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19TH CENTURY LOGIC BETWEEN PHILOSOPHY AND MATHEMATICS

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Abstract. The history of modern logic is usually written as the history of mathematical or, more general, symbolic logic. As such it was created by mathematicians. Not regarding its anticipations in Scholastic logic and in the rationalistic era, its continuous development began with George Boole's *The Mathematical Analysis of Logic* of 1847, and it became a mathematical subdiscipline in the early 20th century. This style of presentation cuts off one eminent line of development, the philosophical development of logic, although logic is evidently one of the basic disciplines of philosophy. One needs only to recall some of the standard 19th century definitions of logic as, e.g., the art and science of reasoning (Whateley) or as giving the normative rules of correct reasoning (Herbart).

In the paper the relationship between the philosophical and the mathematical development of logic will be discussed. Answers to the following questions will be provided:

1. What were the reasons for the philosophers' lack of interest in formal logic?
2. What were the reasons for the mathematicians' interest in logic?
3. What did "logic reform" mean in the 19th century? Were the systems of mathematical logic initially regarded as contributions to a reform of logic?
4. Was mathematical logic regarded as art, as science or as both?

Most 19th century scholars would have agreed to the opinion that philosophers are responsible for research on logic. On the other hand, the history of late 19th century logic indicates clearly a very dynamic development instigated not by philosophers, but by mathematicians. The central feature of this development was the emergence of what has been called the "new logic", "mathematical logic", "symbolic logic", or, since 1904, "logistics". This new logic came from Great Britain, and was created by mathematicians in the second half of the 19th century, finally becoming a mathematical subdiscipline in the early 20th century.

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Charles L. Dodgson, better known under his pen name Lewis Carroll (1832–1898), published two well-known books on logic, *The Game of Logic* ([13]) of 1887 and *Symbolic Logic* of 1896 ([14]) of which a fourth edition appeared already in 1897. These books were written "to be of *real* service to the young, and to be taken up, in High Schools and in private families, as a valuable addition of their stock of healthful mental recreations" ([14, p. xiv]). They were meant "to *popularize* this fascinating subject," as Carroll wrote in the preface of the fourth edition of *Symbolic Logic* ([14, p. xiv]). But, astonishingly enough, in both books there is no definition of the term "logic". Given the broad scope of these books the title "Symbolic Logic" of the second book should at least have been explained.

Maybe the idea of symbolic logic was so widely spread at the end of the 19th century in Great Britain that Carroll regarded a definition as simply unnecessary. Some further observations support this thesis. They concern a remarkable interest by the general public in symbolic logic, after the death of the creator of the algebra of logic, George Boole, in 1864.

Recalling some standard 19th century definitions of logic as, e.g., the art and science of reasoning (Whately) or the doctrine giving the normative rules of correct reasoning (Herbart), it should not be forgotten that mathematical or symbolic logic was not set up from nothing. It arose from the old *philosophical* collective discipline logic. The standard presentations of the history of logic ignore the relationship between the philosophical and mathematical side of its development; they sometimes even deny that there has been any development of philosophical logic at all. Take for example William and Martha Kneale's programme in their eminent *The Development of Logic*. They wrote ([32, p. iii]): "But our primary purpose has been to record the first appearances of these ideas which seem to us most important in the logic of our own day," and these are the ideas leading to mathematical logic.

Another example is J. M. Bocheński's assessment of "modern classical logic" which he dated between the 16th and the 19th century. It was for him a noncreative period in logic which can therefore justly be ignored in a problem history of logic ([7, p. 14]). According to Bocheński classical logic was only a decadent form of this science, a dead period in its development ([7, p. 20]).

Such assessments show that the authors adhered to the predominant views on logic of our time, i.e., actual systems of mathematical or symbolic logic. As a consequence, they have not been able to give reasons for the final divorce between philosophical and mathematical logic, because they have ignored the seed from which mathematical logic has emerged. Following Bocheński's view Carl B. Boyer presented a consistent periodization of the development of logic ([11, p. 633]): "The history of logic may be divided, with some slight degree of oversimplification, into three stages: (1) Greek logic, (2) Scholastic logic, and (3) mathematical logic." Note Boyer's "slight degree of oversimplification" which enabled him to skip 400 years of logical development and ignore the fact that Kant's transcendental logic, Hegel's metaphysics and Mill's inductive logic were called "logic", too.

In discussing the relationship between the philosophical and the mathematical development of logic, at least the following questions will be answered:

1. What were the reasons for the philosophers' lack of interest in formal logic?
2. What were the reasons for the mathematicians' interest in logic?
3. What did "logic reform" mean in the 19th century? Were the systems of mathematical logic initially regarded as contributions to a reform of logic?
4. Was mathematical logic regarded as art, as science or as both?

This paper focuses not only on the situation in Great Britain, but also on the development in Germany. British logicians of that time regarded Germany as the logical paragon. John Venn can be regarded as a chief witness. He deplored, in the second edition of his *Symbolic Logic* of 1894, the lack of a tradition in logic in Great Britain which caused problems in creating the collection of books on logic for the Cambridge University Library ([67, p. 533]):

> At the time when I commenced the serious study of Symbolic Logic many of the most important works which bore on the subject were not to be found in any of those great libraries in this country to which one naturally refers in the first place, and could therefore only be obtained by purchase from abroad. [...] I suppose that the almost entire abandonment of Logic as a serious academic study, for so many years in this country at least, had prevented the formation of those private professorial libraries, the frequent appearance of which in the market has kept the second-hand booksellers' shops in Germany so well supplied with works on this subject.

It should be stressed, however, that when speaking of German logic Venn wasn't referring to contemporary German logical sytems, but to the great 18th century rationalistic precursors of the British algebra of logic beginning with Gottfried Wilhelm Leibniz and ending with the Swiss, Johann Heinrich Lambert.

In the following sections surveys are given of the philosophical and mathematical contexts in which the new logic emerged in Great Britain and Germany. The strange collaboration of mathematics and philosophy in promoting the new systems of logic will be discussed, and finally answers to the four questions already posed will be given.

§1. Contexts.

1.1. The philosophical context in Great Britain. The development of the new logic started in 1847, completely independent of earlier anticipations, e.g., by the German rationalist Gottfried Wilhelm Leibniz (1646–1716) and his followers (cf. [45]; [48, ch. 5]). In that year the British mathematician George Boole (1815–1864) published his pamphlet *The Mathematical Analysis of Logic* ([9]). Boole mentioned that it was the struggle for priority concerning the quantification of the predicate between the Edinburgh philosopher William Hamilton (1788–1856) and the London mathematician Augustus De Morgan (1806–1871) which encouraged this study. Hence, he referred to a startling philosophical discussion which indicated a vivid interest in formal logic in Great Britain. This interest was, however, a new interest, not even 20 years old. One can even say that neglect of formal logic could be regarded as a characteristic feature of British philosophy up to 1826 when Richard Whately (1787–1863) published his *Elements of Logic*.[1] In his preface Whately added an extensive report on the languishing research and education in formal logic in England. He complained ([69, p. xv]) that only very few students of the University of Oxford became good logicians and that

> by far the greater part pass through the University without knowing any thing of all of it; I do not mean that they have not learned by rote a string of technical terms; but that they understand absolutely nothing whatever of the principles of the Science.

Thomas Lindsay, the translator of Friedrich Ueberweg's important *System der Logik und Geschichte der logischen Lehren* ([63], translation [64]), was very critical of the scientific qualities of Whately's book, but he, nevertheless, emphasized its outstanding contribution for the renaissance of formal logic in Great Britain ([38, p. 557]):

> Before the appearance of this work, the study of the science had fallen into universal neglect. It was scarcely taught in the universities, and there was hardly a text-book of any value whatever to be put into the hands of the students.

One year after the publication of Whately's book, George Bentham's *An Outline of a New System of Logic* appeared ([6]) which was to serve as a commentary to Whately. Bentham's book was critically discussed by William Hamilton in a review article published in the *Edinburgh Review* ([20]). With the help of this review Hamilton founded his reputation as the "first logical name in Britain, it may be in the world."[2] Hamilton propagated a revival

[1] [53] lists nine editions up to 1848 and 28 further printings to 1908. Van Evra ([66, p. 2]) mentions 64 printings in the USA to 1913.

[2] This opinion can be found in a letter of De Morgan's to Spalding of 26th June, 1857 (quoted in [23, p. xii]) which was, however, not sent. Hamilton was for George Boole one of the "two greatest authorities in logic, modern and ancient" ([9, p. 81]). The other authority

of the Aristotelian scholastic formal logic without, however, one-sidedly preferring the syllogism. His logical conception was focused on a revision of the standard forms by quantifying the predicates of judgements.[3] The controversy about priority arose, when De Morgan, in a lecture "On the Structure of the Syllogism" ([16]) given to the Cambridge Philosophical Society on 9th November 1846, also proposed quantifying predicates. Neither had any priority, of course. Application of the diagrammatic methods of the syllogism proposed e. g., by the 18th century mathematicians and philosophers Leonard Euler, Gottfried Ploucquet, and Johann Heinrich Lambert, presupposed quantification of the predicate. The German psychologistic logician Friedrich Eduard Beneke (1798–1854) suggested quantifying the predicate in his books on logic [4] and [5], the latter of which he sent to Hamilton. In the context of this paper it is irrelevant to solve the priority question. It is, however, important that a dispute of this extent arose at all. It indicates, there was new interest in *research* on formal logic.

This interest represented only one side of the effect released by Whately's book. Another line of research stood in the direct tradition of Humean empiricism and the philosophy of inductive sciences: the inductive logic of John Stuart Mill (1806–1873), Alexander Bain (1818–1903) and others. Boole's logic was in clear opposition to inductive logic. It was Boole's follower William Stanley Jevons (1835–1882; cf. [29]) who made this opposition explicit.

Boole referred to the controversy between Hamilton and De Morgan, but this influence should not be overemphasized. In his main work on the *Laws of Thought* ([10]) Boole went back to the logic of Aristotle by quoting from the Greek original. This can be interpreted as indicating that the influence of contemporary philosophical discussion was not as important as his own words might suggest. In writing a book on logic he was doing philosophy, and it was thus a matter of course that he related his results to the philosophical discussion of his time. This does not mean, of course, that his thoughts were really influenced by this discussion.

1.2. The philosophical context in Germany. It seems clear that, in regard to the 18th century dichotomy between German and British philosophy represented by the philosophies of Kant and Hume, Hamilton and Boole stood on the Kantian side. There are some analogies between the situations in Great Britain and Germany, where philosophical discussion on logic after Hegel's death was determined by the Kantian influence. In the preface to the second edition of his *Kritik der reinen Vernunft* of 1787 ([31]), Immanuel Kant (1723–1804) wrote that logic has followed the safe course of a science since earliest times. For Kant this was evident because of the fact that

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is Aristotle. This reverence to Hamilton might not be without irony because of Hamilton's disregard of mathematics.

[3] Cf. [22, vol. 4, p. 287].

logic had been prohibited from taking any step backwards from the time of Aristotle. But he regarded it as curious that logic hadn't taken a step forward either ([31, B VIII]). Thus, logic seemed to be closed and complete. Formal logic, in Kant's terminology the analytical part of general logic, did not play a prominent rôle in Kant's system of transcendental philosophy. In any case, it was a negative touchstone of truth, as he stressed ([31, B 84]). Georg Wilhelm Friedrich Hegel (1770–1831) went further in denying any relevance of formal logic for philosophy ([24, I, Introduction, pp. XV–XVII]). Referring to Kant, he maintained that from the fact that logic hadn't changed since Aristotle one could infer that it needed a complete rebuilding ([24, p. XV]). Hegel created a variant of logic as the foundational science of his philosophical system, defining it as "the science of the pure idea, i.e., the idea in the *abstract element of reasoning*" ([25, p. 27]. Hegelian logic thus coincides with metaphysics ([25, p. 34]).

This was the situation when after Hegel's death philosophical discussion on formal logic in Germany started again. This discussion on logic reform stood under the label of "the logical question", a term coined by the Neo-Aristotelian Adolf Trendelenburg (1802–1872). In 1842 he published a paper entitled "Zur Geschichte von Hegel's Logik und dialektischer Methode" with the subtitle "Die logische Frage in Hegel's Systeme". But what is the logical question according to Trendelenburg? He formulated this question explicitly towards the end of his article: "Is Hegel's dialectical method of pure reasoning a scientific procedure?" ([62, p. 414]). In answering this question in the negative, he provided the occasion of rethinking the status of formal logic within a theory of human knowledge without, however, proposing a return to the old (scholastic) formal logic. In consequence the term "the logical question" was subsequently used in a less specific way. Georg Leonard Rabus, the early chronicler of the discussion on logic reform, wrote that the logical question emerged from doubts concerning the justification of formal logic ([49, p. 1]).

Although this discussion was clearly *connected* to formal logic, the so-called reform did not *concern* formal logic. The reason was provided by the Neo-Kantian Wilhelm Windelband who wrote in a brilliant survey on 19th century logic ([70, p. 164]):

> It is in the nature of things that in this enterprise [i.e., the reform of logic] the lower degree of fruitfulness and developability power was on the side of formal logic. Reflection on the rules of the correct progress of thinking, the technique of correct thinking, had indeed been brought to perfection by former philosophy, presupposing a naive world view. What Aristotle had created in a stroke of genius, was decorated with the finest filigree work in Antiquity and the Middle Ages: an art of proving and disproving which culminated in a theory of reasoning, and after this constructing

> the doctrines of judgements and concepts. Once one has accepted the foundations, the safely assembled building cannot be shaken: it can only be refined here and there and perhaps adapted to new scientific requirements.

Windelband was very critical of English mathematical logic. Its quantification of the predicate allows the correct presentation of extensions in judgements, but it "drops hopelessly" the vivid sense of all judgements, which tend to claim or deny a material relationship between subject or predicate. It is "a logic of the conference table", which cannot be used in the vivid life of science, a "logical sport" which has, however, its merits in exercising the final acumen ([70, pp. 166–167]).

The philosophical reform efforts concerned primarily two areas:

1. the problem of a foundation of logic which itself was approached by psychological and physiological means, leading to new discussion on the question of priority between logic and psychology, and to various forms of psychologism and anti-psychologism (cf. [50], [33]);
2. the problem of logical applications focusing interest on the methodological part of traditional logic. The reform of applied logic attempted to bring philosophy in touch with the stormy development of mathematics and sciences of the time.

Both reform procedures had a destructive effect on the shape of logic and philosophy. The struggle with psychologism led to the departure of psychology (especially in its new, experimental form) from the body of philosophy at the beginning of the 20th century. Psychology became a new, autonomous scientific discipline. The debate on methodology resulted in the creation of the philosophy of science which was separated from the body of logic. The philosopher's ignorance of the development of formal logic caused a third departure: Part of formal logic was taken from the domain of the competence of philosophy and incorporated into mathematics where it was instrumentalized for foundational tasks.

1.3. The mathematical context in Great Britain. As mentioned earlier, the influence of the philosophical discussion on logic in Great Britain on the emergence of the new logic should not be overemphasized. Of greater importance were mathematical influences. Most of the new logicians can be related to the so-called "Cambridge Network" ([12, pp. 29–71]), i.e., the movement which aimed at reforming British science and mathematics which started at Cambridge. One of the roots of this movement was the foundation of the Analytical Society in 1812 (cf. [17]) by Charles Babbage (1791–1871), George Peacock (1791–1858) and John Herschel (1792–1871). In regard to mathematics Joan L. Richards called this act a "convenient starting date for the nineteenth-century chapter of British mathematical development" ([51, p. 13]). One of the first achievements of the Analytical Society was a revision of the Cambridge Tripos by adopting the Leibnizian notation for the calculus and abandoning the customary Newtonian theory of fluxions: "the principles of pure D-ism in opposition to the Dot-age of the University" as Babbage wrote in his memoirs ([1, p. 29]). It may be assumed that this successful movement triggered off by a change in notation might have stimulated a new or at least revived interest in operating with symbols. This new research on the calculus had parallels in innovative approaches to algebra which were motivated by the reception of Laplacian analysis. Firstly the development of symbolical algebra has to be mentioned. It was codified by George Peacock in his *Treatise on Algebra* ([42]) and further propagated in his famous report for the British Association for the Advancement of Science ([43], especially pp. 198–207). Peacock started by drawing a distinction between arithmetical and symbolical algebra, which was, however, still based on the common restrictive understanding of arithmetic as the doctrine of quantity. A generalization of Peacock's concept can be seen in Duncan F. Gregory's (1813–1844) "calculus of operations". Gregory was most interested in *operations* with symbols. He defined symbolical algebra as "the science which treats of the combination of operations defined not by their nature, that is by what they are or what they do, but by the laws of combinations to which they are subject" ([18, p. 208]). In his much praised paper "On a General Method in Analysis" ([8]) Boole made the calculus of operations the basic methodological tool for analysis. However in following Gregory, he went further, proposing more applications. He cited Gregory who wrote that a symbol is defined algebraically "when its laws of combination are given; and that a symbol represents a given operation when the laws of combination of the latter are the same as those of the former" ([19, pp. 153–154]). It is possible that a symbol for an arbitrary operation can be applied to the same operation ([19, p. 154]). It is thus necessary to distinguish between arithmetical algebra and symbolical algebra which has to take into account symbolical, but non-arithmetical fields of application. As an example Gregory mentioned the symbols a and $+a$. They are isomorphic in arithmetic, but in geometry they need to be interpreted differently. a can refer to a point marked by a line whereas the combination of the signs $+$ and a additionally expresses the direction of the line. Therefore symbolical algebra has to distinguish between the symbols a and $+a$. Gregory deplored the fact that the unequivocity of notation didn't prevail as a result of the persistence of mathematical practice. Clear notation was only advantageous, and Gregory thought that our minds would be "more free from prejudice, if we never used in the general science symbols to which definite meanings had been appropriated in the particular science" ([19, p. 158]).

Boole adopted this criticism almost word for word. In his *Mathematical Analysis of Logic* of 1847 he claimed that the reception of symbolic algebra and its principles was delayed by the fact that in most interpretations of mathematical symbols the idea of quantity was involved. He felt that these connotations of quantitative relationships were the result of the context of the emergence of mathematical symbolism, and not of a universal principle of mathematics ([9, pp. 3–4]). Boole read the principle of the permanence of equivalent forms as a principle of independence from interpretation in an "algebra of symbols". In order to obtain further affirmation, he tried to free the principle from the idea of quantity by applying the algebra of symbols to another field, the field of logic. As far as logic is concerned this implied that only the principles of a "true Calculus" should be presupposed. This calculus is characterized as a "method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation" ([9, p. 4]). He stressed ([9, p. 4]):

> It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its objects and in its instruments it must at present stand alone.

Boole expressed logical propositions in symbols whose laws of combination are based on the mental acts represented by them. Thus he attempted to establish a psychological foundation of logic, mediated, however, by language. The central mental act in Boole's early logic is the act of election used for building classes. Man is able to separate objects from an arbitrary collection which belong to given classes, in order to distinguish them from others. The symbolic representation of these mental operations follows certain laws of combination which are similar to those of symbolic algebra. Logical theorems can thus be proven like mathematical theorems. Boole's opinion has of course consequences for the place of logic in philosophy: "On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics" ([9, p. 13]).

Although Boole's logical considerations became increasingly philosophical with time, aiming at the psychological and epistemological foundations of logic itself, his initial interest was not to reform logic but to reform mathematics. He wanted to establish an abstract view on mathematical operations without regard to the objects of these operations. When claiming "a place among the acknowledged forms of Mathematical Analysis" ([9, p. 4]) for the calculus of logic, he didn't simply want to include logic in traditional mathematics. The superordinate discipline was a *new* mathematics. This is expressed in Boole's writing: "It is not of the essence of mathematics to be conversant with the ideas of number and quantity" ([10, p. 12]).

1.4. The mathematical context in Germany. The results of this examination of the British situation at the time when the new logic emerged—a reform of mathematics, with initially a lack of interest in a reform of logic, by establishing an abstract view of mathematics which focused not on mathematical objects, but on symbolic operations with arbitrary objects—these results could be transferred to the situation in Germany without any problem.

The most important representative of the German algebra of logic was the mathematician Ernst Schröder (1841–1902) who was regarded as having completed the Boolean period in logic (cf. [7, p. 314]). In his first pamphlet on logic, *Der Operationskreis des Logikkalkuls* ([55]), he presented a critical revision of Boole's logic of classes, stressing the idea of the duality between logical addition and logical multiplication introduced by William Stanley Jevons in 1864. In 1890 Schröder started on the large project, his monumental *Vorlesungen über die Algebra der Logik* ([56], [57], [58], [59]) which remained unfinished although it increased to three volumes with four parts, of which one appeared only posthumously. Contemporaries regarded the first two volumes alone as completing the algebra of logic (cf. [68, p. 196]).

Schröder's opinion concerning the question as to the end to which logic is studied (cf. [44], [46]) can be drawn from an autobiographical note, published in 1901 (and written in the third person), the year before his death. It contains Schröder's own survey of his scientific aims and results. Schröder divided his scientific production into three fields:

1. A number of papers dealing with some of the current problems of his science.
2. Studies concerned with creating an "absolute algebra," i.e., a general theory of connections. Schröder stressed that such studies represent his "very own object of research" of which only little was published at that time.
3. Work on the reform and development of logic.

Schröder wrote ([60]) that his aim was

> to design logic as a calculating discipline, especially to give access to the exact handling of relative concepts, and, from then on, by emancipation from the routine claims of spoken language, to withdraw any fertile soil from "cliché" in the field of philosophy as well. This should prepare the ground for a scientific universal language that, widely differing from linguistic efforts like Volapük [a universal language like Esperanto, very popular in Germany at that time], looks more like a sign language than like a sound language.

Schröder's own division of his fields of research shows that he didn't consider himself a logician: His "very own object of research" was "absolute algebra," which was similar to modern abstract or universal algebra in respect to its basic problems and fundamental assumptions. What was the connection between logic and algebra in Schröder's research? From the passages quoted one could assume that these fields belong to two separate fields of research, but this is not the case. They were intertwined in the framework of his heuristic idea of a general science. In his autobiographical note he stressed ([60]):

> The disposition for schematizing, and the aspiration to condense practice to theory advised Schröder to prepare physics by perfecting mathematics. This required deepening—as of mechanics and geometry—above all of arithmetic, and subsequently he became by the time aware of the necessity for a reform of the source of all these disciplines, logic.

Schröder's universal claim becomes obvious. His scientific efforts served to provide the requirements to found physics as the science of material nature by "deepening the foundations," to quote a famous metaphor later used by David Hilbert ([26, p. 407]) in order to illustrate the objectives of his axiomatic programme. Schröder regarded the formal part of logic that can be formed as a "calculating logic," using a symbolical notation, as a *model* of formal algebra that is called "absolute" in its last state of development.

But what is "formal algebra"? The theory of formal algebra "in the narrowest sense of the word" includes "those investigations on the laws of algebraic operations [...] that refer to nothing but general numbers in an unlimited number field without making any presuppositions concerning its nature" ([54, p. 233]). Formal algebra therefore prepares "studies on the most varied number systems and calculating operations that might be invented for particular purposes" ([54, p. 233]).

It has to be stressed that Schröder wrote his early considerations on formal algebra and logic without any knowledge of the results of his British predecessors. His sources were the textbooks of Martin Ohm, Hermann Günther Graßmann, Hermann Hankel and Robert Graßmann. These sources show that Schröder was a representative of the tradition of German combinatorial algebra and algebraic analysis (cf. [48, ch. 6]).

Like the British tradition, but independent of it, the German algebra of logic was connected to new trends in algebra. It differed from its British counterpart in its combinatorial approach. In both traditions, algebra of logic was invented within the enterprise to reform basic notions of mathematics which led to the emergence of structural abstract mathematics. The algebraists wanted to design algebra as "pan-mathematics", i.e., as a general discipline embracing all mathematical disciplines as special cases. The independent attempts in Great Britain and Germany were combined when Schröder learned about the existence of Boole's logic in late 1873, early 1874. Finally he enriched the Boolean class logic by adopting Charles S. Peirce's theory of quantification and adding a logic of relatives according to the model of Peirce and De Morgan.

The main interest of the new logicians was to utilize logic for mathematical and scientific purposes, and it was only in a second step, but nevertheless an indispensable consequence of the attempted applications, that the reform of logic came into the view. What has been said of the representatives of the algebra of logic also holds for the proponents of competing logical systems such as Gottlob Frege or Giuseppe Peano. They wanted to use logic in their quest for mathematical rigor, something questioned by the stormy development in mathematics.

§**2. Accepting the new logic.** Although created by mathematicians, the new logic was widely ignored by fellow mathematicians. In Germany Schröder was only known as the algebraist of logic, and regarded as rather exotic. George Boole was respected by British mathematicians, but his ideas concerning an algebraical representation of the laws of thought received very little published reaction. He shared this fate with Augustus De Morgan, the second major figure of symbolic logic at that time. In 1864, Samuel Neil, the early chronicler of British mid 19th century logic, expressed his thoughts about the reasons for this negligible reception: "De Morgan is esteemed crotchety, and perhaps formalizes too much. Boole demands high mathematic culture to follow and to profit from" ([41, p. 161]). One should add that the ones who had this culture were usually not interested in logic.

The situation changed after George Boole's death in 1864. In the following comments only some ideas concerning the reasons for this new interest are hinted at. In particular the rôles of William Stanley Jevons and Alexander Bain are stressed which exemplify "the strange collaboration of mathematics and philosophy in promoting the new systems of logic" mentioned in the introduction.

2.1. William Stanley Jevons. A broader international reception of Boole's logic began when William Stanley Jevons made it the starting point for his influential *Principles of Science* ([28]). He used his own version of the Boolean calculus introduced in his *Pure Logic* of 1864. Among his revisions were the introduction of a simple symbolical representation of negation and the definition of logical addition as inclusive "or". He also changed the philosophy of symbolism ([27, p. 5]):

> The forms of my system may, in fact, be reached by divesting his [Boole's] of a mathematical dress, which, to say the least, is not essential to it. The system being restored to its proper simplicity, it may be inferred, not that Logic is a part of Mathematics, as is almost implied in Professor Boole's writings, but that the Mathematics are rather derivatives of Logic. All the interesting analogies or samenesses of logical and mathematical reasoning which may be pointed out, are surely reversed by making Logic the dependent of Mathematics.

Jevons' interesting considerations on the relationship between mathematics and logic representing an early logicistic attitude will not be discussed. Similar ideas can be found not only in Gottlob Frege's work, but also in that of

Hermann Rudolf Lotze and Ernst Schröder. In the context of this paper, it is relevant that Jevons abandoned mathematical symbolism in logic, an attitude which was later taken up by John Venn. Jevons attempted to free logic from the semblance of being a special mathematical discipline. He used the symbolic notation only as a means of expressing general truths. Logic became a tool for studying science, a new language providing symbols and structures. The change in notation brought the new logic closer to the philosophical discourse of the time. The reconciliation was supported by the fact that Jevons formulated his *Principles of Science* as a rejoinder to John Stuart Mill's *A System of Logic* of 1843, at that time the dominating work on logic and the philosophy of science in Great Britain. Although Mill called his logic *A System of Logic Ratiocinative and Inductive*, the deductive parts played only a minor rôle, used only to show that all inferences, all proofs and the discovery of truths consisted of inductions and their interpretations. Mill claimed to have shown "that all our knowledge, not intuitive, comes to us exclusively from that source" ([40, Bk. II, ch. I, §1]). Mill concluded that the question as to what induction is, is the most important question of the science of logic, "the question which includes all others." As a result the logic of induction covers by far the largest part of this work, a subject which we would today regard as belonging to the philosophy of science.

Jevons defined induction as a simple inverse application of deduction. He began a direct argument with Mill in a series of papers entitled "John Stuart Mill's Philosophy Tested" ([29]). This discourse proved that symbolic logic could be of importance not only for mathematics, but also for philosophy.

Another effect of the attention caused by Jevons was that British algebra of logic was able to cross the Channel. In 1877, Louis Liard (1846–1917), at that time professor at the Faculté de lettres at Bordeaux and a friend of Jevons, published two papers on the logical systems of Jevons and Boole ([34], [35]). In [36] he added a booklet entitled *Les logiciens anglais contemporaines* which had five editions until 1907, and was translated into German in [37]. Although Herman Ulrici had published a first German review of Boole's *Laws of Thought* as early as 1855 ([65]; cf. [47]), the knowledge of British symbolic logic was conveyed primarily by Alois Riehl, then professor at the University of Graz, in Astria. He published a widely read paper "Die englische Logik der Gegenwart" ("English contemporary logic") in 1877 ([52]) which reported mainly Jevons' logic and utilized it in a current German controversy on the possibility of scientific philosophy.

2.2. Alexander Bain. Finally a few words on Alexander Bain (1818–1903): This Scottish philosopher was an adherent of Mill's logic. Bain's *Logic*, first published in 1870, had two parts, the first on deduction and the second on induction. He made explicit that "Mr. Mill's view of the relation of Deduction and Induction is fully adopted" ([2, I, p. iii]). Obviously he shared the "[...] general conviction that the utility of the purely Formal

Logic is but small; and that the rules of Induction should be exemplified even in the most limited course of logical discipline" ([2, p. v]). The minor rôle of deduction showed up in Bain's definition "*Deduction* is the application or extension of Induction to *new cases*" ([2, p. 40]).

Despite his reservations about deduction, Bain's *Logic* was quite important for the reception of symbolic logic because of a chapter of 30 pages entitled "Recent Additions to the Syllogism." In this chapter the contributions of William Hamilton, Augustus De Morgan and George Boole were introduced. Presumably many more people became acquainted with Boole's algebra of logic through Bain's report than through Boole's own writings. One example is Hugh MacColl (1837–1909), the pioneer of the calculus of propositions (statements) and of modal logic. He created his ideas independently of Boole, eventually realizing the existence of the Boolean calculus by means of Bain's report. Even in the early parts of his series of papers "The Calculus of Equivalent Statements" he quoted from Bain's presentation when discussing Boole's logic ([39]). In 1875 Bain's logic was translated into French, in 1878 into Polish. Tadeusz Batóg and Roman Murawski ([3]) have shown that it was Bain's presentation which motivated the first Polish algebraist of logic, Stanisław Piątkiewicz (1848–?) to begin his research on symbolic logic.

The remarkable collaboration of mathematics and philosophy can be seen in the fact that a broader reception of symbolic logic commenced only when its relevance for the philosophical discussion of the time came to the fore.

§**3. Conclusions.** Finally, these are the answers to the initial questions:

1. What were the reasons for the philosophers' lack of interest in formal logic?

In Germany philosophers shared Kant's opinion that formal logic was a completed field of knowledge. They were interested primarily in the foundations and application of logic. In Great Britain there was hardly any vivid logical tradition. Philosophy was dominated by empiricist conceptions. New systems of formal logic therefore had difficulties in gaining a footing in the philosophical discussion.

2. What were the reasons for the mathematicians' interest in logic?

Foundational problems and problems in grasping new mathematical objects forced some mathematicians to look intuitively at the logical foundations of their subject. The interest in formal logic was thus a result of the dynamic development of late 19th century mathematics. One should not assume, however, that this was a general interest. Most mathematicians did not (and still do not) care about foundations.

3. How did the mathematicians' logical activities fit into the reform of logic conceptions of the time?

In Germany in the second half of the 19th century, logic reform meant overcoming the Hegelian identification of logic and metaphysics. In Great Britain it meant enlarging the scope of the syllogism or elaborating the philosophy of science. Mathematicians were initially interested in utilizing logic for mathematical means, or they used it as a language for structuring and symbolizing extra-mathematical fields. Applications were, e.g., the foundation of mathematics (Boole, Schröder, Frege), the foundation of physics (Schröder), the preservation of rigour in mathematics (Peano), the theory of probabilities (Boole, Venn), the philosophy of science (Jevons), the theory of human relationships (Alexander Macfarlane), and juridical questions. The mathematicians' preference for the organon aspect of formal logic seems to be the point of deviation between mathematicians and philosophers who were not interested in elaborating logic as a tool.

4. Was mathematical logic regarded as art or as science?

From the applicational interest it follows that it was mainly regarded as an art. The scientific aspect grew, however, with the insight into the power of logical calculi. Nevertheless, in an institutional sense the new logic was established only in the beginning of the 20th century as an academic subject, i.e., as an institutionalized domain of science.

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