1] the notion of Σ-definability of a model $\mathfrak{A}$ in the admissible set $\mathrm{HF}(\mathfrak{B})$ over a model $\mathfrak{B}$. We say that a model $\mathfrak{A}$ is Σ-reducible to a model $\mathfrak{B}$ and write $\mathfrak{A} \leq_\Sigma \mathfrak{B}$ (or simply $\mathfrak{A} \leq \mathfrak{B}$), if a model $\mathfrak{A}$ is Σ-definable in $\mathrm{HF}(\mathfrak{B})$.

From results of Ershov [2, p. 179] we obtain that the relation $\le$ on the class K of all models is a preorder with the least elements. If we use $\le_\Sigma$ to generate a equivalence $\equiv_\Sigma$ on class K, where $\mathfrak{A} \equiv \mathfrak{B}$ if and only if $\mathfrak{A} \le \mathfrak{B}$ and $\mathfrak{B} \le \mathfrak{A}$, then the model $L_\Sigma = \langle K/_\equiv, \le\rangle$ is a partially ordered set. The class $\overline{\mathfrak{A}} \in L_\Sigma$ containing a model $\mathfrak{A}$ is called the Σ-degree of the model $\mathfrak{A}$. Let $\mathfrak{A} \oplus \mathfrak{B}$ be the disjoint union of models $\mathfrak{A}$ and $\mathfrak{B}$. Then the class $\overline{\mathfrak{A} \oplus \mathfrak{B}}$ is the least upper bound of classes $\overline{\mathfrak{A}}$ and $\overline{\mathfrak{B}}$, i.e., we have the following

PROPOSITION 1. *The model L_Σ is an upper semilattice with the least element.*

REMARK. From results of S. S. Goncharov [3] we obtain that a Σ-degree of $\overline{\mathfrak{A} \times \mathfrak{B}}$ is not always equal to $\overline{\mathfrak{A} \oplus \mathfrak{B}}$.

From results of [4, 6] we obtain

PROPOSITION 2. *If models $\mathfrak{A}$ and $\mathfrak{B}$ have degrees a and b respectively and $\mathfrak{A} \le_\Sigma \mathfrak{B}$ then $a \le_T b$.*

PROPOSITION 3. *There are models $\mathfrak{A}$ and $\mathfrak{B}$ whose universe is the set of natural numbers and degrees of their open diagrams are equal, but $\mathfrak{B}$ is not Σ-definable in* $\mathrm{HF}(\mathfrak{A})$.

Assume that a map φ from a set of T-degrees to L_Σ is defined by $\varphi(a) = \overline{\mathfrak{A}}$, where $\mathfrak{A}$ is a finitely generated model of degree a. Then we obtain

THEOREM. *The map φ is an embedding of the upper semilattice of T-degrees into the upper semilattice of Σ-degrees.*

[1] YU. L. ERSHOV, *Σ-definability in admissible sets*, ***Dokl. Akad. Nayk SSSR***, vol. 285 (1985), no. 4, pp. 792–795.

[2] ———, ***Definability and computability***, Plenum, New York, 1996.

[3] S. S. GONCHAROV, *Constructivization of Cartesian powers of models*, 15th all-union algebraic conference, Krasnoyarsk, 1979.

[4] A. N. KHISAMIEV, *On definability of a model in a hereditary finite admissible set*, in ***Generalized definability and computability***, Vychislitel'nye sistemy, no. 161, 1998, pp. 15–20.

[5] ———, *Numbering and definability in the hereditary finite superstructure of a model*, ***Siberian Advances in Mathematics***, vol. 7, 1997, no. 3, pp. 63–74.

[6] ———, *Strong Δ-definability of a model in an admissible set*, ***Siberian Mathematics Journal***, vol. 39 (1998), no. 1, pp. 168–176.

► HIROTAKA KIKYO, *Generic automorphisms and the DMP*.
Department of Mathematical Sciences, Tokai University, 1117 Kitakaname, Hiratsuka 259-1292, Japan.
E-mail: `kikyo@ss.u-tokai.ac.jp`.

This is joint work with Anand Pillay. Given a complete theory T with quantifier elimination in a language L, we consider the (incomplete) theory $T \cup \{$"σ is an L-automorphism"$\}$ in the language $L \cup \{\sigma\}$ and denote its model companion by TA if it exists. Similarly, we denote the model companions for T with two automorphisms and for T with countably many automorphisms by TA_2 and TA_ω respectively.

When T is the theory ACF of algebraically closed fields, TA exists and called $ACFA$. One axiomatization of $ACFA$ is based on the fact that ACF has the DMP (definable multiplicity property).

THEOREM 1. *Let T be a strongly minimal theory with quantifier elimination. The following are equivalent*:

(1) *TA_2 exists*;
(2) *TA_ω exists*;
(3) *T has the DMP.*

Chatzidakis pointed out to us that (1) implies (2).

THEOREM 2. *Let T be a strongly minimal theory with quantifier elimination which is a finite cover of a strongly minimal theory which does have the DMP. Then TA exists if and only if T has the DMP.*

In general, there are many unstable theories T with no TA. For example, if T is unstable without independence property then there is no TA. Also, if T has the fcp then T_σ has no model completion ([1]). As far as I know, every known theory T which has TA is stable.

[1] H. KIKYO, *Model companions of theories with an automorphism*, MSRI Preprint #1998-039, 1998 (to appear in ***The Journal of Symbolic Logic***).

[2] H. KIKYO and A. PILLAY, *The definable multiplicity property and generic automorphisms*, to appear in ***Annals of Pure and Applied Logic***.

► MOSES KLEIN, *Embedding groups into the Baer-Specker group*.
Department of Mathematics, 480 Lincoln Dr., Madison, Wisconsin 53706, USA.
E-mail: klein@math.wisc.edu.

A torsion-free abelian group is called countably separable if every countable subgroup is contained in a countable free direct summand. In 1989 Dugas and Irwin proved the following statement independent of *ZFC*: Every countably separable group is isomorphic to a pure subgroup of the Baer-Specker group $\mathbf{Z}^\omega$. Their proof used $\diamond$ and *MA*. We extend this result by showing that the statement is also independent of any axioms about cardinal exponentiation.

► NURLAN T. KOGABAEV, *Strictly computability of class of constructive I-algebras*.
Novosibirsk State University, Pirogova Street 20/1, room 604, Novosibirsk 630090, Russia.
E-mail: kogabaev@mail.ru.

In Construcive model theory the question of existence of computable numeration for the class of constructive models is closely connected with the problem of choice of most "natural" computable numeration. To solve the above mentioned problem the family of all computable numerations of this class is provided with the additional structure consisting of preorder of reducibility. Using given approach it is reasonable to turn to the greatest with respect to this preorder computable numeration. The existence of such a greatest numeration leads to the notions of principal computable numeration and strictly computable class of models [1].

There are a number of results concerning the above area of problems in the case of various particular classes of constructive models. The examples of strictly computable, computable but not strictly computable, and non-computable classes of models can be found in [1, 2].

In the present paper we study Boolean algebras with distinguished ideals. Throughout, these algebras are called I-algebras and are considered in the signature $\sigma = \langle \vee, \wedge, C, 0, 1, I_1, \dots, I_\lambda \rangle$, where $I_1, \dots, I_\lambda$ are unary predicate symbols. A theory T_λ of I-algebras contains axioms of Boolean algebras and sentences stating that $I_1, \dots, I_\lambda$ are ideals; λ is a fixed natural number.

In the paper we show that there is a principal computable numeration for the class of all constructive I-algebras. That is, there exists a computable sequence α_0 of constructive I-algebras such that any other computable sequence α of constructive I-algebras is reducible to α_0. Thus, the class of all constructive I-algebras is strictly computable.

[1] V. P. DOBRITSA, *Computable classes of constructive models*, in ***Handbook of recursive mathematics***, vol. 1, Elsevier, 1998, pp. 183–233.

[2] S. S. GONCHAROV, ***Countable Boolean algebras and decidability***, Plenum, New York, 1997.

► OREN KOLMAN AND SAHARON SHELAH, *Infinitary axiomatizabiliy of slender and cotorsion-free groups*.
King's College London, Strand, London WC2R 2LS, United Kingdom.

E-mail: okdman@e-math.ams.org.
Institute of Mathematics, Hebrew University of Jerusalem, Jerusalem, Israel.

This talk contains some results on the axiomatizability in higher-order logic of the classes of slender and cotorsion-free infinite abelian groups.

Recall that the *Baer-Specker group* $\mathbb{Z}^\omega$ is the product of countably many copies of $\mathbb{Z}$ (the integers). A torsion-free group G is *slender* if whenever $h\colon \mathbb{Z}^\omega \to G$ is a homomorphism, then $h(e_n) = 0$ for all but finitely many $n \in \mathbb{N}$, where e_n is the standard nth unit vector $(0, \dots, 0, 1, 0, \dots) \in \mathbb{Z}^\omega$. A group A is *cotorsion* if $\operatorname{Ext}(J, A) = 0$ for all torsion-free groups J. A group G is *cotorsion-free* if G does not contain any non-zero subgroup which is cotorsion.

In 1974, Paul Eklof used infinitary logic to generalize some classical theorems of infinite abelian group theory. He characterized the strongly $\aleph_1$-free groups as exactly those abelian groups which are $L_{\infty\omega_1}$-equivalent to free abelian groups, used his criterion to deduce that the class of free abelian groups is not $L_{\infty\omega_1}$-definable, and showed that the Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a free abelian group, strengthening a theorem of Baer that $\mathbb{Z}^\omega$ is not free. The present talk continues in the tradition allying infinite abelian group theory with infinitary logic. Its main results are the following:

THEOREM 1. *The class of slender abelian groups is axiomatizable in the infinitary logic $L_{\infty\omega_1}$.*

COROLLARY 2. *The Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a slender group.*

COROLLARY 3. *The class of cotorsion-free groups is $L_{\infty\omega}$-definable.*

► KEPA KORTA AND JESUS M. LARRAZABAL, *On the logic of collective intention*.
I. L. C. L. I., The University of the Basque Country, Av. J. Elosegi 275, E-20015 Donostia, Spain.
E-mail: ylpkocak@sc.ehu.es.
E-mail: y1plaanj@sc.ehu.es.

In recent years collective action has become one of the main topics of interest in very different fields, with many applications to the design of multiagent systems, group interactions, models of conversation, etc. At the basis of that notion there is a clear idea of intentional action, taking intention as a mental cause of action. The traditional BDI framework for a singular agent has to be enriched with at least the concepts of mutual belief, collective intention ([4], [3]) and the determination of the relationships among them.

There is a strong logical tradition of using modalities for representing knowledge, belief and other mental states with a clear "possible worlds" semantics and its corresponding formal systems. For collective action we have to adopt a multimodal semantics fixing the properties of mutual belief [1] and collective intention [2], considering also the "rationality" conditions giving coherence to the system.

Concerning mutual belief, we adopt Colombetti's modal operator, that has a fixpoint semantics, which places it between KD4 and KD45. With respect to collective intention, we introduce it by a KD modal operator with axioms 4 and 5 in relation with belief introspection and axiom 4 in relation with mutual belief. Further properties will be analyzed in the present work.

[1] M. COLOMBETTI, *Formal semantics for mutual belief*, ***Artificial Intelligence***, vol. 62 (1993), pp. 341–353.

[2] K. KORTA, *Hacia un modelo del diálogo*: *intención, acción, comunicación*, ***Memoria de tesis doctoral***, Donostia-San Sebastián, UPV-EHU, 1996.

[3] K. KORTA and J. M. LARRAZABAL, *Diálogo y acción colectiva*, ***Manuscrito***, vol. 21 (1998), no. l, pp. 107–148.

[4] J. SEARLE, *Collective intentions and actions*, in ***Intentions in communication*** (P. R.

Cohen, J. Morgan, and M. E. Pollack, editors), MIT Press, Cambridge, Massachusetts, 1990, pp. 401–415.

► ANDREI KOUZNETSOV, *Multidimensional logic on the generalized consequences*.
Novokuznetsk Branch of Kemerovo State University, Department of Philosophy, 654080 Zaporojskaya St., 1-50 Novokuznetsk, Russia.
E-mail: kuzfam@nvkz.kuzbass.net.

The idea of the multidimensional logics has the following pre-history. Parallel with the papers of L. Brouwer and of J. Łukasiewicz in Russia in 1910–1913 there appear three papers of N. Vasiliev ([1], [2]) with the program of construction of the non-aristotelian logics. Vasiliev has suggested to subdivide the logical laws into two levels: external and internal; the first level is gnosiological—it submits to the laws of non-contradiction and of the excluded middle, the second one depends on the ontological assumptions about the cognizable world.

In the one-dimensional world the only positive statements are possible—the negative ones are not atomic, but presents the result of the inference: one can't ingenuously see that the subject S does not possess the predicate P.

But we can imagine the world, where one can fix not only the presence, but also the absence of the situation. In this case the positive and the negative facts will be independently from each other presented in the experiment. Vasiliev constructs the logic without the laws of non-contradiction and of the excluded middle, but with the law of the "excluded fourth". Then he generalizes this approach on the case of n dimensions—there can be constructed n-dimensional logics with n types of the atomic statements.

V. Smirnov ([2], [3]) has suggested the construction of the n-dimensional logics in the form of the algebras of classes.

The finite consequence of classes $\langle S1, \dots, Sn \rangle$ in which the intersection of any pair of classes is empty, and the union of all classes is the universum, is put for the predicative symbol. The relation of the inclusion is put as follows:

$$\langle S1, \dots, Sn \rangle \subseteq \langle P1, \dots, Pn \rangle \rightleftharpoons Si \subseteq P1 \cup \dots \cup Pi \text{ and } Pi \subseteq Si \cup \dots \cup Sn$$

This approach can be generalized by considering the generalized consequences of classes with the partial order. In the talk we will present the multidimensional—pseudo-Boolean—algebra (which elements represent pseudo-Boolean lattices). After entering the relation of inclusion, operations of intersection, union, negation and implication we prove the

THEOREM 1. *$\langle W, \subseteq, \cap, \cup, 0, 1, \implies, \neg \rangle$ is a pseudo-Boolean algebra.*

We will also bring some informal illustrations of the multidimensional logics. Finally, we will compare n-dimensional logics in the form of algebra of classes with ones in the form of syllogistics.

[1] N. VASILIEV, ***Imaginary logic***, Selected works (Russian), Nauka, Moscow, 1989.

[2] V. SMIRNOV, ***The logical ideas of N. A. Vasiliev and modern logic***, Studies in Logic and the Foundations of Mathematics, vol. 126, Amsterdam, 1989, pp. 625–640.

[3] ———, ***Multidimensional logics***, Logical Studies (Russian), vol. 2, Moscow, 1993, pp. 259–278.

► LARS KRISTIANSEN, *Honest and semi-honest subrecursive degrees*.
Oslo University College, Faculty of Engineering, Norway.
E-mail: larskri@iu.hioslo.no.

Let $T_0 \preceq T_1$ denote that each computable function, which is provable total in the first order theory T_0, is also provable total in the first order theory T_1. The relation $\preceq$ induces a degree structure on the sound finite Π_2 extensions of EA (Elementary Arithmetic). [1] is devoted to the study of this structure. However, we do not study the structure directly. Rather we define an isomorphic subrecursive degree structure $\langle \leq, \mathcal{S} \rangle$, and then we study $\langle \leq, \mathcal{S} \rangle$ by subrecursive and computability-theoretic means. Furthermore, we introduce and investigate some operators on the degrees of $\langle \leq, \mathcal{S} \rangle$. These operators correspond to inference rules in formal arithmetic. One operator (the collection operator) corresponds to the Σ_1 collection rule. Another operator (the jump operator) corresponds to the Σ_1 induction rule.

The structure $\langle \leq, \mathcal{S} \rangle$ is called the structure of semi-honest degrees. The structure of honest degrees is a strict substructure of $\langle \leq, \mathcal{S} \rangle$. In [2] and [3] we study the jump operator on the honest degrees.

[1] L. Kristiansen, *Subrecursive degrees and fragments of Peano arithmetic*, accepted for publication in ***Archive for Mathematical Logic***.

[2] ———, *Low$_n$, high$_n$, and intermediate subrecursive degrees*, in ***Combinatorics, computation and logic*** (Proceedings) (Calude and Dinneen, editors), Springer, Singapore, 1999, pp. 286–300.

[3] ———, *A jump operator on honest subrecursive degrees*, ***Archive for Mathematical Logic***, vol. 37 (1998), pp. 105–125.

► OLIVER KULLMANN, *On minimally unsatisfiable conjunctive normal forms*.
Department of Computer Science, University of Toronto, Toronto, Ontario M5S 3G4, Canada.
E-mail: kullmann@cs.toronto.edu.

Supported by the Natural Sciences and Engineering Research Council of Canada and by the Communications and Information Technology Ontario.

We give an overview on recent progress towards an understanding of the structure of *minimally unsatisfiable clause-sets*, focussing on the notion of *deficiency* $\delta(F) = c(F) - n(F)$ ([4]), where $c(F)$ is the number of clauses and $n(F)$ is the number of variables, and considering also generalisations like *lean clause-sets* ([5]), which are clause-sets without non-trivial *autarkies*.

Starting with [1], where $\delta(F) \geq 1$ for minimally unsatisfiable F has been shown, and the class of "saturated" minimally unsatisfiable F with $\delta(F) = 1$ has been characterised, one central problem has been the *classification* of minimally unsatisfiable clause-sets with small deficiency. In [3] the class of all minimally unsatisfiable F with $\delta(F) = 1$ is characterised, and in [2] an upper bound for tree-like resolution depending on the deficiency is given. In [6] for all k the decision problem for minimally unsatisfiable F with $\delta(F) \leq k$ has been shown to be in *polynomial time* (see also [7] for further developments).

[1] Ron Aharoni and Nathan Linial, *Minimal non-two-colorable hypergraphs and minimal unsatisfiable formulas*, ***Journal of Combinatorial Theory***, Series A, vol. 43 (1986), pp. 196–204.

[2] Hans Kleine Büning, *An upper bound for minimal resolution refutations*, in ***Computer science logic 12th international workshop, CSL '98*** (Georg Gottlob, Etienne Grandjean, and Katrin Seyr, editors), Lecture Notes in Computer Science, vol. 1584, Springer, 1999, pp. 171–178.

[3] Gennady Davydov, Inna Davydova, and Hans Kleine Büning, *An efficient algorithm for the minimal unsatisfiability problem for a subclass of CNF*, ***Annals of Mathematics and Artificial Intelligence***, vol. 23 (1998), pp. 229–245.

[4] John Franco and Allen Van Gelder, *A perspective on certain polynomial time solvable classes of satisfiability*, to appear in ***Discrete Applied Mathematics***, 2000.

[5] Oliver Kullmann, *Investigations on autark assignments*, ***Discrete Applied Mathematics***, 2000, to appear.

[6] ———, *An application of matroid theory to the SAT problem*, ***Technical Report TR00-018***, Electronic Colloquium on Computational Complexity (ECCC), March 2000.

[7] ———, *On structural properties of minimally unsatisfiable clause-sets*, January 2000, in preparation.

► BEIBUT SH. KULPESHOV, *Binary $\aleph_0$-categorical weakly O-minimal theories*.
Informatics and Control Problems Institute, 480100, ul. Pushkina 125, Almaty, Kazakhstan.
E-mail: kulpesh@usa.net.

Let $M = \langle M, <, \dots \rangle$ be a linearly ordered structure. A subset B of M is *convex* if whenever $a < x < b$ and $a, b \in B$, we have $x \in B$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, <, \dots \rangle$ such that any parametrically definable subset of M is a finite union of convex sets. We say that a complete *theory* is weakly o-minimal if all its models are weakly o-minimal. Here we prove some properties of binary $\aleph_0$-categorical weakly o-minimal theories. For the obtaining of these results, we use Baizhanov's technique which he has elaborated for classification of one-types in weakly o-minimal theories.

[1] B. S. BAIZHANOV, *Classification of one-types in weakly o-minimal theories*, preprint, 1996.

[2] B. SH. KULPESHOV, *Weakly o-minimal structures and some of their properties*, ***Journal of Symbolic Logic***, vol. 63 (1998), pp. 1511–1528.

[3] A. PILLAY and C. STEINHORN, *Definable sets in ordered structures I*, ***Transactions of the American Mathematical Society***, vol. 295 (1986), pp. 565–592.

► SATORU KURODA, *Integer division and second order weak bounded arithmetic*.
Gumma Prefectural Women's University, 1395-1 Kaminote, Tamamura-machi, Sawagun, Gunma, 370-1193, Japan.
E-mail: satoru@math.human.nagoya-u.ac.jp.

The theory T^{pol} is a second order theory whose $\Delta_1^{1,w}$ definable predicates (intuitively, provably $NP \cap \text{co-}NP$ predicates) are exactly those decidable in polynomial time. This theory, however, has a very poor ability in defining functions. Namely, Takeuti [3] proved that T^{pol} cannot define the function $\lfloor a/b \rfloor$.

In the present talk, we will improve this result in the following way: let $T^{\text{pol}+}$ be the theory T^{pol} extended by the function $\text{MSP}(x, y) = \lfloor x/2^{|y|} \rfloor$. Then we have

THEOREM 1. *For $k \in \omega$, $T^{\text{pol}+} \vdash \forall x\, \exists y\, (y = \lfloor x/k \rfloor$ if and only if k is not a power of* 2.

Contrary to the theorem of Takeuti, our proof is model-theoretical and makes use of the method of length initial substructures developed by Johannsen [2].

[1] P. CLOTE and G. TAKEUTI, *Bounded arithmetic for NC, AlogTIME, L and NL*, ***Annals of Pure and Applied Logic***, vol. 56 (1992), pp. 73–117.

[2] J. JOHANNSEN, *A remark on independence results for sharply bounded arithmetic*, ***Mathematical Logic Quarterly***, vol. 44 (1998), pp. 569–571.

[3] G. TAKEUTI, *A second order version of S_2^i and U_2^1*, ***Journal of Symbolic Logic***, vol. 56 (1991), pp. 1038–1063.

► PAUL LARSON, *On Gödel's argument for $\mathfrak{c} = \omega_2$*.
Kobe University, Department of Computer and Systems Engineering, Kobe University, Kobe 657-8501, Japan.
E-mail: larson@alan.scitec.kobe-u.ac.jp.

In 1970, Gödel submitted a paper to the ***Proceedings of the National Academy of Sciences of the USA*** entitled *Some considerations leading to the probable conclusion that the true power of the continuum is $\aleph_2$*, in which he presented four axioms which he found plausible, and claimed that they settled the value of the continuum. As shown by Martin and Solovay, and admitted by Gödel, the argument was flawed, and it was not published until its recent appearance in Gödel's collected works. In joint work with Jörg Brendle and Stevo Todorčević, we start with three axioms that appear either implicitly or explicitly in Gödel's argument, and using his argument, derive from them that $\mathfrak{c} \leq \omega_2$. In doing so, we show that there is a theorem for which Gödel's argument is the best known proof. The argument, aside from its historical interest, raises several issues which are interesting in their own right.

[1] J. BRENDLE, P. LARSON, and S. TODORČEVIĆ, *Revisiting Gödel's argument for the true power of the continuum*, in preparation.

[2] S. Feferman, J. W. Dawson, Jr., W. Goldfarb, C. Parsons, R. Solovay (editors), ***Kurt Gödel, Collected works, Volume III, Unpublished essays and lectures***, Oxford University Press, 1995.

► GÉRARD LELOUP, *Valuations with cyclically ordered exponents*.
UPRESA 7056 (Equipe Logique Paris VII) et Departement de Mathematiques, U. F. R. Sciences, Universite du Maine, avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France.
E-mail: leloup@logique.jussieu.fr.

(Joint work with M. Giraudet and F.-V. Kuhlmann.)

We say that a set C is *cyclically ordered* if it is equipped with a ternary relation $(\cdot,\cdot,\cdot)$ on C that satisfies (1), (2), (3) below.

(1) $(\cdot,\cdot,\cdot)$ enjoys $\forall x, y, z,\ (x, y, z) \Longrightarrow x \neq y \neq z \neq x$
(2) $(\cdot,\cdot,\cdot)$ enjoys $\forall x, y, z,\ (x, y, z) \Longrightarrow (y, z, x)$
(3) for all $x \in C$, $(x, \cdot, \cdot)$ is antisymetric and transitive.

Let C be an abelian group with a cyclic order $(\cdot,\cdot,\cdot)$, we say that C is a *cyclically ordered group* if $(\cdot,\cdot,\cdot)$ is compatible, i.e., $\forall x, y, z, u,\ (x, y, z) \Longrightarrow (x+u, y+u, z+u)$.

Any totally ordered set is a cyclically ordered one once equipped with the ternary relation (x, y, z) if and only if $x < y < z$ or $y < z < x$ or $z < x < y$. With the same relation, any totally ordered group is a cyclically ordered one. Such a cyclically ordered group is called a *linear cyclically ordered* one.

Let k be a commutative ring (or field), and C be a cyclically ordered group.

We generalize the usual definitions of $k[\![C]\!]$, the ring of formal power series, and $k[C]$, the ring of polynomials, in such a way that they coincide with classical one when C is a linear cyclically ordered group. We can prove that $k[C]$ is integral if and only if k is integral and C is torsion free, and that $k[\![C]\!]$ is a field if and only if k is a field and C a linear cyclically ordered group. Then, we define the cyclic valuation on $k[\![C]\!]$, and point out some of its properties.

Next, we define cyclically valued groups, we give two first order properties and prove that the subgroups of Hahn products of abelian groups indexed by C are exactly the cyclically valued groups satisfying those properties, and such that the supports of all the elements are well ordered.

If we assume that C is a cyclically ordered group, and extend the definition of cyclically valued group to cyclically valued ring, there exist some properties analogous to the properties of the cyclically valued groups, with formal power series rings with cyclically ordered exponents instead of Hahn products.

► OLIVIER LESSMANN, *Simplicity inside a homogeneous model*.
Dept of Mathematics (m/c 249), University of Illinois at Chicago, Chicago, Illinois 60607-7045, USA.
E-mail: lessmann@uic.edu.

We work inside a homogeneous model M, i.e., any partial elementary embedding from M into M of size less than the size of M can be extended to an automorphism of M. The following are examples of abstract situations where each model has a homogeneous extension: first order theories, Robinson theories, e. c. models of a universal f. o. theory, theories in various logics (n-variable, infinitary logics) with amalgamation over sets. More concrete examples are Euclidean spaces. In a homogeneous model M, complete types correspond to orbits of the group of automorphisms, and Lascar strong types to orbits of strong automorphisms. We introduce a dependence relation inside M, which coincides with dividing when M is compact. We say that the homogeneous model M is *simple* if this dependence relation has local character and independent sequences exist for types with an unbounded number of realizations. When M is simple, we prove all the usual first order properties of dividing for this dependence relation—extension, symmetry, transitivity, amalgamation for Lascar strong types. The proofs are short and provide nice simplifications even in the first order case. When M is also stable—i.e., M does not have the order property witnessed by indiscernibles—then each complete type has a bounded number of independent extensions.

This work is joint with Steve Buechler.

► ØYSTEIN LINNEBO, *Field's nominalism and the concept of an object*.
Harvard University, Department of Philosophy, Emerson Hall, Cambridge, Massachusetts 02138, USA.
E-mail: linnebo@fas.harvard.edu.

In this paper I present a challenge to Hartry Field's nominalism, based on the notion of an object that is at work in his philosophy of mathematics. Although Field admits that the axioms of Peano arithmetic and of NBG set theory describe logical possibilities, he denies that there are such objects as numbers and sets. But this position is incompatible with a famous view of mathematical objects, according to which all that is required for the existence of mathematical objects is that the theory that describes these objects be a coherent logical possibility. I sketch an account of one such notion of an object. This account holds (a) that the notion of an object is given content only in connection with modern quantificational logic, (b) that what is correct use of this notion should be assessed by naturalistic or scientific criteria, and (c) that there may be different criteria of existence associated with different kinds of objects. This position allows for mathematical objects that are very thin in the sense that their existence amounts to nothing more than the coherence of the mathematical theory describing them. I then argue that Field is unable to reject this notion of an object and that he thus is forced to accept this thin form of platonism. For (c) can be rejected only by accepting a Quinean form of holism, which is unappealing to Field. And to reject (b) would be incompatible with Field's naturalism. So Field's most promising response would be to reject (a). But I argue that doing so would amount to adopting an unexplicated and inexplicable notion of an object, and that doing so would be incompatible with Field's naturalism.

► BENEDIKT LÖWE, *A measure-theoretic Wagde hierarchy*.
Mathematisches Institut, Rheinische Friedrich-Wilhelms-Universität Bonn, Beringstraße 6, D-53115 Bonn, Germany.
E-mail: loewe@math.uni-bonn.de.

We shall look at a relation $\leq_{\mathrm{BlW}}$ on sets of reals that is closely related to Wadge reducibility (cf. [4] and [1]). For two sets of reals A and B we say that $A \leq_{\mathrm{BlW}} B$ holds if and only if the Wadge game $G_{\mathrm{W}}(A, B)$ has probabilistic value 0 (cf. [2]). Of course, this is a weaker reducibility relation that Wadge reducibility in the sense that $A \leq_{\mathrm{W}} B$ implies $A \leq_{\mathrm{BlW}} B$. By a theorem of Neeman (cf. [3]), the two relations have to coincide under the assumption of Blackwell Determinacy throughout the $\mathbf{L}(\mathbb{R})$ hierarchy. It is an open question whether they are equal everywhere—this question is equivalent to "Does Blackwell Determinacy imply Wadge Determinacy?".

In this talk we shall show (assuming Blackwell Determinacy) that the relation $\leq_{\mathrm{BlW}}$ exhibits the very same structural properties as $\leq_{\mathrm{W}}$ under AD. They can only fail to be the same if $\leq_{\mathrm{W}}$ is not wellfounded, i.e., if Wadge Determinacy is false.

[1] Alessandro Andretta, *The secret lore of the cabal*, monograph in preparation.
[2] Donald A. Martin, *The determinacy of Blackwell games*, ***Journal of Symbolic Logic***,

vol. 63 (1998), pp. 1565–1581.

[3] DONALD A. MARTIN and ITAY NEEMAN, *The strength of Blackwell determinacy*, in preparation.

[4] ROBERT VAN WESEP, *Wadge degrees and descriptive set theory*, in ***Cabal Seminar 76–77, Proceedings, Caltech-UCLA logic seminar 1976–1977*** (Alexander S. Kechris and Yiannis N. Moschovakis, editors), Lecture Notes in Mathematics, vol. 689, Berlin, 1978, pp. 151–170.

► F. LUCAS, *First-order theories of r-projectable ℓ-groups and Priestley products*.
Equipe Algèbre et géométrie (UPRESA 6093), Université d'Angers, 2 bd Lavoisier 49045 Angers Cedex.
E-mail: Lucas@univ-angers.fr.

(A large part of the following is joint work with D. Gluschankof.)

Boolean products where used in the study of first order properties of projectable ℓ-groups.

We introduce the Priestley products as a generalisation of both the Boolean products, and of the Hahn products on a root system.

We also introduce the r-projectability property as a generalisation of the projectability property. It is more suitable in the case of Hahn products on a root system, when this root system is neither a chain nor an antichain.

Using these generalisations we obtain results in two different directions.

1. A first order classification of a large class of r-projectable divisible ℓ-groups, (containing special valued ones), in terms of a lattice which encodes the maximal supports of the elements of the ℓ-group when carefully embedded in a Hahn product.
2. A transfert result of the elementary equivalence for Priestley products on a given Priesley space, and for Priestley powers of totally ordered groups on different Priestley spaces, in terms of the elementary equivalence of two structures involved: on the one hand the totally ordered groups, on the other hand lattices related to the Priestley spaces.

[1] D. GLUSCHANKOF and F. LUCAS, *Hyper-regular lattice-ordered groups*, ***Journal of Symbolic Logic***, vol. 58 (1993), pp. 1342–1358.

[2] ———, *The representation of ℓ-groups over Priestley spaces*, Séminaire Structures Algébriques Ordonnées, no. 49.

[3] F. LUCAS, *First order theories of subgroups of divisible Hahn products*, Séminaire Structures Algébriques Ordonnées, no. 56.

[4] V. WEISPFENNING, *Model theory of abelian l-groups*, in ***Lattice-ordered groups*** (A. M. W. Glass and W. Charles Holland, editors), Kluwer Academic Press, pp. 41–79.

► KERKKO LUOSTO, *Four variables and finite groups*.
Department of Mathematics, P. O. Box 4 (Yliopistonkatu 5), FIN–00014 University of Helsinki, Finland.
E-mail: Kerkko.Luosto@Helsinki.Fi.

We consider the strength of the infinitary logic $\mathcal{L}^4_{\infty\omega}$ with four variables on finite groups. A simple game-theoretic argument shows that $\mathcal{L}^4_{\infty\omega}$ cannot distinguish elementary abelian groups of different dimensions. In contrast, every completely reducible centreless group can be characterized by a single sentence of $\mathcal{L}^4_{\infty\omega}$, and, consequently, by a single sentence of the first order logic $\mathcal{L}^4_{\omega\omega}$ with four variables, by the result of Dawar, Lindell and Weinstein.

► ALEXANDER V. LYALETSKI, *On Herbrand theorem*.
Cybernetics Department, Taras Shevchenko University of Kyiv, 6, Glushkov avenue, 03127 Kyiv, Ukraine.
E-mail: lav@tc.unicyb.kiev.ua.

1st-order classical logic (with functional symbols and without equality) is considered. A new formulation of Herbrand theorem is suggested in such a form that the forms A, B, and C [1] can be obtained as consequences of the one. For this, an original notion of admissible substitution [2] is generalized in case of a set of arbitrary formulas.

Let S be a finite set of formulas. Herbrand quasi-universe $\mathrm{HQ}(S)$ for S is the following minimal set of terms:

(i) for any constant c from formulas belonging to S and for any variable x of a positive quantifier from formulas belonging to S it is true that $c, x \in \mathrm{HQ}(S)$;

(ii) if $t_1, \ldots, t_m \in S$ and a m-arity functional symbol f occurs in formulas from S, then $f(t_1, \ldots, t_m) \in \mathrm{HQ}(S)$.

THEOREM. *A finite set S of formulas is unsatisfiable if and only if there are a finite set S' of variants $F_1, \ldots, F_n$ of formulas from S and a substitution σ of terms from* $\mathrm{HQ}(S')$ *for all variables bounded by negative quantifiers from formulas belonging to S' such that*:

(1) *formulas F_i and F_j have common bounded variables only if they are variants of a same formula from S and satisfy a certain condition*;

(2) *σ is an admissible substitution for S'*;

(3) *a formula $(M_1 * \sigma) \,\&\, \cdots \,\&\, (M_n * \sigma)$ considered as a formula of the proposition calculus is false, where $M_1, \ldots, M_n$ are obtained by elimination of all quantifiers in $F_1, \ldots, F_n$, respectively, and $M_i * \sigma$ denotes the result of application of σ to M_i $(i = 1, \ldots, n)$.*

[1] J. HERBRAND, *Recherches sur la theorie de la demonstration*, ***Travaux de la Societe des Sciences et de Lettres de Varsovie, Class III***, Sciences Mathematiques et Physiques, vol. 33, 1930.

[2] A. V. LYALETSKI, *Variant of Herbrand theorem for formulas in prefix form* (in Russian), ***Kibernetika***, vol. 1 (1981), pp. 112–116.

► LARISA MAKSIMOVA, *Intuitionistic logic and projective Beth property*.
Institute of Mathematics, Siberian Branch of Russian Academy of Sciences, 630090 Novosibirsk, Russia.
E-mail: `lmaksi@math.nsc.ru`.

We consider the family $E(\mathrm{Int})$ of all superintuitionistic logics. It is well known that there is a duality between $E(\mathrm{Int})$ and the family of varieties of Heyting algebras.

Let $\mathbf{x}, \mathbf{q}, \mathbf{q}'$ be disjoint lists of variables not containing y and z, $\Gamma(\mathbf{x}, \mathbf{q}, y)$ a set of formulas. We say that a logic L has the *projective Beth property PBP* if $\Gamma(\mathbf{x}, \mathbf{q}, y), \Gamma(\mathbf{x}, \mathbf{q}', z) \vdash_L y \leftrightarrow z$ implies $\Gamma(\mathbf{x}, \mathbf{q}, y) \vdash_L y \leftrightarrow C(\mathbf{x})$ for some formula $C(\mathbf{x})$; the *Beth Property BP* arises from PBP by deleting $\mathbf{q}$ and $\mathbf{q}'$.

It is known [Maksimova 77] that there exist exactly eight extensions of Int with Craig's interpolation property CIP, and $L \in E(\mathrm{Int})$ has CIP if and only if its associated variety $V(L)$ has the Amalgamation Property AP. All extensions of Int have BP [Kreisel 60], and CIP implies PBP in $E(\mathrm{Int})$.

We say that a variety V has the property *SES* if for any $\mathbf{B}$ in V and its subalgebra $\mathbf{A}$, for any $b \in \mathbf{B} - \mathbf{A}$ there exist $\mathbf{C} \in V$ and homomorphisms $\beta \colon \mathbf{B} \to \mathbf{C}$ and $\gamma \colon \mathbf{B} \to \mathbf{C}$ such that $\beta(x) = \gamma(x)$ for all $x \in \mathbf{A}$ and $\beta(b) \neq \gamma(b)$.

LEMMA 1. *For each logic L in $E(\mathrm{Int})$, L satisfies PBP if and only if $V(L)$ has SES.*

PROPOSITION 2. *Let L be any logic in $E(\mathrm{Int})$.*

(i) *If L has CIP then L and $\Delta(L)$ have PBP.*

(ii) *If L has PBP then $L + \neg p \vee \neg\neg p$ has PBP.*

Here $\Delta(L) = \mathrm{Int} + \{\, p \vee (p \to A) \mid A \in L \text{ and } p \text{ is not in } A \,\}$ [Hosoi 67]. By this proposition, at least fifteen logics in $E(\mathrm{Int})$ have PBP. These logics are finitely axiomatizable and have the finite model property.

THEOREM 3. *There exist exactly sixteen logics with PBP in $E(\mathrm{Int})$. Their list contains* *eight logics with CIP and, moreover, five logics of the form* $\Delta(L)$ *and two logics of the form* $\Delta(L) + \neg p \vee \neg\neg p$, *where L has CIP, and one more logic which is characterized by four-element linearly ordered Heyting algebra.*

It follows that there are exactly sixteen varieties of Heyting algebras with SES.

THEOREM 4.

(i) *There exist an algorithm which for any extension of* Int *by finitely many axiom schemes decides whether this extension possesses PBP or not.*
(ii) *SES is base-decidable on the family of varieties of Heyting algebras.*

► GIAN ARTURO MARCO, *Omitting types and the logic action.*
C. so S. Martino n' 8, Turin, Italy.
E-mail: giannimarco@tin.it.

This research is on the omitting types property for Borel and Analytic sets in Polish G spaces, where G is any subgroup of the symmetric group S_∞ endowed with the pointwise topology.

S_∞ and its subgroups act on the space of countable structures for a given theory in a countable language. This space is universal for the S_∞-action.

We give a topological characterization of the notion of omitting types as a special requirement of orbit-separation or of boundedness of the orbital rank.

We say that a countable structure M strongly omits a non isolated orbit p if no suborbit q of p is locally dense; we show that this is equivalent to the following: q is not separable from p by a q dominating subtype s such that q is splittable with, respect to s. In general we prove that omitting types fails for a countable structure M whenever we can produce a G embedding between orbits that is not in the automorphism group of M.

We show that the existence of such a G-embedding depends on the presence of a group-stabilizer that is not a Jordan group.

Let M be a countable structure in a countable language; the main statement of this paper is the following:

THEOREM 1. *If* $G \subset S_\infty$ *is closed in the Baire space* ω^ω, $G = \mathrm{Aut}(M)$, *and every* M*-type is a Jordan set, then the* G *omitting types holds for* M.

As a corollary we have:

COROLLARY 2. *The Scott sentence of* M *has no model of height* ω_1, $L_{\omega_1\omega}$*-elementary equivalent to* M.

► JUAN CARLOS MARTÍNEZ, *A forcing construction for thin-very tall boolean algebras.*
Facultat de Matemàtiques, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain.
E-mail: martinez@mat.ub.es.

A Boolean algebra B is *superatomic*, if every subalgebra of B is atomic. It is known that the Cantor-Bendixson process for topological spaces can be transferred to the context of Boolean algebras. If B is a Boolean algebra, we define for every ordinal α *the ideal* I_α as follows: $I_0 = \{0\}$; if $\alpha = \beta + 1$, $I_\alpha =$ the ideal generated by $I_\beta \cup \{\, b \in B : b/I_\beta$ is an atom in $B/I_\beta \,\}$; and if α is a limit, $I_\alpha = \bigcup\{\, I_\beta : \beta < \alpha \,\}$. Then, B is superatomic if and only if there is an ordinal α such that $B = I_\alpha$.

We define *the height* of a superatomic Boolean algebra B by $\mathrm{ht}(B) =$ the least ordinal α such that B/I_α is finite (which means $B = I_{\alpha+1}$). For every $\alpha < \mathrm{ht}(B)$, we denote by $\mathrm{wd}_\alpha(B)$ the cardinality of the set of atoms of B/I_α, and then we define *the width* of B by $\mathrm{wd}(B) = \sup\{\, \mathrm{wd}_\alpha(B) : \alpha < \mathrm{ht}(B) \,\}$.

It was shown by Juhász and Weiss that for every ordinal $\alpha < \omega_2$, there is a superatomic Boolean algebra of width ω and height α. Also, Baumgartner and Shelah proved that Con(ZFC) $\rightarrow$ Con(ZFC + "there is a superatomic Boolean algebra of width ω and height

ω_2"). Then, we will show that we can improve Baumgartner-Shelah's theorem, showing that Con(ZFC) $\rightarrow$ Con(ZFC+"for every $\alpha < \omega_3$ there is a superatomic Boolean algebra of width ω and height α").

► MARIA DA PAZ N. MEDEIROS AND LUIZ CARLOS PEREIRA, *Translations and normalization procedures*.
Department of Philosophy, UFRN, Rio de Janeiro, Brazil.
E-mail: mpaz@ufrnct.br.
Department of Philosophy, PUC-Rio/UFRJ, Rio de Janeiro, Brazil.
E-mail: luiz@inf.puc-rio.br.

The aim of the present paper is to explore some interesting connections between translations among logics and different normalization procedures. It can be easily verified that the family of negative translations (Kolmogorov, Gentzen, Gödel) can be proof-theoretically justified by the normalization procedure for classical logic used by Prawitz (see Prawitz [1]). Prawitz shows, as a preliminary result to normalization, that every application of the classical absurd rule in the fragment $\{\wedge, \rightarrow, \perp, \forall\}$ of the natural deduction system C can be reduced to an atomic application. The moral is: in the given fragment, classical reasoning can be restricted to atomic formulas. In 1989, J. Seldin presented a new normalization strategy where he showed that every derivation in the fragment $\{\wedge, \vee, \rightarrow, \perp, \exists\}$ could be transformed into a derivation with at most one application of the classical absurd rule. Moreover, if this application occurred, it would be the last rule in the resulting derivation. The moral now is: in this new fragment, classical reasoning, if used at all, is used as the last step of an argument. This normalization procedure gives rise (1) to a new translation from classical logic into intuitionistic logic, and (2) to an extension of Glivenko's theorem to first order logic. In the final part of the paper, we extend this analysis to the translation proposed by P. Krauss and to translations from intuitionistic logic into classical linear logic.

[1] D. Prawitz, ***Natural deduction***, Almqvist and Wiksell, Stockholm, 1965.

[2] J. Seldin, *Normalization and excluded middle I*, ***Studia Logica***, vol. 48 (1989), pp. 193–217.

[3] ———, *On the proof-theory of the intermediate logic MH*, ***Journal of Symbolic Logic***, vol. 51 (1986), pp. 626–647.

► HEIKE MILDENBERGER AND SAHARON SHELAH, *The relative consistency of* $\mathfrak{g} <$ cf(Sym(ω)).
Institute of Mathematics, The Hebrew University of Jerusalem, Givat Ram, 91904 Jerusalem, Israel.
E-mail: heike@math.huji.ac.il.
Institute of Mathematics, The Hebrew University of Jerusalem, Givat Ram, 91904 Jerusalem, Israel.

We give a forcing construction for a model where the groupwise density number $\mathfrak{g}$ is $\aleph_1$ and the bounding number, $\mathfrak{b}$ and the cofinality of the symmetric group, cf(Sym(ω)), are both $\aleph_2$.

Simon Thomas asked whether $\mathfrak{g} \neq$ cf(Sym(ω)) is consistent [1, Question 3.1].

Though the iterands have the countable chain condition and iteration is with finite supports, the construction works so far only for length $\aleph_2$ because of the preservation theorems we prove along the way.

[1] Simon Thomas, *Groupwise density and the cofinality of the symmetric group*, ***Archive for Mathematical Logic***, vol. 37 (1998), pp. 483–493.

► DAVID MILLER, *Some restricted Lindenbaum theorems equivalent to the axiom of choice*.
Department of Philosophy, University of Warwick, Coventry CV4 7AL, United Kingdom.
E-mail: d.w.miller@warwick.ac.uk.

$\langle S, \mathbf{Cn}\rangle$ is a *deductive calculus* if $\mathbf{Cn}$ is a function from $\wp(S)$ to $\wp(S)$ for which

(a) if $X \subseteq S$ then $X \subseteq \mathbf{Cn}(X) \subseteq S$;
(b) if $X \subseteq S$ then $\mathbf{Cn}(\mathbf{Cn}(X)) = \mathbf{Cn}(X)$; and
(c) if $X \subseteq S$ then $\mathbf{Cn}(X) = \bigcup\{ \mathbf{Cn}(Z) \mid Z \subseteq X \text{ and } |Z| < \aleph_0 \}$.

A set $Y \subseteq S$ is a *theory* in $\langle S, \mathbf{Cn}\rangle$ if $Y = \mathbf{Cn}(Y)$; a theory $\mathbf{Y}$ is y-*saturated* in $\langle S, \mathbf{Cn}\rangle$ if

(a) $y \notin \mathbf{Y}$ and
(b) if $x \notin \mathbf{Y}$ then $y \in \mathbf{Cn}(\mathbf{Y} \cup \{x\})$.

The generalized Lindenbaum extension theorem LET (for whose proof the axiom of choice AC is needed) says that if $y \notin \mathbf{Y}$ then $\mathbf{Y}$ has a y-saturated extension.

Dzik [1] gives a direct proof of AC from a premise *let* that attributes only a weak version of the Lindenbaum property, namely the existence of one saturated theory, to a restricted set of deductive calculi (in particular $\mathbf{Cn}(Y)$ takes only the values Y and S). Within ZF, plainly, *let* and LET are equivalent to AC. The question arises whether the conditional *let* $\Longrightarrow LET$ can be proved in ZF without in effect first proving AC from *let*, and then proving LET from AC. The paper provides a partial answer to this question.

[1] W. DZIK, *The existence of Lindenbaum's extensions is equivalent to the axiom of choice*, ***Reports on Mathematical Logic***, vol. 12 (1981), pp. 29–31.

► ROBERT MILNIKEL, *Monotone proofs in predicate default logic with the domain closure assumption*.
Wellesley College, 668 Washington St. #7, Wellesley, 2482, Massachusetts, USA.
E-mail: rmilnikel@wellesley.edu.

The addition of infinitary rules to the base logic of a default logic system can have a significant impact on the the complexity of monotone proofs in the system. In particular, proofs are no longer necessarily finite, but have depths bounded by recursive ordinals. It is suggested in [1] that open defaults in predicate default theories be dealt with via a domain closure assumption, instantiated as a rule of inference with infinitely many premises. We will examine the impact of such infinitary rules on proof schemes (as defined in [2]) and on tableau proof systems for skeptical consequence (as defined in [3]).

[1] M. KAMINSKI, J. A. MAKOWSKY, and M. TIOMKIN, *Extensions for open default theories via the domain closure assumption*, ***Journal of Logic and Computation***, vol. 8 (1998), pp. 169–187.

[2] W. MAREK, A. NERODE, and J. B. REMMEL, *Nonmonotonic rule systems I*, ***Annals of Mathematics and Artificial Intelligence***, vol. 1 (1990), pp. :241–273.

[3] R. MILNIKEL, *Tableau proof systems for skeptical consequence*, ***Proceedings, FLoC workshop on complexity-theoretic and recursion-theoretic methods in databases, artificial intelligence, and finite model theory*** (T. Eiter, G. Gottlob, V. W. Marek, and J. B. Remmel, editors).

► VLADIMIR A. MOLCHANOV, *Nonstandard theory of many-sorted finite models and its application to semigroups and automata on infinite words*.
Department of Mathematics and Physics, Pedagogical Institute of Saratov State University, Michurina 92, Saratov 410028, Russia.
E-mail: souslin@sgu.ssu.runnet.ru.

The new approach to pseudovarieties of finite algebraic systems based on the nonstandard analysis methods was elaborated in the paper [1]. The purpose of this talk is to generalize this approach on pseudovarieties of many-sorted finite models and to apply it to the theory of arbitrary words (both finite and infinite in any direction) on finite automata and semigroups. This includes

(i) the introduction of the nonstandard lower predicate language with many-sorted variables and its interpretation in finite many-sorted algebraic systems;

(ii) the characterization of such pseudovarieties as equational classes defined by identities of this language and
(iii) the investigation of the interplay between nonstandard varieties of four-sorted semigroup algebras and varieties of generally regular languages of arbitrary words on semigroups and automata.

By a four-sorted semigroup algebra (shortly 4-semigroup) we mean a four-sorted algebra $\mathcal{S} = (S, S^{\rightarrow}, S^{\leftarrow}, S^{\leftrightarrow})$ equipped with six operations: an associative product on S, mixed products $S \times S^{\rightarrow} \to S^{\rightarrow}$, $S^{\leftarrow} \times S \to S^{\leftarrow}$, $S^{\leftarrow} \times S^{\rightarrow} \to S^{\leftrightarrow}$ and maps $S \to S^{\rightarrow}$, $S \to S^{\leftarrow}$ (denoted $s \mapsto s^{+\omega}$, $s \mapsto s^{-\omega}$), satisfying, for every $s, t \in S$, $x \in S^{\leftarrow}$, $y \in S^{\rightarrow}$ and $n \in \mathbf{N}$, the equations $x(st) = (xs)t$, $(st)y = s(ty)$, $(xs)y = x(sy)$, $s(ts)^{+\omega} = (st)^{+\omega}$, $(ts)^{-\omega}t = (st)^{-\omega}$, $(s^n)^{+\omega} = s^{+\omega}$, $(s^n)^{-\omega} = s^{-\omega}$.

An important example of 4-semigroup gives us the algebraic system $\mathcal{W} = (W_{\text{fin}}(A), A^{\mathbf{N}}, A^{-\mathbf{N}}, A^{\mathbf{Z}})$ equipped with the usual concatenation product, where $W_{\text{fin}}(A)$ is the semigroup of finite words over a finite alphabet A, $\mathbf{Z}$ is the set of integers, $\mathbf{N}$ and $-\mathbf{N}$ are the sets of positive and negative integers, respectively.

Using methods of the nonstandard analysis, we elaborate a general approach to the theory of arbitrary words and generalize the well-known Kleene's theorem proving that the class of recognizable languages (i.e., those recognized by either a finite Büshi automaton, or a finite semigroup, or a finite 4-semigroup), coincides with the class of generally rational languages (i.e., the smallest set of languages over an alphabet A containing finite languages and closed under three special operations).

[1] V. A. MOLCHANOV, *Nonstandard characterization of pseudovarieties*, ***Algebra Universalis***, vol. 33 (1995), pp. 533–547.

▶ SRIRAM NAMBIAR AND JOHN CORCORAN, *Boole's elimination Summation problem: an overlooked breakthrough*.
Department of Philosophy, University of Buffalo, SUNY-Buffalo, New York 14260-4150, USA.
E-mail: nambiar@acsu.buffalo.edu.
Department of Philosophy, University of Buffalo, SUNY-Buffalo, New York 14260-4150, USA.

Before the time of George Boole (1815–1864), logic restricted itself very largely to determining the validity and invalidity of arguments taken from a limited class. Boole is noted for having expanded this class to include arguments whose "categorical" propositions involved arbitrarily large numbers of terms in "Boolean combinations", rather than just two. Boole is not noted for an equally important innovation that may have contributed more towards altering the course of the development of logic: Boole set radically different types of problems for logic to treat. Previously logic treated problems of the following type: given a set of premises and a conclusion, to determine whether or not the premise set implies the conclusion. In sharp contrast Boole proposed several new problem classes one of which is *Boole's elimination summation problem class* composed of problems of the following type: given a set of propositions involving a certain set of non-logical terms, given an arbitrary subset of these terms, to find a proposition that (1) follows from the given set, that (2) contains only terms from the given subset and that (3) "summarizes" all of the consequences of the given set involving only these terms, i.e., that implies every consequence of the given set involving only the terms from the given subset. In other words, Boole's problem was to find a proposition which is a summation of all the consequences of the given set expressible using only the given subset of terms. This paper discusses Boole's treatment of these problems, their importance in the history of logic, and the current status of the general problem. Examples are given from modern equational logic, first-order logic, and higher-order logic.

► MAURO DI NASSO, ZFC[Ω]: *a nonstandard set theory where all sets have nonstandard extensions*.
Dipartimento di Matematica Applicata, via Bonanno 25/b, 56126 Pisa, Italy.
E-mail: dinasso@dma.unipi.it.

Let ZFC[Ω] be the first order theory in the language of set theory plus a binary relation symbol Ω, where all axioms of ZFC except regularity are assumed—separation and replacement schemata are considered for *all* formulas—and the following are postulated.

(1) Ω is a functional correspondance whose domain is the class of all sequences (i.e., functions defined on some infinite cardinal κ).
(2) $\Omega(\varphi) = \{\, \Omega(\psi) : \psi(i) \in \varphi(i) \text{ for all } i \,\}$.
(3) If $\vartheta(i) = \{\varphi(i), \psi(i)\}$ for all i, then $\Omega(\vartheta) = \{\Omega(\varphi), \Omega(\psi)\}$.
(4) If $\Omega(\varphi) = \Omega(\psi)$ then $\Omega(f \circ \varphi) = \Omega(f \circ \psi)$ (provided such compositions are defined).

For each set A, define its *κ-nonstandard extension* $J_\kappa(A) = \{\, \Omega(\varphi) : \varphi \colon \kappa \to A \,\}$, κ any infinite cardinal. If $\varphi(i) \neq \psi(i)$ for all i, then $\Omega(\varphi) \neq \Omega(\psi)$.

(5) For every κ, the *κ-nonstandard embedding* J_κ satisfies κ^+-saturation (a set x is internal relative to J_κ if $x \in J_\kappa(A)$ for some A).

By restricting to countable sequences, the above axioms can be arranged so to get a truly elementary presentation of nonstandard analysis, suitable even at the freshman level (see [1]).

THEOREM 1 (Transfer principle). *For every κ, J_κ is a nontrivial bounded elementary embedding of the universe into itself.*

In analogy to the von Neumann hierarchy $\bigcup_{\alpha \in ON} V_\alpha(\emptyset)$ of wellfounded sets, define:

$$W_0 = \emptyset;$$
$$W_{\alpha+1} = \mathcal{P}\left(\bigcup_{\kappa \leq \alpha} J_\kappa(W_\alpha) \cup W_\alpha\right);$$
$$W_\lambda = \bigcup_{\alpha < \lambda} W_\alpha \quad \text{if } \lambda \text{ limit}.$$

As a substitute of regularity, we postulate:

(6) Every set is in some W_α. In other words, all sets in ZFC[Ω] are obtained by transfinite iterations of power-set and nonstandard extension operations. As a consequence, reflection properties hold.

THEOREM 2. ZFC[Ω] *is equiconsistent with* ZFC.

Symbol Ω is used for historical reasons. In fact, ZFC[Ω] can be presented as a strengthening in the general framework of set theory of the pre-Robinsonian approach to nonstandard analysis by C. Schmieden and D. Laugwitz [2], where Ω denoted a new "infinite" natural number.

[1] V. BENCI and M. DI NASSO, *A truly elementary presentation of nonstandard analysis by axiomatic approach*, preprint.

[2] C. SCHMIEDEN and D. LAUGWITZ, *Eine erweiterung der infinitesimalrechnung*, ***Mathematische Zeitschrift***, vol. 69 (1958), pp. 1–39.

► SARA NEGRI, *Systems of natural deduction for intuitionistic linear logic*.
Department of Philosophy, PL 24, Unioninkatu 40 B, 00014 University of Helsinki, Finland.
E-mail: negri@helsinki.fi.

We present a system of natural deduction with implicit structural rules for intuitionistic linear logic. Each connective as well as the modality ! has an introduction and an elimination rule. No explicit weakening or contraction rules for !-formulas are needed. These structural rules are absorbed into a generalized form of the rule of introduction for the !-modality.

General elimination rules were introduced in von Plato [2] and systematically employed in Negri and von Plato [1] in order to obtain a close correspondence between natural deduction and sequent calculus and between normalization and cut-elimination for intuitionistic and classical logic.

Here the method is extended to intuitionistic linear logic by employing general elimination rules for all connectives and quantifiers, and also for the !-modality. Normalization thus follows through translation to sequent calculus and cut-elimination. The subformula property obtains for all normal derivations. Other methods of absorbing the structural rules into rules that discharge assumptions are presented.

[1] S. Negri and J. von Plato, *Structural proof theory*, Cambridge University Press, 2001, in press.

[2] J. von Plato, *Natural deduction with general elimination rules*, ***Archive for Mathematical Logic***, in press.

► LISA O'NEILL AND JOHN SEREMBUS, *Leblanc and the development of twentieth century logic*.
Department of Philosophy, Saint Peter's College, 2641 Kennedy Blvd., Jersey City, New Jersey 07306, USA.
E-mail: LAONEILL@EROLS.COM.
Humanities Division, Widener University, Chester, Pennsylvania 19013, USA.

Twentieth century logic witnessed the development of alternative logics, among them logics that sought to address philosophical problems more concretely than classical approaches. Within these alternatives are two interesting and important areas: free logic and probability logic. A key figure in the development of these fields was logician and philosopher Hugues Leblanc (1924–1999).

Along with others Leblanc developed free logic, an alternative to classical logic and a kind of logic that claims to address certain philosophical problems more directly and extensively than classical logic. Additionally, he was a pioneer in the development of probability logic, in particular, probabilistic semantics. Probabilistic semantics is an alternative to standard model-theoretic semantics and an approach to semantics that also claims to boast useful philosophical solutions. In recognition of his numerous contributions we present a paper that focuses on Leblanc's most important contributions in these two areas. Specific attention will be given to the usefulness of these alternatives in addressing key philosophical problems.

► MATTI PAUNA, *On non-wellfounded sets as fixed points of substitutions*.
Department of Mathematics, University of Helsinki, Finland.
E-mail: pauna@cs.helsinki.fi.

(Joint work with Tapani Hyttinen.)

We work in set theory with urelements, denoted by $\mathcal{U}$, and without the foundation axiom, ZFC^-. In [1] non-wellfounded sets and the antifoundation axiom, AFA are studied. We denote by $V_{\mathrm{afa}}[\mathcal{U}] \cup \mathcal{U}$ the model of $ZFC^- + AFA$. Substitution operations $\mathrm{sub}(s, b)$, also denoted by $b[s]$, are studied in [1]. Here s is a function, called substitution, such that $\mathrm{dom}(s) \subseteq \mathcal{U}$, and $b \in V_{\mathrm{afa}}[\mathcal{U}] \cup \mathcal{U}$, such that the following conditions hold:

(i) If $x \in dom(s)$, then $\mathrm{sub}(s, x) = s(x)$.
(ii) If $x \in \mathcal{U} - \mathrm{dom}(s)$, then $\mathrm{sub}(s, x) = x$.
(iii) For all sets b, $\mathrm{sub}(s, b) = \{\, \mathrm{sub}(s, p) \mid p \in b \,\}$.

There is a unique substitution operation $\mathrm{sub}(s, b)$ defined for all substitutions s and $b \in V_{afa}[\mathcal{U}] \cup \mathcal{U}$.

Substitutions can be composed in the following way. A substitution s is proper if for all $x \in \mathrm{dom}(s)$, $s(x) \in V_{\mathrm{afa}}[\mathcal{U}]$ whenever $s(x) \neq x$. If s and t are substitutions, then $t \star s$ is the substitution whose domain is $\mathrm{dom}(s)$ and for every $x \in \mathrm{dom}(s)$, $(t \star s)(x) = s(x)[t]$. Now

AFA is equivalent to: for every proper substitution e there is a unique proper substitution s such that $s = s \star e$.

We study the non-wellfounded sets as fixed points of substitutions. If f is a function, let f^* be the class function such that if $x \in \mathrm{dom}(f)$, $f^*(x) = f(x)$, otherwise $f^*(x) = x$. If f and g are functions, let $f[g]$ be the function such that $\mathrm{dom}(f[g]) = \mathrm{dom}(g)$ and $f[g](x) = f^*(x)$, if x is an urelement, and otherwise $f[g](x) = \{\, g^*(y) \mid y \in f^*(x)\,\}$. Note that if $f(x)$ is a set, then $g(x) = \{\, g(y) \mid y \in f(x)\,\}$. We say that g is a *fixed point* of f, $(\mathrm{FP}(g, f))$, if $\mathrm{dom}(f) \cup \bigcup f^*[\mathrm{dom}(g)] \subseteq \mathrm{dom}(g)$ and $g = f[g]$. The *fixed point axiom*, *FA* is: every function has a fixed point. We have shown that assuming the strong extensionality axiom, *FA* and *AFA* are equivalent. In *ZFC*, let X be a set and $f : X \to \mathcal{P}(X)$, be such that $f(x) = x \cap X$. Then f has a unique fixed point g, namely the Mostowski collapse of X. We will show that from the *FA* it follows that for every substitution f there is g such that $g = g \star f$. However, g is not necessarily unique, and for arbitrary f, g does not necessarily exist.

[1] Jon Barwise and Lawrence Moss, ***Vicious circles***, CSLI Lecture Notes, vol. 60, CSLI Publications, Stanford, 1996.

► CARLOS PELTA, *Wide sets, deep many-valuedness and sorites arguments*.
Department of Linguistics, Logic and Philosophy of Science, Universidad Autónoma de Madrid, Campus de Cantoblanco, 28049 Madrid, Spain.
E-mail: `lvr@step.es`.

In this contributed talk, I show how to obtain a powerful and truthful explanation of the failure of Sorites Arguments combining an adaptation of the Wide set theory formulated by professors Formato and Gerla (1998) and the concept of deep many-valuedness established by professor Hubert Marraud (1998 and 1999). Our conclusion is that if the premises of a Sorites Argument are conceived like a succession of indexed consequence operators (where indices express the accuracy of the inferences) prefixing sentences and forming wide data sets, the failure of the Argument is because of the transitive property for the inclusion between those wide data sets is not always holded.

In a general sense, a wide set is a finite set as big as we will. Given a binary predicate EQ interpreted as a multivalued extension of the classical relation of equipotence between finite sets, and given a finite set X, $\mathrm{Wide}(X)$ is denoted by the formula $\mathrm{EQ}(X, X \cup \{X\})$. An important aspect is that an infinite set can be reduced to the existence of wide sets because an infinite set is a set equipotent to its successor. On page 383 of their article, Formato and Gerla declare that sorites paradoxes are based on the notion of wide set. In fact, this is the main objective of our contributed paper: the explanation of the genesis and failure of that kind of arguments not finding out the difficulty on the gradual character of the assertion of truth of fuzzy predicates but using the theory of Formato and Gerla under the influence of the idea of deep many-valuedness developed by Marraud. Until now in the investigations about logical many-valuedness has prevailed the construction of superficial many-valued logics, that is, logics with an arbitrary number (bigger than two) of truth-values but always incorporating a bivalent consequence relation. We can wonder why we hold the existence of logics with a number bigger than two of truth-values but in which the sentences of the kind "S_{n+1} follows from $S_1, \ldots, S_n$" only can take the assignation of two truth values. By this reason one can speak about deep many-valued logics, that is, many-valued logics whose consequence relation is $n \geq 2$-valued. From a deeply many-valued perspective, v.g. considering several consequence operators, the matter is not that, for example, the third premise in a sorites argument be less true (with a determined degree of truth) than the second one: it will happen that in the third premise is displayed a different consequence relation weaker than the preceding relation. So, for consequence operations C_i, a theory $C_1(D)$ is more trustworthy than a theory $C_2(D)$ if and only if the quantity of information derivable from a data set D is bigger under C_2, that is, C_2 is more lax than C_1.

Using tools of the Wide set theory and conditional probabilities, I show that a precise explanation of the failure of sorites arguments is located on the failure of the transitivity for the inclusion between sets of premises conceived like sentences prefixed by consequence operators and having indices of accuracy progressively more defeasible in order to the presence of new information added

[1] F. Formato and G. Gerla, *Grasping infinity by finite sets*, ***Mathematical Logic Quarterly***, vol. 44 (1998), no. 3, pp. 383–393.

[2] H. Marraud, *Razonamiento aproximado y grados de consecuencia*, ***Endoxa***, vol. 10 (1998), pp. 55–70.

[3] ———, *Sorites paradox from a logical point of view*, in ***Proceedings of the International Congress 'Analytic philosophy at the turn of the millenium'***, Santiago de Compostela, 1999.

[4] C. Pelta, *Deep many-valuedness*, 1999, to appear.

► DAVID A. PIERCE, *Model-theoretic differential geometry*.
McMaster University, Department of Mathematics and Statistics, 1280 Main Street West, Hamilton, Ontario, Canada, L8S 4K1.
E-mail: pierced@math.mcmaster.ca.

A *differential field* is here a structure $(k, \partial_0, \dots, \partial_{m-1})$, where k is a field of characteristic zero, and the unary functions ∂_i are commuting derivations. The model-completion of the theory of differential fields is axiomatized by the Frobenius Theorem of differential geometry. Similar arguments go through formally in case the derivations span a non-commutative Lie algebra.

The Frobenius Theorem is usually stated in terms of a real manifold, but it has a purely algebraic content. If K is a field-extension of k, then the possible extensions of the derivations ∂_i to K span a vector space, whose dual may be called the space of *differential* 1*-forms of K over k*. There are 1-forms $\mathrm{d}T^i$, dual to the derivations ∂_i. A system of 1-forms ω of K has a *generic solution* if and only if the 2-forms $\mathrm{d}\omega$ are in the space generated by the forms ω, *and* the forms $\mathrm{d}T^i$ are linearly disjoint from the forms ω. (That the system has a generic solution means that the kernel of the forms ω is spanned by particular *commuting* extensions of the derivations ∂_i.)

Tracey McGrail [1] and Yoav Yaffe [2] have shown independently that the theory of differential fields has a model-completion, so their axioms must be equivalent to the Frobenius Theorem; but the latter can also be translated directly into an infinite set of first-order axioms.

[1] Tracey McGrail, *The model theory of partial differential fields*, ***Journal of Symbolic Logic***, to appear.

[2] Yoav Yaffe, *Model completion of Lie differential fields*, ***Annals of Pure and Applied Logic***, to appear.

► AHTI PIETARINEN, *Some remarks on games logic plays*.
School of Cognitive and Computing Sciences, University of Sussex, Falmer, Brighton BN1 9QH, United Kingdom.
E-mail: ahtivp@cogs.susx.ac.uk.

Various logical methods have recently been employed in analysing games, and conversely, game-theoretic methods have been used in evaluating logical statements. I argue that this latter connection can be made tighter than before, by describing some new game-theoretic phenomena behind some logical notions. Primarily they arise in (i) first-order formulas having non-linear (partial or non-transitive) informational independencies, traditionally corresponding to sequential games of imperfect information. (ii) One can also have more semantic dependence that in classical logic, requiring extensive form of games with cycles, while not all imperfect information games can correspond to logical languages. These considerations suggest that Henkin quantifiers should be interpreted with (genuine) simultaneous games rather than sequential games.

Some of the further connections can be described as follows. (iii) Informationally independent logics give rise to sequential games of imperfect recall (retrieval failure). (iv) Informational independence of negation can correspond to games of incomplete information (hidden chance moves). (v) Informationally independent formulas incorporate signalling and team playing, related to imperfect recall. (vi) Semantic games can be viewed as extensive form games where information sets denote (priority) scope relations, and skolemised formulas as strategic forms of games. (vii) Even linearly ordered classical logic can be seen as a logic of imperfect information and imperfect recall, as well as asymmetirc in information (discriminating games). (viii) In game-theoretically interpeted modal logics, there is a need for distinguishing between two notions of information sets, suggesting a dynamisation of information sets. (ix) all games for logic so far are strictly competitive, but not necessarily determinable, giving rise to partial logics, as a product of entirely natural assumptions about the game. (x) Games that are non-strictly competitive can amount to forms of non-coherence: by distinguishing between different notions of consistency, paraconsistencies receive game-theoretic import.

► JAN VON PLATO, *Classical natural deduction*.
Department of Philosophy, PL 24, Unioninkatu 40 B, 00014 University of Helsinki, Finland.
E-mail: `vonplato@helsinki.fi`.

Prawitz [2] added to intuitionistic natural deduction a rule of indirect proof restricted to atomic formulas. The resulting system admits the normalization of derivations for the $\vee$, $\exists$-free fragment of the language of predicate logic. In von Plato [3] and Negri and von Plato [1], Prawitz' rule is generalized into a *rule of excluded middle* permitting to conclude a formula C in propositional calculus if C is derived from an atom P and from its negation. The rule is admissible for arbitrary propositional formulas.

A generalization of the rule of excluded middle is given that applies to quantified formulas also. The result is a normalizing system of classical natural deduction for the full language of predicate logic. Normal derivations are well-founded, possibly infinitary trees. An example shows how to apply classical natural deduction in the proof theory of arithmetic.

[1] S. NEGRI and J. VON PLATO, ***Structural proof theory***, Cambridge University Press, 2001, in press.

[2] D. PRAWITZ, ***Natural deduction: A proof-theoretical study***, Almqvist and Wicksell, Stockholm, 1965.

[3] J. VON PLATO, *Proof theory of full classical propositional logic*, 1998, submitted.

► VALERI PLISKO, *A predicate logic based on the Gödel interpretation and the system of hereditary effective operations*.
Department of Mathematical Logic and Theory of Algorithms, Faculty of Mechanics and Mathematics, Moscow State University, Moscow, 119899, Russia.
E-mail: `plisko@ttk.ru, plisko@lpcs.math.msu.ru`.

Let T be an intuitionistic arithmetic theory, i.e., an arbitrary set of arithmetic sentences closed under derivability in the intuitionistic arithmetic HA. A closed predicate formula will be called T-valid if each its closed arithmetic instance is in T. The set of all T-valid predicate formulas will be denoted by $L(T)$. An arithmetic theory T is called an IS-theory if there are arithmetic Σ_1-formulas $A(x)$ and $B(x)$ such that the formula

$$\forall x\ \neg(A(x)\ \&\ B(x))\ \&\ \neg\forall x\ \exists y\ ((A(x) \supset y = 0)\ \&\ (B(x) \supset y \neq 0))$$

is in T. An arithmetic formula is said to be negative if it does not contain $\vee$ and $\exists$. The negative fragment of a theory T is denoted by T^-.

THEOREM 1. *If T is an intuitionistic IS-theory, then $T^- \leq_1 L(T)$.*

The theory T consists of arithmetic sentences A such that its Dialectica interpretation A^D in the language of arithmetic in all finite types is true if variables in A^D are interpreted as hereditary effective operations. One can prove that T is an intuitionistic IS-theory and T^- is not recursively enumerable. The following theorem is a consequence of this fact and Theorem 1:

THEOREM 2. *The logic $L(\mathsf{T})$ is not recursively enumerable.*

An analogous result for the predicate logic based on the Gödel interpretation and the system of hereditary recursive operations was proved in [1].

The author is partially supported by RFBR grant 98-01-00249.

[1] V. PLISKO, *Two semantics and logics based on the Gödel interpretation*, in ***Computational logic and proof theory, Proceedings of the 5th Kurt Gödel colloquium, KGC'97, Vienna, Austria, August 1997***, Lecture Notes in Computer Science, vol. 1289, Springer-Verlag, Berlin, pp. 233–240.

► VALERI PLISKO AND DMITRI VITER, *Equality from the constructive point of view*.
Department of Mathematical Logic and Theory of Algorithms, Faculty of Mechanics and Mathematics, Moscow State University, Moscow, 119899, Russia.
E-mail: `plisko@ttk.ru, plisko@lpcs.math.msu.ru`.

We consider equality in the framework of the constructive model theory developed in [1]. Let (M, ν) be an enumerated set. A constructive equality structure compatible with (M, ν) is a function $\mathcal{E} \colon \mathbf{N}^2 \to 2^{\mathbf{N}}$ such that $\mathcal{E}(m, n) \neq \emptyset \iff \nu(m) = \nu(n)$. The elements of $\mathcal{E}(m, n)$ are treated as the realizations (in the Kleene sense) of the sentence $\nu(m) = \nu(n)$. The set of realizations of a sentence in the language with the only predicate symbol $=$ is defined in accordance with Kleene's recursive realizability. The theory of a constructive equality structure is defined as the set of all sentences whose set of realizations is not empty.

In the classical logic, the equality theory of a set M depends only of the cardinality of M. In the constructive case the situation is rather different.

An equality structure is said to be normal if the sentence $\forall x\ (x = x)$ is realizable. It is proved that all normal constructive equality structures compatible with a given enumerated set have the same theory.

The next step is extracting structures with decidable equality, i.e., equality structures where the sentence $\forall x\ \forall y\ (x = y \vee \neg x = y)$ is realizable. It is proved that the theory of a normal constructive equality structure with decidable equality compatible with a given enumerated set (M, ν) is just the classical equality theory of M.

There are different constructive equality theories with undecidable equality even in the case of a 2-element enumerated set.

The first author is partially supported by RFBR grant 98-01-00249.

[1] V. PLISKO, *On languages with constructive logical connectives*, ***Dokl. Akad. Nauk SSSR***, vol. 296 (1987), pp. 35–38.

► CHRIS POLLETT AND RANDALL PRUIM, *Strengths and weaknesses of LH arithmetic*.
Department of Mathematics, 405 Hilgard Ave., Box 951555, University of California, Los Angeles, California 90095, USA.
E-mail: `cpollett@willow.math.ucla.edu`.
Department of Mathematics and Statistics, Calvin College, Grand Rapids, Michigan, USA.
E-mail: `rpruim@calvin.edu`.

In Pollett [3] a bounded arithmetic theory Z was shown not to be able to prove the collapse of the polynomial hierarchy. This theory also had the property that if $Z \subseteq S_2^i$ for any $i \geq 1$ then the polynomial hierarchy collapses. Here S_2^i are the theories of Buss [1]. Unfortunately, despite this property Z seemed too weak a theory to formalize many of the arguments that have been used in computational complexity. In this talk, we give a new arithmetic characterization of the levels of log-time hierarchy. Using this characterization, we propose a variant of the theory TAC^0 of Clote and Takeuti [2]. This variant has nice deductive fragments which in some sense correspond to the levels of the log-time hierarchy. We show that this theory (like Z) cannot prove the collapse of the polynomial hierarchy. Furthermore, we give some evidence that this theory may be strong enough to prove that the log-time hierarchy is infinite, so unlike Z it can carry out useful complexity arguments.

[1] S. R. Buss, ***Bounded arithmetic***, Bibliopolis, Napoli, 1986.

[2] P. Clote and G. Takeuti, *First order bounded arithmetic and small boolean circuit complexity classes*, in ***Feasible mathematics II*** (P. Clote and J. Remmel, editors), Birkhäuser, Boston, 1995, pp. 154–218.

[3] C. Pollett, *Multifunction algebras and the provability of PH↓*, ***Annals of Pure and Applied Logic***, vol. 104 (Aug. 2000), no. 1–3, pp. 279–303.

► ANDREJA PRIJATELJ, *Linear logic in view of bounded structural rules*.
Department of Mathematics, University of Ljubljana, Slovenia.
E-mail: `Andreja.Prijatelj@fmf.uni-lj.si`.

In this paper we shall present an alternative living space of linear logic in view of bounded structural rules. The main purpose being to explore the famous completeness problem of linear logic from a different angle. It is well-known that the main difficulty of the problem mentioned is caused by the presence of modalities 'of course' and 'why not'. In order to avoid this obstacle we shall consider particular axiomatic systems of a modality free linear logic enriched with some bounded structural rules instead. To ensure the cut-elimination property the systems are firstly based on two-fold sequents following the idea of Girard's LU [1]. A two-fold sequent consists of a linear part and a separated structural part. The former is inhabited with usual linear logic formulas (without explicit modalities) and the latter with modalized formulas in disguise. A bounded structural operation may act solely on formulas within a structural part, while the usual linear two-premiss 'tensor' and 'par' rules are suitably modified to help recovering the visual absence of both modalities. The two parts may also communicate with each other by the so-called switching rules. We shall show that there are faithfull embeddings of the systems considered into linear logic. Ultimately the two-fold systems can be transformed into equivalent linear systems mimicing the intended effects of linear logic modalities.

[1] J.-Y. Girard, *Linear logic: a survey*, in ***Logic and algebra of specification*** (Bauer, Brauer, and Schwichtenberg, editors), Heidelberg, 1993, pp. 63–112.

[2] J.-Y. Girard, Y. Lafont, and L. Reigner (editors), ***Advances in linear logic***, London Mathematical Society Lecture Notes, vol. 222, Cambridge University Press, 1995.

[3] A. Prijatelj, *From bounded structural rules to linear logic modalities*, manuscript, Paris, 1999.

► VERA PUNINSKAYA, *Modules with few models over a serial ring*.
Chair of Algebra, Moscow State University, 119899, Moscow, Russia.
E-mail: `punins@orc.ru`.

An R-module M is called *uniserial* if the lattice of submodules of M is a chain. A module that is a direct sum of uniserial modules is called *serial*. A ring R is called *serial* if both R_R and ${}_RR$ are (serial) modules. A module (over a countable ring) is said to have *few types* if its theory has only countably many n-types for every natural number n. A module is said to have *few models* if its theory has less than continuum many countable models.

It is proved

THEOREM 1. *Let M be a right module over a countable serial ring R. The following conditions are equivalent*:

1. *M has few types*;
2. *M has few models*;
3. *M is Σ-pure-injective.*

It gives a generalisation of the previous result of the author [1] that every module with few models over a countable serial ring is Σ-pure-injective.

[1] V. A. PUNINSKAYA, *Vaught's conjecture for modules over a serial ring*, ***Journal of Symbolic Logic***, to appear.

► CHRISTIAN RENTERIA, *Natural deduction for CTL-proof theory*.
Pontificia Universidade Catolica, Rio de Janeiro, Rua Joaquim Murtinho 675, Santa Teresa, Rio de Janeiro, CEP:20241, RJ, Brasil.
E-mail: chren@ibm.net, chren@inf.puc-rio.br.

CTL (Computation Tree Logic) [1] is a logic used both for the synthesis and for the verification of concurent programs. Usualy presented in the Hilbert style (axiomatic) [2], it has been adapted for sequent calculus [3], but without normalization. The aim of the present work is to present a Natural Deduction system for CTL. For that reason the original language has been modified, and labeled formulae have been used. It has been proved that the system is correct and complete. Then the possibility of normalizing the proofs is studied and it is seen, as expected, that strong normalization is achieved in the system without the induction rule.

[1] CLARKE AND EMERSON, ***Design synthesis of synchronization skeletons using branching-time temporal logic***, Lecture Notes in Computer Science, vol. 131, Springer-Verlag, Berlin.

[2] GOLDBLATT, ***Logics of time and computation***, CSLI Lecture Notes, vol. 7.

[3] ECHAGUE, *A Gentzen-system for computation tree logic*, in ***Proceedings of the XIX CLEI-PANEL Latin American informatics conference, Buenos Aires***, 1993.

[4] PRAVITZ, ***Natural deduction***, Stockholm, 1965.

[5] ***Proceedings of the second Scandinavian logic symposium***, Studies in Logic and the Foundation of Mathematics, vol. 63.

► MICHAEL RESCORLA, *Brandom's philosophy of logic*.
Department of Philosophy, Harvard University, Cambridge, Massachusetts 02138, USA.
E-mail: rescorla@fas.harvard.edu.

Robert Brandom's ***Making it explicit*** advances a novel conception of logic. According to Brandom, logical locutions allow speakers to discuss linguistic practice by making explicit the inferential norms which implicitly govern that practice. For instance, a speaker can signal acceptance of the inference from p to q by using the conditional, in particular by affirming the sentence "If p, then q." Of course, the classical material conditional is not well-suited for this role. But, for precisely that reason, Brandom denies that natural language "if-then" corresponds to the classical material conditional. Brandom introduces a new kind of conditional, one more appropriate for his purposes. He then extends his analysis to other logical connectives, such as negation, identity, and the quantifiers.

In my paper, I criticize Brandom's conception of logic. In my view, logical locutions do not serve merely to facilitate discussion of linguistic practice. They serve a much more general and important purpose: they allow us to express complex views concerning any topic whatsoever. I develop this line of criticism by reflecting upon the compositionality of linguistic meaning, a phenomenon which I argue that Brandom fails to explain satisfactorily. I further buttress my case by considering Brandom's own paradigmatic example, the conditional. I argue that this example is far more problematic than Brandom acknowledges. In particular, the actual linguistic data undermine Brandom's claim that conditional locutions serve primarily to facilitate discussion of inferential norms. I conclude with some general remarks about my discussion's import for Brandom's overall philosophical project.

[1] ROBERT BRANDOM, ***Making it explicit***, Harvard University Press, Cambridge, 1998.

► ANDRZEJ KRZYSZTOF ROGALSKI, *Recent approaches to the theory of truth*.
Faculty of Theology, Lugano, Switzerland.
E-mail: andrzej_rogalski@hotmail.com.

Tarski's famous paper on the concept of truth [6] has become a paradigm of all 20th century's researches on truth. It has greatly influenced the logical semantics, various formal (standard and non-standard) approaches to linguistics and also the modern logical philosophy. The present paper is focused on:

(1) Tarski's method to interpret his achievements from current perspectives (cf. [2] and [4]),
(2) Pointing out shortcomings of the Tarski's definition of truth and the different new ways of approaches, e.g., Davidson's approach to meaning by truth-conditions [1] and recent deflationist formulations by Soames [5] and, on the other hand, on
(3) A non-standard approach, based on the notion of denotation, that can be applied to all languages whose expressions have established meanings. This approach enables to formulate:
    (a) the syntactic definition of a true sentence which corresponds to the Tarski's convention (T), and
    (b) the semantic definition of truthfulness of sentences in which the notion of satisfaction in Tarski's sense is formally replaced by the use of the notion of denotation (cf. [7] and [3]).

[1] D. DAVIDSON, ***Inquiries into truth and interpretation***, Clarendon Press, Oxford, 1984.

[2] J. PEREGRIN, ***Truth and its nature (if any)***, Kluwer, Synthese Library, vol. 284, 1999.

[3] A. K. ROGALSKI and U. WYBRANIEC-SKARDOWSKA, *On truth and denotation*, ***Proceedings of the logic colloquium 1999***, 2000, submitted.

[4] A. K. ROGALSKI, E. BRYNIARSKI, and U. WYBRANIEC-SKARDOWSKA, *The notion of truth in systems of knowledge*, in ***Studies in grammar, logic and rethorics***, 2000, to appear.

[5] S. SOAMES, ***Understanding truth***, Oxford University Press, New York-Oxford, 1999.

[6] A. TARSKI, *Pojecie prawdy w jezykach nauk dedukcyjnych* (English translation: *The concept of truth in formalized languages*, in ***Logic, semantics, metamathematics: Papers from 1923 to 1938***, Clarendon Press, Oxford, 1956), 1933.

[7] U. WYBRANIEC-SKARDOWSKA, *Logical and philosophical ideas in certain formal approaches to language*, ***Synthese*** vol. 116 (1998), pp. 231–277.

► V. V. RYBAKOV, *Barwise's information frames and modal logics*.
Mathematics Department, Science Faculty of Hacettepe University, Beytepe-Ankara, Turkey, and Mathematical Department, Krasnoyarsk University, pr. Svobodnyi 79, 660 062, Krasnoyarsk, Russia.
E-mail: rybakov@math.kgu.krasnoyarsk.su, rybakov@eti.cc.hun.edu.tr.

The aim of our research is to develop mathematical theory of Barwise's information frames [1]. These frames, in mathematical sense, are special multi-based models, or algebraic systems, with the relations $\Vdash_s$ which express the context's depending consequence relations on classification types, i.e., they are aimed to handle information about situations which occur as a base set in these frames. We answer the Jon Barwise question from [1]: to find axiomatization for modal logics generated by information frames:

THEOREM 1.

(i) *Modal logic* $L(\mathcal{K}_c)$ *of all complete information frames is the logic* K.
(ii) *Modal logic* $L(\mathcal{K}_{c,s})$ *of all complete and sound information frames is the logic* T.
(iii) *Modal logic* $L(\mathcal{K}_{c,h})$ *of all complete and hereditary information frames is the logic* $K4$.
(iv) *Modal logic* $L(\mathcal{K}_{c,h,s})$ *of all complete hereditary and sound information frames is the logic* $S4$.
(v) *Modal logic* $L(\mathcal{K}_{c,con})$ *of all complete and consistent information frames is the logic* D.

THEOREM 2.

(i) *The weak modal logic* $L_w(\mathcal{K}_c)$ *of all information frames is the logic* K.
(ii) *The weak modal logic* $L_w(\mathcal{K}_h)$ *of all hereditary information frames is the logic* $K4$.

THEOREM 3. *Any Kripke complete normal modal logic is complete with respect to the appropriate class of complete information frames, i.e., is information frames complete.*

THEOREM 4. *For any rarefied information frame IF the context's logical consequence relation* $\Vdash_s$ *can be presented by accessibility context's consequence of usual Kripke frames.*

THEOREM 5. *All modal logics generated by arbitrary classes of complete, rarefied and fully classified information frames are Kripke complete.*

[1] J. BARWISE, *Information and impossibilities*, ***Notre Dame Journal of Formal Logic***, vol. 38 (1997), no. 4, pp. 488–515.

[2] J. BARWISE and J. SELIGMAN, ***Information flow in distributed systems***, Tracts in Theoretical Computer Science, vol. 44, 1997.

[3] F. DRETSKE, ***Knowledge and the flow of information***, Bradford Books, MIT Press, 1981.

► SAEED SALEHI, *A generalized realizability for constructive arithmetics*.
Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, P. O. Box 137, 00-950 Warsaw, Poland.
E-mail: salehi@impan.gov.pl.

We introduce a generalized notion of realizability, appropriate for constructive arithmetics, in particular Basic Arithmetic, **BA**. This arithmetic is equivalent of **HA** on intuitionistic logic and of **PA** on classical logic, based on Basic Logic, the propositional counterpart of which, was invented by Visser, and is developed for the quantifier case by Ardeshir and Ruitenburg [1].

The crucial different cases from the Kleene's one, are $\rightarrow$ and $\forall$. We restrict the recursive functions, associated for these cases, to a definable subclass, for example total recursives or primitive recursives. So the primitive recursive (PR-)realizability for $A \rightarrow B$ is:

$$n\,\mathbf{r}^{\mathrm{PR}}\,A \rightarrow B \;\equiv\; \mathrm{PR}(n) \wedge \forall x\,(x\,\mathbf{r}^{\mathrm{PR}}\,A \longrightarrow \exists z\,\{\mathbf{T}(n,x,z) \wedge \mathbf{U}z\,\mathbf{r}^{\mathrm{PR}}\,B\}).$$

The (primitive recursive) formula $\mathrm{PR}(x)$, intuitively says that in the "program" of the recursive function with code x, there is no use of (unbounded) minimalization. So if $\mathbb{N} \models \mathrm{PR}(n)$ then φ_n is primitive recursive (but not necessary vice versa.)

One can prove the soundness of PR-realizability for **BA**, in the usual way. In the "usual" proof for **HA**, one can not restrict the partial recursives to a proper subclass, so it seems that Kleene's definition, for *HA*, can not be optimized in this way.

A generalized form of *Church's Thesis*, $\mathbf{GCT}^{\mathrm{PR}}$:

$$\forall x\,\big(\exists z\,(z\,\mathbf{r}^{\mathrm{PR}}\,A(x)) \rightarrow \exists y\,\phi(x,y)\big)$$
$$\Longrightarrow \exists u\,\big[\mathrm{PR}(u) \wedge \forall x\,\big(\exists z\,(z\,\mathbf{r}^{\mathrm{PR}}\,A(x)) \rightarrow \exists w\,\{\mathbf{T}(u,x,w) \wedge \phi(x,\mathbf{U}w)\}\big)\big]$$

is realizable in **BA**, and the completeness theorem of Troelstra [2], holds for $\mathbf{BA} + \mathbf{GCT}^{\mathrm{PR}}$ and $\mathbf{r}^{\mathrm{PR}}$:

$$\mathbf{BA} \vdash \exists x\,(x\,\mathbf{r}^{\mathrm{PR}}\,\psi) \iff \mathbf{BA} + \mathbf{GCT}^{\mathrm{PR}} \vdash \psi.$$

As applications, we deduce that **BA** enjoys the traditional constructive properties: Disjunction Property and Explicit Definability by Numbers, also each provably total function of **BA** is primitive recursive.

[1] WIM RUITENBURG, *Basic predicate calculus*, ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), no. 1, pp. 18–46.

[2] A. S. TROELSTRA, editor, ***Metamathematical investigations of intuitionistic arithmetic and analysis***, Springer-Verlag, Berlin, 1974.

► THOMAS SCANLON, *Groups definable in compact complex manifolds*.
University of California, Berkeley, Department of Mathematics, Evans Hall, Berkeley, California 94720-3840, USA.
E-mail: `scanlon@math.berkeley.edu`.

A compact complex manifold considered as a first-order structure with the closed analytic subvarieties of its Cartesian powers as the basic definable sets is a totally transcendental structure of finite Morley rank. We classify the groups interpretable in compact complex manifolds. Our main results are the following theorems.

THEOREM 1. *A complex Lie group G is definable in a compact complex manifold if and only if it has the structure of a meromorphic group in the sense of Fujiki: there is a compact complex manifold $\overline{G}$ into which G embeds as a Zariski dense subset and for which the group operation on G extends to meromorphic operations on $\overline{G}$ and the left and right regular actions of G on itself extend to holomorphic actions of G on $\overline{G}$.*

THEOREM 2. *If G is a strongly minimal group interpretable in a compact complex manifold, then G is definably isomorphic to a one-dimensional connected algebraic group or to a non-algebraic simple complex torus.*

THEOREM 3. *If G is a connected group interpretable in a compact complex manifold, then G has a definable normal subgroup $L \leq G$ which is definably isomorphic to a linear algebraic group and for which G/L is definably isomorphic to a complex torus.*

This is a report on joint work with Anand Pillay.

[1] A. FUJIKI, *On automorphism groups of compact Kähler manifolds*, ***Inventiones Mathematicae***, vol. 44 (1978), no. 3, pp. 225–258.

► PAVEL SCHREINER, *Fragment of logic of finite constant domains without disjunction and existential quantifier*.
Novosibirsk State Pedagogical University, Department of Mathematics, Novosibirsk, Russia.
E-mail: `paul@nspu.nsu.ru`.

It is known that the fragment of predicate intuitionistic logic in the language without the disjunction and existential quantifier is complete under Kripke frames with finite domains ([2]). On the other hand, D. Gabbay has proved [1] that the fragment of predicate intuitionistic logic without the disjunction and existential quantifier coincides with a similar fragment of logic of constant domains.

In this paper, we prove that it is impossible to combine of these two results, i.e., the fragment of logic of finite constant domains without the disjunction and existential quantifier does not coincide with a similar fragment of the logic of constant domains and with a similar fragment of the logic of finite domains.

Namely, we construct a formula which contains neither disjunction nor existential quantifier and is valid in all Kripke frames with finite constant domains but refutable in the logic of constant domains and in the logic of finite domains.

[1] D. GABBAY, ***Semantical investigations in Heyting's intuitionistic logic***, D. Reidel Publ. Co., Dordrecht, Holland, 1981.

[2] P. SCHREINER, *Fragment of predicate intuitionistic logic complete under Kripke frames with finite domains*, Abstract of Invited and Contributed Talks, Logic Colloquium, Utrecht, 1999, p. 55.

► PHILIP SCOWCROFT, *Easy proofs of some theorems of Ax-Kochen-Ershov*.
Department of Mathematics, Wesleyan University, Middletown, Connecticut 06459, USA.
E-mail: pscowcroft@wesleyan.edu.

Emulating Abraham Robinson's proof, via model completeness, that the theory of real-closed ordered fields is complete, one may transfer model completeness and completeness from the theories of the residue field and of the value group to the theory of a corresponding Henselian valued field. This approach applies in all the familiar cases: residue characteristic zero; mixed characteristic with finite ramification; algebraically maximal Kaplansky fields. Where Robinson uses real closure to verify that his test for model completeness applies, the present approach uses Henselization—or in the last case, algebraically maximal immediate algebraic extension—to show that Robinson's test applies to immediate extensions; then it applies to arbitrary extensions by a general model-theoretic argument using no valuation theory. A simple modification of the prime-model test then allows one to handle completeness as well as model completeness.

► MASAHIRO SHIOYA, *Partitioning pairs of uncountable sets*.
Institute of Mathematics, University of Tsukuba, Tsukuba, 305-8571, Japan.
E-mail: shioya@math.tsukuba.ac.jp.

Let κ be a regular cardinal and λ a cardinal greater than κ. Using the method of minimal walks, Todorčević gave a coloring of the pairs of $x \subset y$ from $[\lambda]^\kappa$ with λ colors such that every cofinal subset of $[\lambda]^\kappa$ should contain a pair for each color. We will show how to push up the number of colors to λ^+ in case $\kappa^{\mathrm{cf}\lambda} = \kappa$.

► VLADIMIR SOTIROV, *Leibnitz style arithmetization of monadic predicate calculus with equality*.
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 8, Acad. G. Bonchev Str., 1113 Sofia, Bulgaria.
E-mail: vlsot@math.bas.bg; sotirov@fmi.uni-sofia.bg.
Web site: http://www.math.bas.bg/~vlsot

Leibnitz's idea to translate syllogistic into arithmetic was realized in [2]. In this paper, Leibnitz style translation is extended on the whole monadic predicate calculus including equality. To build an *arithmetical* model, let $N > 1$ be an integer *without multiple factors*. Any predicate P_i is interpreted by arbitrary divisor of N (possibly 1 or N) denoted with $d(P_i)$, and any individual variable x_i is evaluated by $d(x_i)$, a *prime* divisor of N. Following the construction of a formula F, its *arithmetical statement* $\mathcal{AR}[F]$ corresponding to given evaluation will be obtained. Namely, for atomic formulas, $\mathcal{AR}[P_i(x_j)]$ is "$d(x_j)$ divides $d(P_i)$" and $\mathcal{AR}[x_i = x_j]$ is "$d(x_i) = d(x_j)$"; for a subformula G, $\mathcal{AR}[(\forall x)G]$ is "for any prime divisor d, $\mathcal{AR}_d^x[G]$" where $\mathcal{AR}_d^x$ differs from $\mathcal{AR}$ attaching d to x; finally, all propositional connectives are replaced with their non-formal analogues. If $\mathcal{AR}[F]$ is a true arithmetical sentence for any N under arbitrary evaluation, F is named *arithmetically true*.

THEOREM 1. *Any formula of the monadic predicate calculus with equality is a tautology if and only if it is arithmetically true.*

First of all, a predicate formula is a tautology *if and only if* its universal closure is a tautology. Because of the decidability of the monadic predicate calculus with equality, the domains may be supposed to be finite [1, Theorem 25.1]. Any such model can be transformed in an isomorphic arithmetical model: if all elements of the domain D are $a_1, \ldots, a_n$, take an integer $N = p_1 \cdot \cdots \cdot p_n$ where $p_1, \ldots, p_n$ are *different* prime numbers. If the interpretation of P_i is $\{a_{i_1}, \ldots, a_{i_k}\} \neq \emptyset$, let $d(P_i)$ be $p_{i_1} \cdot \cdots \cdot p_{i_k}$; if the subset is empty, $d(P_i)$ is 1. The evaluation of individual variables is obvious.

[1] G. Boolos and R. Jeffrey, ***Computability and logic***, 3rd edition, Cambridge University Press, 1989.

[2] V. Sotirov, *Arithmetizations of syllogistic à la Leibnitz*, ***Journal of Applied Non-Classical Logics***, vol. 9 (1999), no. 2–3, pp. 387–405.

► THOMAS STRAHM, *Bounded applicative theories*.
Institut für Informatik und angewandte Mathematik, University of Bern, Neubrückstr. 10, CH-3012 Bern, Switzerland.
E-mail: strahm@iam.unibe.ch.

Theories with self-application form the operational core of systems of explicit mathematics, which have been introduced by Feferman in [3]. In an applicative universe of discourse, all objects may be thought of as operations or rules, which can freely be applied to each other: self-application is meaningful, though not necessarily total. It has turned out that applicative theories provide a natural framework for studying notions of abstract computability, especially from a proof-theoretic perspective, cf. [4] for a recent survey.

This contribution is concerned with the study of (unramified) *bounded* applicative systems which have a strong relationship to classes of computational complexity. We provide new applicative theories whose provably total functions coincide with the functions computable in polynomial time, polynomial space, polynomial time *and* linear space, as well as linear space. Our theories can be seen as natural applicative analogues of systems of bounded arithmetic, cf. Buss' monograph [1]. We are also interested in higher type aspects of our systems; in particular, it is shown that Cook and Urquhart's system PV^{ω} (cf. [2]) is directly contained in a natural applicative theory of polynomial strength.

[1] S. R. Buss, ***Bounded arithmetic***, Bibliopolis, Napoli, 1986.

[2] S. A. Cook and A. Urquhart, *Functional interpretations of feasibly constructive arithmetic*, ***Annals of Pure and Applied Logic***, vol. 63 (1993), no. 2, pp. 103–200.

[3] S. Feferman, *A language and axioms for explicit mathematics*, in ***Algebra and logic*** (J. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer-Verlag, Berlin, 1975, pp. 87–139.

[4] G. Jäger, R. Kahle, and T. Strahm, *On applicative theories*, in ***Logic and foundations of mathematics*** (A. Cantini, E. Casari, and P. Minari, editors), Kluwer, 1999, pp. 83–92.

► THOMAS STUDER, *Impredicative overloading in explicit mathematics*.
Institut für Informatik und angewandte Mathematik, Universität Bern, Neubrückstr. 10, CH-3012 Bern, Switzerland.
E-mail: tstuder@iam.unibe.ch.

Castagna, Ghelli and Longo [1] provide a theoretical foundation for object-oriented programming via λ-calculi with overloading. However, their full calculus exposes a new form of impredicativity for which no denotational semantics was available.

In this talk we introduce the system OTN of explicit mathematics based on elementary separation, product, join and weak power types. In such theories types are represented by names, and these are first order values. Hence they can be used in computations which is the key to model overloading. We present a set-theoretical model for OTN, and we develop in OTN a theory of impredicative overloading. Together this yields a solution to the problem of impredicativity mentioned above.

The use of power types in explicit mathematics has always been doubted. Glass [2] showed that weak power types add nothing to the proof-theoretic strength of various systems of explicit mathematics without join, and Jäger [3] proved the inconsistency of strong power types with elementary comprehension. Our work also provides a first example of an application of power types in explicit mathematics.

[1] GIUSEPPE CASTAGNA, GIORGIO GHELLI, and GIUSEPPE LONGO, *A calculus for overloaded functions with subtyping*, ***Information and Computation***, vol. 117 (1995), no. 1, pp. 115–135.

[2] THOMAS GLASS, *On power set in explicit mathematics*, ***Journal of Symbolic Logic***, vol. 61 (1996), no. 2, pp. 468–489.

[3] GERHARD JÄGER, *Power types in explicit mathematics?*, ***Journal of Symbolic Logic***, vol. 62 (1997), no. 4, pp. 1142–1146.

► Z. TALASBAEVA, *Positive numberings of the families of* Σ_n^{-1} *sets*.
KazSNU, Almaty, Kazakhstan.

We refer to [3]–[6] for the basic notions of the computability theory and the theory of numberings. Let S be a family of the sets from the class Σ_n^{-1}, $n \in \omega$, of the hierarchy of Ershov [2].

A numbering $\alpha : \omega \to S$ is called Σ_n^{-1}-*computable* if

$$\{ (x, m) \mid x \in \alpha m \} \in \Sigma_n^{-1}.$$

This definition is a slight modification of the notion of Σ_n^0-computable numbering given in [4].

A numbering α is called *positive*, [5], if

$$\{ (n, m) \mid \alpha n = \alpha m \} \in \Sigma_1^0.$$

The conjecture on existence of any positive Σ_n^{-1}-commputable numbering of the class of the all Σ_n^{-1} sets, $n \geq 2$, was formulated in [1]. This conjecture was proved for second level of the hierarchy of Ershov. $(n = 2.)$

THEOREM ([7]). *The class* Σ_2^{-1} *has a positive* Σ_2^{-1}-*computable numbering.*

The following theorem is a more essential result.

THEOREM. *For every* $n \geq 2$ *the class* Σ_n^{-1} *sets has an infinite number of nonequivalent positive* Σ_n^{-1}-*computable numberings.*

[1] S. A. BADAEV and S. S. GONCHAROV, *Theory of numbering*: *Open problems*, in ***Procceeding of the applications, USA, Boulder, June 13–17***, to appear.

[2] U. L. ERSHOV, *About one hierarchy of sets*, ***Algebra and logic***, vol. 7 (1968), no. 1, pp. 65–71.

[3] ———, ***Theory of numberings***, Moskow, Science, 1977.

[4] S. S. GONCHAROV and A. SORBI, *Factorally-computable numberings and nontrivial semilattics of Rodjers*, ***Algebra and logic***, vol. 36 (1997), no. 6, pp. 621–641.

[5] A. I. MALCEV, ***Algorithms and recursive functions***, Moskow, Science, 1965.

[6] H. ROGERS, ***Theory of recursive functions and effective computability***, Moskow, World, 1972.

[7] Z. T. TALASBAEVA, *The positive computability of the classes of hierarchy of Ershov*, Almaty, Vestnik, KazSU, to appear in 2000.

► MAKOTO TATSUTA AND SACHIO HIROKAWA, *Long D-normal form yields uniqueness of proofs*.
Department of Mathematics, Kyoto University, Kyoto 606-8502, Japan.
E-mail: tatsuta@kusm.kyoto-u.ac.jp.
Computing and Communications Center, Kyushu University, Fukuoka 812-8581, Japan.
E-mail: hirokawa@cc.kyushu-u.ac.jp.

There have been much interest in the conditions for uniqueness of normal form proofs in intuitionistic logic. [3] introduced a notion of D-normal form proof and showed that D-normal form proof is unique for any implicational formula. But this result means the uniqueness of D-normal form proof. There may be some other normal form proof which is not D-normal. We show that D-normality yields the uniqueness of proofs as long as the long-normal form proofs are concerned.

For a formula, core(A) is the rightmost variable in A. A normal form proof is D-normal if core$(A) \neq$ core(B) for any initial sequent $\Gamma, A, B \vdash A$.

THEOREM. *If a formula A has a long D-normal form proof, then long normal form of A is unique.*

[1] S. HIROKAWA, *Principal types of BCK-λ-terms*, ***Theoretical Computer Science***, vol. 107 (1993), pp. 253–276.

[2] M. TATSUTA, *Uniqueness of normal form proofs of minimal formulas*, ***Journal of Symbolic Logic***, vol. 58 (1993), no. 3, pp. 789–799.

[3] ———, *Uniqueness of D-normal proofs*, Asian Logic Conference, June 7–10, 1999.

► KATRIN TENT, *BN-pairs of finite Morley rank*.
Mathematisches Institut, Universität Würzburg, 97074 Würzburg, Germany.
E-mail: tent@mathematik.uni-wuerzburg.de.

We report on recent progress on the Cherlin-Zil'ber Conjecture, viz. that a simple group of finite Morley rank is an algebraic group over an algebraically closed field. The conjecture implies in particular that any such group must have a definable 'split' BN-pair. The following is a partial analog of the famous paper by Fong and Seitz on finite groups with split BN-pairs:

THEOREM ([1]). *Let G be a simple group with a definable BN-pair of rank* 2 *where $B = U.T$ for $T = B \cap N$ and a normal subgroup U of B with $Z(U) \neq 1$. It was shown in* [3] *that the Weyl group $W = N/B \cap N$ has cardinality* $2n$ *with* $n = 3, 4, 6, 8$ *or* 12. *If G has finite Morley rank then furthermore the following holds*:

(i) *If $n = 3$, then G is definably isomorphic to* $\mathrm{PSL}_3(K)$ *for some algebraically closed field K.*
(ii) *If U is nilpotent, then $n \neq 12$, and for $n = 3, 4$, or 6 the Cherlin-Zil'ber Conjecture holds.*
(iii) *If $Z(U)$ contains a B-minimal subgroup A with* $\mathrm{RM}(A) \geq \mathrm{RM}(P_i/B)$ *for both parabolic subgroups P_1 and P_2, then $n = 3, 4$ or 6 and G is definably isomorphic to* $\mathrm{PSL}_3(K)$, $\mathrm{PSp}_4(K)$ *or $G_2(K)$ for some algebraically closed field K.*

Note that the conditions in (iii) are necessarily satisfied if the Cherlin-Zil'ber Conjecture is true.

[1] K. TENT, *Split BN-pairs of finite Morley rank*, preprint.

[2] K. TENT and H. VAN MALDEGHEM, *Cherlin's conjecture for buildings with panels of Morley rank* 1, preprint.

[3] ———, *On irreducible (B, N)-pairs of rank* 2, preprint.

► DMITRY TISHKOVSKY, *On algebraic counterparts of some properties of superintuitionistic predicate and intuitionistic polymodal logics*.
Institute Mathematics of Siberian Branch of Russian Academy of Sciences, ul. Arbuzova 5 kv. 150, Novosibirsk 630117, Russia.
E-mail: tishkov@math.nsc.ru.

It was shown in [2] that for each superintuitionistic predicate logic L there exists a intuitionistic polymodal propositional logic ML such that every polymodal formula of ML is a metatheorem of the logic L. Using this fact it is possible to construct algebraic semantics for the class of superintuitionistic first-order logics [2].

In the paper we discuss derivation interpolation property (IPD), implicative interpolation property (IP), ordinary Beth property (B2) and projective Beth property (PB2) [1] of the logics mentioned above.

Algebraic counterparts of these properties were found for superintuitionistic first-order logics and for intuitionistic polymodal logics and special equivalence theorems were proved for these logics.

A corollary of the theorems is that if a polymodal propositional logic ML corresponds to some superintuitionistic predicate logic L in above sense then possession by ML of property IPD (IP, B2, PB2) implies possession by L of property of the same name.

[1] L. MAKSIMOVA, *Explicit and implicit definability in modal and related logics*, ***Bulletin of the Section of Logic***, vol. 27 (1998), no. 1/2, pp. 36–39.

[2] D. E. TISHKOVSKY, *On algebraic semantics for superintuitionistic predicate logics*, ***Algebra i logika*** (in Russian), vol. 38 (1999), no. 1, pp. 68–95.

► CARLO TOFFALORI, *On the Ziegler spectrum of a group ring*.
Dipartimento di Matematica e Fisica, Università di Camerino, Via Madonna delle Carceri, 62032 Camerino, Italy.
E-mail: toffalori@campus.unicam.it.

This is an abstract of joint work with Mike Prest (Manchester) and Annalisa Marcja (Florence) [1].

Let R be a Dedekind domain, G be a finite group. We consider R-torsionfree modules over the group ring RG and we study their Ziegler topology. First we show that, for every R-torsionfree N in the Ziegler space ZG_{RG}, either N is injective and simple, or there is some maximal ideal P of R such that N is a reduced point in $Zg_{R_P}G$ where R_P denotes the completion of the localization of R at P. In the latter case, the induced topology over RG or over R_PG is the same. So we examine R_PG for a given P. We see that a reduced torsionfree N in Zg_{R_PG} is in the closure of finitely generated points (i.e., R_PG-lattices) and each indecomposable R_PG-lattice is "almost" isolated and closed. We analyse the case when R_PG-lattices are a classe of finite representation type, in particular when the order of G is not in P. Finally we discuss the role of RG-lattices within Zg_{RG}.

[1] A. MARCJA, M. PREST, and C. TOFFALORI, *The torsionfree part of the Ziegler spectrum of RG when R is a Dedekind domain and G is a finite group*, preprint.

► VLADIMIR TOLSTYKH, *On the automorphism groups of free two-step nilpotent groups*.
Department of Mathematics, Kemerovo State University, Krasnaja, 6, 650043 Kemerovo, Russia.
E-mail: vlad@corvette.kuzb-fin.ru.

Our main result is that the elementary theory of the group $\mathrm{Aut}(N_\varkappa)$, where $N_\varkappa$ is a free two-step nilpotent group of infinite rank $\varkappa$, interprets 'set theory', or, more precisely, the full second-order theory of the set $\varkappa$ (with no relations). The research is concerned with a general question whether set theory is interpretable in the automorphism groups of free algebras of (possibly 'large') infinite rank from a given variety [1]. The answer is positive, for example, for any variety of vector spaces over a fixed division ring [2] and for the variety of all groups [3].

The main steps in the interpretation of set theory in the elementary theory of $\mathrm{Aut}(N_\varkappa)$ are as follows. Suppose that N is an arbitrary non-abelian free two-step nilpotent group. It is first proved the subgroup $\mathrm{IA}(N)$ of all IA-automorphisms of N, which induce the trivial automorphism on the abelianization $A = N/[N, N]$ of N, is definable in $\mathrm{Aut}(N)$. The next is a proof of definability of conjugations by powers of primitive elements in $\mathrm{Aut}(N)$, and then the subgroup $\mathrm{Inn}(N)$ of all conjugations. The structure $\langle \mathrm{Aut}(A), A\rangle$, that is the automorphism group of A with the action on the elements of A, is therefore interpretable in $\mathrm{Aut}(N)$. In the case when N is infinitely generated one can interpret in $\langle \mathrm{Aut}(A), A\rangle$ the endomorphism semi-group of A; it is known that this semi-group interprets set theory [1].

Let A_2 be a two-generator free abelian group. Since the ring of integers $\mathbf{Z}$ is interpretable with parameters in $\langle \mathrm{Aut}(A_2), A_2 \rangle$, the elementary theory of the automorphism group of any non-abelian free two-step nilpotent group is unstable and undecidable. The fact that the ring of integers $\mathbf{Z}$ can be reconstructed in the automorphism group $\mathrm{Aut}(N_2)$ of a two-generator free two-step nilpotent group N_2 seems to be quite interesting, because $\mathrm{Aut}(N_2)$ is 'close' in some sense to the group $\mathrm{GL}(2, \mathbf{Z})$, which is isomorphic to the automorphism group of a two-generator free abelian group.

[1] S. Shelah, *Interpreting set theory in the endomorphism semi-group of a free algebra or in a category*, ***Annales Scientifiques de L'universite de Clermont***, fasc. 13 (1976), pp. 1–29.

[2] V. Tolstykh, *Elementary equivalence of infinite-dimensional classical groups*, ***Annals of Pure and Applied Logic***, to appear.

[3] ———, *Set theory is interpretable in the automorphism group of an infinitely generated free group*, ***Journal of the London Mathematical Society***, to appear.

► LORENZO TORTORA DE FALCO, *Coherent obsessional experiments for linear logic proof-nets*.
Équipe Preuves Programmes, Systèmes [EP 2025], Université Paris VII, 2 place Jussieu, 75251 Paris Cedex 05, France.
E-mail: `tortora@logique.jussieu.fr`.

This talk is a contribution to the research field of proof-theory initiated in [1] with the discovery of linear logic (LL) and (1) its syntax: proof-nets (2) its underlying notion of computation: cut-elimination for proof-nets (3) the equivalence relation on proof-nets induced by the coherent semantics.

The cut-elimination procedure for proof-nets induces a (syntactical) equivalence relation on proof-nets, which can be roughly defined by: R and R' are syntactically equivalent if and only if they have the same normal form.

Denotational semantics gives an "extensional" description of proofs as functions, which is invariant with respect to cut-elimination: if π' is obtained from π by applying some steps of cut-elimination, then π and π' are interpreted by the same function. Every semantics induces then an equivalence relation on proofs, which can be roughly defined by: π and π' are semantically equivalent if and only if they have the same interpretation.

Clearly, two syntactically equivalent proof-nets are also semantically equivalent. In [2], we try to "measure" the quality of the representation of proofs as proof-nets by asking whether the converse holds for coherent semantics (which gave birth to LL): in case of positive answer we say that the semantics is injective.

Starting from the definition of experiment of [1], we introduce the key-notion of "injective obsessional experiment", which allows to give a positive answer to our question for certain fragments of LL, and to build counter-examples to the injectivity of coherent semantics in the general case.

[1] J. Y. Girard, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–101.

[2] L. Tortora de Falco, *Réseaux, cohérence et expériences obsessionnelles*, ***Ph. D. thesis***, University Paris 7, January 2000.

► SERGEI TUPAILO, *Realization of intuitionistic KPM in explicit mathematics*.
Institut für Informatik und angewandte Mathematik, Universität Bern, Neubrückstrasse 10, CH 3012 Bern, Switzerland.
E-mail: `sergei@iam.unibe.ch`.

We continue the study of realizability of different intuitionistic theories in Feferman's Explicit Mathematics began in [1] and [2]. It's easily seen that intuitionistic Kripke-Platek Set Theory **IKP** is a subsystem of Aczel's Constructive Set Theory **CZF**, so that embedding of **IKP** into $\mathbf{T}_0$ is already contained in [2]. However, analogous relations don't obviously hold for much stronger theories **IKPi** and **IKPM**, which are intuitionistic versions of familiar

KPi and **KPM**, respectively. Nevertheless, we can modify our realizability interpretation to take care of these theories as well.

The key question is how to realize admissibility. If we take it to be expressed by a new predicate symbol *Ad*, then we define $\mathfrak{r}\ \underline{\mathrm{rn}}\ \mathrm{Ad}(\alpha) \equiv \mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ \mathrm{Ad}(\alpha)$ as

$$\begin{aligned}
&\mathsf{p}_{000}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ \mathrm{Tran}[\alpha] \wedge \mathsf{p}_{001}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ \mathrm{Pair}^{\alpha} \wedge \mathsf{p}_{010}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ \mathrm{Union}^{\alpha} \wedge \mathsf{p}_{011}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ \mathrm{Infinity}^{\alpha} \\
&\qquad \wedge \forall x \,\varepsilon\, \alpha\, \forall \mathfrak{x}\, (\mathsf{ln}x = 1 \rightarrow \mathsf{p}_{100}\mathfrak{r}x\mathfrak{x}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ (s[\mathsf{str}(\alpha, x), \mathfrak{x}] \in \alpha)) \\
&\qquad \wedge \forall x \,\varepsilon\, \alpha\, \forall \mathfrak{x}\, (\mathsf{ln}x = 1 \wedge \forall y \,\varepsilon\, \mathsf{str}(\alpha, x)\, (\mathsf{ln}y = 1 \rightarrow \mathfrak{x}y \,\varepsilon\, \alpha \wedge \mathsf{ln}(\mathfrak{x}y) = 1) \\
&\qquad\qquad \rightarrow \mathsf{p}_{101}\mathfrak{r}x\mathfrak{x}\ \underline{\mathrm{rn}}^{\mathcal{E}}\ (sc[\mathsf{str}(\alpha, x), \mathfrak{x}] \in \alpha)),
\end{aligned}$$

where $\mathsf{p}_{\mathsf{klm}}\mathfrak{r}$ is $\mathsf{p}_{\mathsf{k}}(\mathsf{p}_{\mathsf{l}}(\mathsf{p}_{\mathsf{m}}\mathfrak{r}))$, $\mathsf{k}, \mathsf{l}, \mathsf{m} = 0, 1$,

$$\begin{aligned}
s[\beta, \mathfrak{x}] &:\doteq \{\, c \,\varepsilon\, \mathsf{seq} \mid \mathsf{ln}c = 0 \vee (\mathsf{ln}c \neq 0 \wedge c = \mathsf{sm}(\mathsf{sm}_0 c, \mathfrak{x}) \wedge \mathsf{sm}_0 c \,\varepsilon\, \beta) \,\}, \\
sc[\beta, \mathfrak{x}] &:\doteq \{\, c \,\varepsilon\, \mathsf{seq} \mid \mathsf{ln}c = 0 \vee (\mathsf{ln}c \neq 0 \wedge \mathsf{head}c \,\varepsilon\, \beta \wedge \mathsf{bodyt}c \,\varepsilon\, \mathsf{j}(\bar{\beta}, \lambda y.\mathsf{str}(\alpha, \mathfrak{x}y))) \,\}, \\
\bar{\beta} &:\doteq \{\, y \,\varepsilon\, \beta \mid \mathsf{ln}y = 1 \,\},
\end{aligned}$$

and the rest of definitions are the same as in [2]. With these definitions *Ad*-axioms are realized and all corresponding theorems of [2] still hold, including

THEOREM 4. **IKPM** *is realizable in* $\mathbf{T_m}$.

[1] S. TUPAILO, *Realization of analysis into explicit mathematics*, ***Technical Report IAM-00-003***, University of Bern, Switzerland; submitted for publication, 1999.

[2] ———, *Realization of constructive set theory into explicit mathematics*: *a lower bound for impredicative Mahlo universe*, ***Technical Report IAM-00-004***, University of Bern, Switzerland; submitted for publication, 2000.

► JOUKO VÄÄNÄNEN AND BOBAN VELIČKOVIĆ, *Winning strategies and σ-closed sets of partial isomorphisms*.

Department of Mathematics, University of Helsinki, Helsinki, Finland.
E-mail: `jvaanane@cc.helsinki.fi`.
UFR de Mathèmatiques, Universitè de Paris 7, Paris, France.

Let I_κ be the set of all partial isomorphisms of cardinality $< \kappa$ between two first order structures **A** and **B** of the same vocabulary. $X \subseteq I$ has the κ-back-and-forth property if for all $\lambda < \kappa$ and for all $p \in X$: $\forall a \in {}^{\lambda}A\ \exists b \in {}^{\lambda}B\ (p \cup \{\langle a(i), b(i)\rangle : i < \lambda\} \in X)$ and $\forall b \in {}^{\lambda}B\ \exists a \in {}^{\lambda}A\ (p \cup \{\langle a(i), b(i)\rangle : i < \lambda\} \in X)$. X is σ-closed, if whenever $p_0 \subseteq p_1 \subseteq \dots$ are in X, also $\bigcup_{n<\omega} p_n \in X$. Let $I^*_{\aleph_1}$ be the largest $X \subseteq I$ with the $\aleph_1$-back-and-forth property. If $I^*_{\aleph_1} \neq \emptyset$, then $\mathbf{A} \equiv_{\infty\aleph_1} \mathbf{B}$. The following theorem is a special case of a more general result:

THEOREM 1. *The following conditions are equivalent*:

(1) *The Nonempty Player has a winning strategy in the Banach-Mazur game on* $\langle I^*_{\aleph_1}, \subseteq\rangle$.

(2) $I^*_{\aleph_1}$ *contains a σ-closed dense subset*.

Another measure of the largeness of I^*_λ is the Ehrenfeucht-Fraïssé-game.

THEOREM 2. *Suppose* **A** *and* **B** *have cardinality* $\leq \lambda = \kappa^+$. *Then the following conditions are equivalent*:

(3) *Player II has a winning strategy in the Ehrenfeucht-Fraïssé-game of length* ω_1 *between* **A** *and* **B**, *players playing* $\leq \kappa$ *elements at a time.*

(4) I^*_λ *has a σ-closed subset with the λ-back-and-forth property.*

The condition (4) was introduced by Dickmann in 1975. It implies isomorphism among structures of cardinality $\leq \aleph_1$. It is an open problem, whether (4) is transitive for all **A** and **B**.

COROLLARY 3. (4) *is transitive among structures of cardinality* $\leq \lambda = \kappa^+$.

► VIKTOR V. VERBOVSKIY, *On unary functions definable in coset-minimal ordered groups*.
Institute of Control and Informatics Problems, Ministry of Education and Science, 480100, ul. Pushkina 125, Almaty, Kazakhstan.
E-mail: vvver@mail.ru.

A totally ordered group G is *coset-minimal* [1] if every definable subset of G is a finite union of cosets of definable subgroups intersected with intervals with endpoints in $G \cup \{-\infty, +\infty\}$. We study definable unary functions in coset-minimal ordered groups. Main results: Every unary definable function in a discrete coset-minimal ordered group is piecewise linear. Every unary definable function in a dense coset-minimal ordered group is piecewise monotone. Let G be dense and $\mathcal{F}$ the family of all definable unary functions $f : G_1 \to G_1$, where $G_1 \leq G$ is convex definable and o-minimal and G/G_1 is discrete. Then any G-definable unary function is definable in the reduct to the language $\{<, +, \mathcal{F}\}$. Dense coset-minimal ordered groups have the Exchange Property.

Definition 1 ([1]). Let f be a definable function from a group G to a group H. Then f is *piecewise linear* if there is a finite partition of G into definable sets $\{ X_i : i \in I \}$, elements $g_i \in G$ and $h_i \in H$ and homomorphisms $\sigma_i : \langle X_i - g_i \rangle \to H$ for all $i \in I$, such that $f(x + g_i) - h_i$ is the restriction of σ_i to $X_i - g_i$ for all $i \in I$.

[1] F. Point and F. Wagner, *Essentially periodic ordered groups*, ***Communications in Algebra***, to appear.

► MAXIM VSEMIRNOV, *Undecidable weak theories of polynomial rings*.
Steklov Institute of Mathematics at St. Petersburg, 27 Fontanka, St. Petersburg, 191011, Russia.
E-mail: vsemir@pdmi.ras.ru.

We show that the theory $\mathrm{Th}(\mathbf{F}_q[X_1, \dots, X_n], S_1, S_{X_1}, \perp)$ is undecidable. Here $\mathbf{F}_q$ is a finite field of q elements, S_1 and S_{X_1} are the natural analogues of the successor function $(S_1(f) = f + 1,\ S_{X_1}(f) = f + X_1)$ and $\perp$ is a coprimeness predicate. The proof uses the truth of the analogues of the Erdös-Woods conjecture for polynomial rings over a finite field, which was obtained in [1].

[1] M. A. Vsemirnov, *The Woods-Erdös conjecture for polynomial rings*, ***Annals of Pure and Applied Logic***, to appear.

► ALEX WILKIE, *Liouville functions*.
Mathematical Institute, University of Oxford, 24-29 St Giles, Oxford OX1 3LB, United Kingdom.
E-mail: wilkie@maths.ox.ac.uk.

A Liouville *number* is an irrational real number that can be well approximated by rational numbers to an accuracy that is just sufficient for Liouville's theorem (on the lack of good rational approximation to algebraic numbers) to imply that it must be transcendental. By analogy, I define a Liouville *function* to be an entire, non-polynomial function that can be very well approximated by polynomials with rational coefficients. The precise degree of accuracy is defined in terms of a growth condition on the heights of the (rational) Taylor coefficients of the function, and is just sufficient to allow application of effective quantifier elimination (for the complex field) and of effective versions of Hilbert's Nullstellensatz in order to show that no hidden relationships hold between values of terms built up from the function using algebraic operations (the 'Schanuel property').On the other hand, all "generic" sets of terms (with integer coefficients)do have zeros. This allows us to apply recent results of Zil'ber, based on Hrushovski's amalgamation method, to conclude that certain substructures of the expansion of the complex field by a Liouville function have nice model-theoretic properties, e.g., omega-stability and almost model completeness.

► DAN E. WILLARD, *Extending the cut-free version of the second incompleteness theorem almost to Robinson's arithmetic Q*.
SUNY at Albany, Computer Science Department, Albany, New York 12222, USA.
E-mail: dew@cs.albany.edu.

Let us recall that Robinson's Arithmetic system Q differs from Peano Arithmetic by containing no Induction axioms [3]. In 1985, Pudlák strengthened a theorem by Bezboruah and Shepherdson [2] to show that the *cut-permissive version* of the Second Incompleteness (that discussed the capacities of the Hilbert-style deduction calculus) generalized from Peano Arithmetic to *all possible extensions* of Q.

The main gap left by the prior literature was it had not resolved to what extent the Second Incompleteness Theorem was valid for weak systems with respect to cut-free deductive methods. In particular, Adamowicz [1]) had shown that $I\Sigma_0 + \Omega_2$ satisfied the Second Incompleteness property for semantic tableaux deduction, but it was not known how much closer to Robinson's Arithmetic Q this result would extend.

We have obtained a partial (albeit not complete) answer to this question. Let d denote any cut-free method of deduction, such as for instance the Herbrand methodology, Semantic Tableaux, Resolution or the Cut-Free Sequent Calculus. For each such d, we have proven there exists a corresponding Π_1 sentence, called V_d, such that if α is any consistent extension of $Q + V_d$ then α cannot prove that the deduction-method d applied to the axiom set α is assured not to produce a proof of $0 = 1$.

Our prior research [5, 6, 7] had documented the existence of some exceptions to the cut-free version of the Second Incompleteness Theorem when an axiom system had a capacity to prove only Addition is a total function (and it viewed Multiplication instead as a 3-way relation). The combination of this prior negative result and the new positive results, reported in the preceding paragraph, tightly complement each other.

[1] Z. ADAMOWICZ, *On the tableaux consistency in weak theories*, circulating manuscript from the Mathematics Institute of the Polish Academy of Sciences, 1999.

[2] A. BEZBORUAH and J. SHEPHERDSON, *Gödel's second incompleteness theorem for Q*, ***Journal of Symbolic Logic***, vol. 41 (1976), pp. 503–512.

[3] E. MENDELSON, ***Introduction to math logic***, Wadsworth-Brooks-Cole Math Series, 1987. See p. 157 for Q's definition.

[4] P. PUDLÁK, *Cuts consistency statements and interpretations*, ***Journal of Symbolic Logic***, vol. 50 (1985), pp. 423–442.

[5] D. WILLARD, *Self-verifying axiom systems*, in ***Third Kurt Gödel symposium***, Lecture Notes in Computer Science, vol. 713, Springer-Verlag, 1993, pp. 325–336.

[6] ———, *The tangibility reflection principle*, in ***Fifth Kurt Gödel symposium***, Lecture Notes in Computer Science, vol. 1289, Springer-Verlag, 1997, pp. 319–334.

[7] ———, *Self-verifying systems, the incompleteness theorem and tangibility reflection principle*, ***Journal of Symbolic Logic***, to appear.

► URSZULA WYBRANIEC-SKARDOWSKA, *Three principles of compositionality*.
University of Opole, Matejki 5/21, 45055 Opole, Poland.
E-mail: uws@uni.opole.pl.

The subject matter of consideration touches the principle of compositionality in a formulation close to the following ([2]):

(C) *The meaning of the compound expression of language is a function of the meanings of its constituents and of the syntactic rules by means of which it is formed.*

Some historical remarks about this principle, its role in philosophy of language, and adherence to cognitive science will be given at the beginning. Three different aspects of compositionality, representing cognitively independent factors are taken into account in three different formulations of the principles. They are: syntactic, semantic, and pragmatic factors constituting language as a tool of communication.

Language is characterized as categorial, i.e., generated by a categorial grammar. Every compound expression of such a language has a functor-argument form and both it and its constituents (main functor and its arguments) have determined: (1) the *syntactic category* (*syntactic form*), (2) *intension* (*meaning*, *connotation*), (3) *extension* (*denotation*).

Considerations are formalized on the ground of the author's general formal-logical theory TCG of categorial syntax ([3]) and categorial semantics ([4]). Categorial language L can be treated as a partial algebra A_L with a carrier as the set of all *wfes* of language L, and functors as partial functions (operators) on this set. *Conceptual* (*intensional*) *model* of L and *semantic* (*referential*, *extensional*) model of L are partial algebras too; they correspond to A_L and are similar to A_L. They are given as homomorphic images of A_L determined by *intension* and, respectively, *extension* understood as functions defined on *wfes* of L. Such an approach is possible by adopting the following scheme of *compositionality* (C_h) for the function h representing:

(1) *syntactic form*;
(2) *intension*;
(3) *extension*:

If $f(e_1, e_2, \ldots, e_n)$ is a functor-argument expression of L with main functor f and its arguments $e_1, e_2, \ldots, e_n$, then

$$(C_h) \qquad h(f(e_1, e_2, \ldots, e_n)) = h(f)(h(e_1), h(e_2), \ldots, h(e_n)).$$

Within the author's theory can be formulated the syntactic notion of *truth* (corresponding to Tarski's T-scheme) and the semantic notion employing the notion of *meaning* (*intension*) and *denotation* (*extension*). By means of *compositionality* (C_h) the author gives some theorems of TCG (1999) of the adequacy of syntax with respect to semantics (both categorial and compositional) and refers to the *true-value principle* (cf. [1]) proving:

THEOREM. *Two expressions of L have the same interpretation* (*intension*, *resp. extension*) *if and only if replacing one of them by another in a sentence of L never alters the true-value of the sentence in* (*intensional*, *resp. extensional*) *model of L*.

[1] W. HODGES, *Compositional semantics for language of imperfect information*, ***Logic Journal of the IGPL***, vol. 5, (1996), no. 4, pp. 539–563.

[2] T. M. V. JANSEN, *Compositionality*, in ***Handbook of logic and language*** (J. van Benthem and A. ter Muelen, editors), Elsevier, 1997.

[3] U. WYBRANIEC-SKARDOWSKA, ***Theory of language syntax***, Kluwer, Dordrecht, Boston, and London, 1991.

[4] ———, *Logical and philosophical ideas in certain approaches to language*, ***Synthese***, vol. 116 (1998), no. 2, pp. 231–277.

► TATIANA YAVORSKAYA (SIDON), *Logic of proof and storage predicates*.
Department of Mathematical Logic, Faculty of Mechanics and Mathematics, Moscow State University, 119899, Moscow, Russia.
E-mail: `tanya@lpcs.math.msu.ru`.

Propositional logic of proofs was introduced and studed by S. Artemov in [1]. He incorporated the proof predicate "*t is a proof of F*" denoted by $t : F$ in the propositional language and found the complete axiom system for the logic of proofs $\mathcal{LP}$. Proofs are represented by proof terms constructed by three elementary operations on proofs that realize application, (positive) proof checking and nondeterministic choice. Artemov proved that $\mathcal{LP}$ is sufficient to realize the modality in the whole Gödel's provability logic $S4$ (see [2]) and thus provided $S4$ with the intended provability reading.

We describe logic of proofs extended by the additional storage predicate $[t]F$ with the informal reading "t *knows about* F". In the intended semantics t is a reference for a text containing the information (knowledge) F. Some of the texts represent proofs. This fact is denoted by $t : F$. Proofs contain only valid information that is reflected via the reflexivity principle $t : F \wedge [t]G \to G$. The presence of the storage predicate allows us to specify operations on proofs depending on the formula arguments. For example, negative proof checking that verifies whether t is a proof of F and if no then returnes the proof of $\neg(t : F)$ depends on both t and F. In our language we can replace F by the reference for $t : F$

$$[a](t : F) \wedge \neg(t : F) \to \mathsf{check}(a) : (\neg t : F).$$

We present the complete axiom system for the logic of proof and storage predicates $\mathcal{LPS}$. The obtained system is sufficient to realize $S4$ in such a way that the constructed realization makes sence for the arbitrary proof predicate.

[1] S. Artemov, *Operational modal logic*, ***Technical Report 95-29***, MSI, Cornell University, 1995.

[2] K. Gödel, *Eine Interpretation des intuitionistischen Aussagenkalkuls*, ***Ergebnisse Math. Kolloq.***, vol. 4 (1933), pp. 39–40.

► ROSTISLAV YAVORSKY, *Provability logics with quantifiers on proofs*.
Steklov Mathematical Institute, Moscow, Gubkina, 8, 117966, Russia.
E-mail: `rey@mi.ras.ru`.

The language qLP of the provability logic with quantifiers on proofs contains propositional letters, proof variables, boolean connectives, quantifiers over the proof variables and binary proof operator $x : F$ ("x is a proof of the formula F") introduced by S. N. Artemov in [1].

This language naturally extends the language of the logic of proofs considered in [1]. The operational properties of provability studied in [2] could be expressed in this language by the corresponding quantified sentences:

$$\forall x\, \forall y\, \exists z\, (x : (A \to B) \to (y : A \to z : B)),$$
$$\forall x\, \exists y\, (x : A \to y : (x : A)), \text{ etc.}$$

Also, this language may be considered as an extension of the propositional modal language, since the modal provability operator $\Box A$ could be expressed by the formula $\exists x\, (x : A)$.

An arithmetical interpretation $*$ assigns arithmetical sentences to propositional variables, commutes with boolean connectives and quantifiers and $(x : A)^* = \mathrm{Prf}(x, \ulcorner A^* \urcorner)$. (We suppose that the set of proof variables coincides with the set of individual variables in arithmetic.)

Given an arithmetical theory T and a class of proof predicates K we define the logic $qLP_K(T)$ as the set of all formulas in the language qLP, which are provable in T under every arithmetical interpretation $*$ based on proof predicates from K.

We present recent results concerning the decidability and arithmetical complexity of the corresponding logics for different T and K. It turned out that for many natural classes K the logic $qLP_K(PA)$ is Π^0_2-hard. For the case of the Gödel proof predicate decidable fragments were found.

[1] S. Artemov, *Logic of proofs*, ***Annals of Pure and Applied Logic***, vol. 67 (1994), pp. 29–59.

[2] ———, *Explicit provability: the intended semantics for intuitionistic and modal logic*, ***Technical Report 98-10***, MSI, Cornell University, 1998.

► VLADIMIR A. ZAKHAROV, *On the approximation relation on dynamic logic models*.
Faculty of Computational Mathematics and Cybernetics, Moscow State University, Moscow,

RU-119899, Russia.
E-mail: zakh@cs.msu.su.

We deal with the equivalence problem for deterministic propositional sequential programs whose semantics is defined by means of dynamic logic models. Let $\mathcal{A}$ and $\mathcal{P}$ be the alphabets of basic actions and propositional variables. A sequential program π over $\mathcal{A}$ and $\mathcal{P}$ is a finite transition system whose nodes are associated with basic actions and transitions are labelled by formulae built of propositional variables by means of connectives $\neg$ and $\wedge$. Dynamic logic models over $\mathcal{A}$ and $\mathcal{P}$ are defined as in [1]. We assume that some initial state s_0 is distinguished in every model σ. The run $\pi(\sigma)$ of a program π on a model σ is defined as usual (see [2]). If $\pi(\sigma)$ terminates and reaches some state s in σ then the result $[\pi(\sigma)]$ of this run is s; otherwise $[\pi(\sigma)]$ is undefined. By a *semantics* we mean any set M of dynamic logic models. If all models in M are based on the same frame $\mathcal{F}$ then M is called a *homogeneous* semantics. Programs π_1 and π_2 are called *equivalent* on M ($\pi_1 \sim_M \pi_2$ in symbols) if and only if $[\pi_1(\sigma)] = [\pi_2(\sigma)]$ holds for each model σ in M. We say that a semantics M_1 *approximates* semantics M_2 ($M_2 \preceq M_1$) if and only if $\pi_1 \sim_{M_1} \pi_2$ implies $\pi_1 \sim_{M_2} \pi_2$ for every pair of programs π_1, π_2. If both $M_2 \preceq M_1$ and $M_1 \preceq M_2$ hold then we write $M_1 \approx M_2$. A semantics M is called *saturated* if and only if $M \not\approx M \cup \{\sigma\}$ holds for every model σ which is not in M. A homogeneous semantics M based on a frame $\mathcal{F}$ is called *homogeneously saturated* if and only if $M \not\approx M \cup \{\sigma\}$ holds for every model σ based on $\mathcal{F}$ which is not in M. We study the lattice-theoretic properties of semantics with respect to approximation relation $\preceq$. Denote by $AP(M_1, M_2)$ the following assertion: each model σ_2 in M_2 is a homomorphic image of some model σ_1 in M_1. Clearly, $AP(M_1, M_2)$ implies $M_2 \preceq M_1$ for arbitrary semantics.

Proposition 3. *Suppose M_1 and M_2 are saturated semantics. Then $M_2 \preceq M_1$ if and only if $AP(M_1, M_2)$ holds.*

Corollary 1. *The set of saturated semantics is a complete lattice with respect to $\preceq$.*

Proposition 4. *There exists a pair M_1, M_2 of homogeneously saturated semantics such that $M_2 \preceq M_1$ but $AP(M_1, M_2)$ does not hold.*

Corollary 2. *The set of homogeneously saturated semantics is a lower semi-lattice but not a lattice.*

Corollary 3. *There exists a saturated semantics M which has not the least homogeneous approximation*.

[1] D. Harel, *Dynamic logics*, in ***Handbook of philosophical logics*** (D. Gabbay and F. Guenthner, editors), 1984, pp. 497–604.

[2] V. A. Zakharov, *An efficient and unified approach to the decidability of equivalence of propositional programs*, Lecture Notes in Computer Science, vol. 1443, Springer-Verlag, 1998, pp. 247–259.

► YI ZHANG, *Adjoining cofinitary permutations*.

Department of Mathematics and Computer Science, Istanbul Bilgi University, Istanbul, 80310, Turkey.
E-mail: zhangbilgi.edu.tr.

We consider two kinds of closely related mathematical structures, cofinitary permutation groups and almost disjoint permutation families in $\mathrm{Sym}(N)$.

We say that two permutations f, $g \in \mathrm{Sym}(N)$ are almost disjoint (a. d.) if and only if $|f \cap g| < \omega$. And an a. d. permutation family $A \subseteq Sym(N)$ is a subset of $\mathrm{Sym}(N)$ such that for any $f, g \in A$, f and g are a. d.

A permutation g is cofinitary if and only if g has only finitely many fixed points. A group $G \leq \mathrm{Sym}(N)$ is cofinitary if and only if every non-identity elements is cofinitary. It is easily seen that $G \leq \mathrm{Sym}(N)$ is cofinitary if and only if G is both an almost disjoint set of permutations and a group. For a discussion of different aspects of cofinitary groups, the reader can consult the well-written survey paper [2].

In several papers, some common features had been proved about these two structures, eg, in [1], we proved that any maximal cofinitary group or any maximal a. d. family in $\mathrm{Sym}(N)$ cannot be meager in N^N.

However, it is also interesting to know the difference between the cardinalities of these two structures. In my talk, I will explain a forcing result in [3] which is my first attempt to work in this direction. I will also ask several open questions concerning the subject.

[1] J. Brendle, O. Spinas, and Y. Zhang, *Uniformity of meager ideals and maximal cofinitary groups*, ***Journal of Algebra***, to appear.

[2] P. Cameron, *Cofinitary permutation groups*, ***Bulletin of the London Mathematical Society***, vol. 28 (March 1996), pp. 113–140.

[3] Y. Zhang, *adjoining cofinitary permutations*, ***Journal of Symbolic Logic***, to appear.

Abstracts presented by title

► B. S. BAIZHANOV, *Some remarks on theories without finite cover property*.
Institute of Informatics and Control Problems, Academy of Sciences of Kazakhstan, 125, Pushkin, Almaty, 480100, Kazakhstan.
E-mail: baijan@ipic.academ.alma-ata.su.

Let M be a model of a language L with complete theory T without the finite cover property, $A \subset M$, and let $M^* = (M, L \cup \{P^1\})$ be a model of language $L^* = L \cup \{P^1\}$, such that $P^1(M^*) = A$. We prove that $\forall l < \omega,\ \forall \bar{\alpha}, \bar{\beta} \in (M \setminus A)^l$,

$$(**) \qquad [\mathrm{tp}(\bar{\alpha}|A) = \mathrm{tp}(\bar{\beta}|A) \iff \mathrm{tp}_*(\bar{\alpha}|A) = \mathrm{tp}_*(\bar{\beta}|A)]$$

if and only if there are not A-definable formulas $E(x, y, \bar{z})$, $D(\bar{z})$ such that

(1) $\forall \bar{\beta} \in D(M^{l(\bar{z})}),\ E(x, y, \bar{\beta})$ is a relation of equivalence.

(2) $\exists \bar{\beta}_1, \bar{\beta}_2 \in (M \setminus A)^{l(\bar{z})} \cap D(M^{l(\bar{z})})$ such that we have

  (a) Every $E(x, y, \bar{\beta}_1)$-class meets with A and there is $E(x, y, \bar{\beta}_2)$-class does not meet with A.

  (b) $\forall a, a' \in A,\ [E(a, a', \bar{\beta}_1) \iff E(a, a', \bar{\beta}_2]$.

E. Bouscaren proved $(**)$, when A is an elementary submodel of M and T is superstable theory ([1], Proposition 2.1).

[1] E. Bouscaren, *Dimensional order property and pairs of models*, ***Annals of Pure and Applied Logic***, vol. 41 (1989), pp. 205–231.

► B. BAIZHANOV AND M. ERIMBETOV, *Extensions of countable models of stable theories*.
Institute of Informatics and Control Problems, Academy of Sciences of Kazakhstan, 125, Pushkin, Almaty, 480100, Kazakhstan.
E-mail: baijan@ipic.academ.alma-ata.su.
Institute of Informatics and Control Problems, Academy of Sciences of Kazakhstan, 125, Pushkin, Almaty, 480100, Kazakhstan.
E-mail: LNS@ipic.academ.alma-ata.su.

In our report we consider various properties of elementary extensions of finite generated models of stable theories. We introduce the notions of convergence of formula to type [1], the equivality of Fraissé-Shelah-types [2] and study the relations of these notions with the two-cardinality of formulas and the notion of admissible set, considered in [3].

We obtained some sufficient conditions for constructing of chains of countable models with given properties.

[1] B. S. Baizhanov, *Expansion of a model of a weakly o-minimal theory by a family of unary predicates*, accepted by ***The Journal of Symbolic Logic***.

[2] ———, *Pairs of models and property NBAM*, in ***Proceedings of Institute of Informatics and Control Problems*** (M. Aidarkhanov and B. Baizhanov, editors), Gylym, Almaty, 1995, pp. 81–89, in Russian.

[3] M. M. Erimbetov, *About one property of countable stable theories*, ***Siberian Mathematical Journal***, vol. XXII, (1981), no. 1, pp. 81–86.

► M. BURGIN, *Fixed points for algorithmic classes*.
University of California, Los Angeles, California 90095, USA.
E-mail: mburgin@math.ucla.edu.

Let **A** be a class of algorithms over a set X and **E** be a set of functions having the form $f: X^m \to X^n$ where n and m are arbitrary natural numbers.

Definition 1. A one-to-one mapping $u: \mathbf{A} \to X$ is called a constructive internal naming of **A** if u^-1 and u are defined by some algorithms in **A**.

Remark. When $X = N$, then u is called an enumeration.

Definition 2. A mapping $F: \mathbf{A} \to \mathbf{E}$ is called a natural projection if for any algorithm A from **A** and any x from X we have $F(A(x)) = A(x)$.

Definition 3. A function $K(z, x)$ from **E** (algorithm $H(z, x)$ from **A**) is called universal in **E** (in **A**) at the level n if for any $g(x)$ from **E** (any $A(x)$ from **A**) for some fixed $z = a$ and any x from X we have $K(a, x) = g(x)$ $(H(a, x) = A(x))$.

Theorem 4. **A** *has a universal algorithm if and only if* **E** *has a universal function*.

Definition 5. A one-to-one mapping $u: X^n \to X$ is called a linearization for the level n. Let us assume that **E** contains universal functions and linearization for each level n and all its iterations $w(z(x)) = K(z, x)$ where is a universal algorithm in **A**.

Algorithmic Fixed Point Theorem. *For any* $A(x_1, \dots, x_n + 1)$ *from* **A** *there is such* $B(x_1, \dots, x_n)$ *from* **A** *that* $w(A(x_1, \dots, x_n, B(x_1, \dots, x_n))) = w(B(x_1, ..., x_n))$.

Corollary 6. *For any* $A(x)$ *from* **A** *there is such a from* X *that* $w(A(a)) = w(a)$.

Let **U** be some universal class of super recursive algorithms.

Corollary 7 (Kleene Fixed Point Theorem). *For any* $A(x_1, \dots, x_n + 1)$ *from* **U** *there is such* $B(x_1, \dots, x_n)$ *from* **U** *that* $w(A(x_1, \dots, x_n, B(x_1, \dots, x_n))) = w(B(x_1, \dots, x_n))$.

Let **V** be some universal class of super recursive algorithms [1].

Corollary 8. *For any* $A(x_1, \dots, x_n + 1)$ *from* **V** *there is such* $B(x_1, \dots, x_n)$ *from* **V** *that* $w(A(x_1, \dots, x_n, B(x_1, \dots, x_n))) = w(B(x_1, \dots, x_n))$.

[1] M. Burgin, *Super-recursive algorithms as a tool for high performance computing*, in ***High performance computing, Proceedings, San Diego***, 1999.

► J. V. TUCKER AND J. I. ZUCKER, *Abstract versus concrete models of computation on partial metric algebras*.
Department of Computer Science, University of Wales, Swansea SA2 8PP, United Kingdom.
E-mail: J.V.Tucker@swansea.ac.uk.
Department of Computing and Software, McMaster University, Hamilton, Ont. L8S 4L7, Canada.
E-mail: zucker@mcmaster.ca.
Please address correspondence to the second author.

A model of computation is abstract if, when applied to any algebra, the resulting programs for computable functions and sets on that algebra are invariant under isomorphisms, and hence do not depend on a representation for the algebra. Otherwise it is concrete. Intuitively, concrete models depend on the implementation of the algebra.

The difference is particularly striking in the case of topological partial algebras, and notably in algebras over the reals. We investigate the equivalence between abstract and concrete models of computation on partial metric algebras. In the course of this investigation, interesting aspects of continuity, extensionality and non-determinism are uncovered.

► A. D. YASHIN, *On the relationship between intuitionistic propositional logic and intuitionistic protothetics*.
Udmurt State University, Department of Algebra and Topology, Universitetskaya str., 1, Izhevsk, Russia 426034.
E-mail: yashin@wing.uni.udm.ru.
This work was partially supported by RFBR grant No. 98-06-80205 "Logical theory of informalizability".

Let **Int** be the intuitionistic propositional logic, Var be a set of propositional variables and **T** be a theory formulated in some rich language $\mathcal{L}$. One may conceive either the 1st or 2nd order language, or the propositional language with propositional quantification, etc.

The language $\mathcal{L}$ is assumed to contain the standard intuitionistic connectives $\rightarrow, \vee, \wedge, \neg$. Also we suppose that the class of all $\mathcal{L}$-formulas includes the subclass $\mathrm{Sent}(\mathcal{L})$ of *sentences*.

A function $t\colon \mathrm{Var} \rightarrow \mathrm{Sent}(\mathcal{L})$ is said to be an *interpretation*. An arbitrary interpretation is expanded on the class of all propositional formulas by means of induction:

$t(A \circ B) = t(A) \circ t(B)$ for binary connective $\circ$, and $t(\neg A) = \neg t(A)$.

Let the inference system of **T** supports the standard axiom schemata and rules of **Int**. Then the following statement is undoubtful:

"*If* $\mathbf{Int} \vdash A$, *then for every interpretation* t, $\mathbf{T} \vdash t(A)$" (*the soundness theorem*). The inverse statement

"*If* $\mathbf{Int} \nvdash A$, *then for some interpretation* t, $\mathbf{T} \nvdash t(A)$" one may treate as *the completeness theorem* for **Int** with respect to **T**.

The completeness of **Int** with respect to the intuitionistic arithmetic HA (Heyting arithmetic) is known as *de-Jongh–Smorynski theorem*.

We present a similar result for *quite intuitionistic prototetics* $\mathbf{Int\,2_i}$, i.e., for intuitionistic propositional logic with propositional quantification, with an additional axiom $\neg\forall p\,(p \vee \neg p)$ (the negation of *tertium non datur*).

THEOREM. *Let* $A(p_1, \ldots, p_n)$ *be a propositional formula, such that* $\mathbf{Int} \nvdash A(p_1, \ldots, p_n)$. *Then there are sentences* $E_1, \ldots, E_n$ *of the language of protothetics, such that* $\mathbf{Int\,2_i} \nvdash A(p_1|E_1, \ldots, p_n|E_n)$.