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1997 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '97

Leeds, England, July 7–12, 1997

The 1997 European Summer Meeting of the Association was held at the University of Leeds in Leeds, England, July 7–12, 1997. The meeting was co-sponsored by the British Logic Colloquium, the London Mathematical Society, and the Division of Logic, Mathematics, and Philosophy of Sciences of the International Union of the History and Philosophy of Science. The program consisted of four short courses of three lectures each in the main conference topics (Computability Theory, Model Theory, Proof Theory, and Set Theory), and 18 other invited one-hour plenary addresses. The latter included the British Logic Colloquium Lecture by Paul Benacerraf, a lecture given by Gerald Sacks in honor of the late Robin Gandy, and lectures by Keith Stenning and David Over in a two-hour colloquium on the Psychology of Proof. There were also 37 half-hour, invited, special session lectures, divided among the main topics; 58 contributed talks; and 23 papers presented by title.

The following short courses were given:

M. Rathjen, *The realm of ordinal analysis*.

R. Shore, *Effective model theory*: *The number of models and their complexity*.

J. Steel, *Determinacy, inner models, and definable scales*.

A. Wilkie, *The model theory of analytic and smooth functions*.

Invited plenary addresses were given by: P. Benacerraf (The British Logic Colloquium Lecture: *What mathematical truth could not be–II*), U. Berger (*Continuous functionals of dependent and transfinite type*), S. Goncharov (*Computable enumerations and recursive models*), R. Jensen (*Remarks on core model theory*), R. Soare (*Martin's invariance conjecture and low sets*), N. Jones (*Computability and complexity from a programming perspective*), B. Kim (*Simple theories*), J. Knight (*Recursive infinitary formulas*), I. Neeman (*Large cardinals and the determinacy of long games*), L. Newelski (*On topological stability conjecture*), D. Over (*Logic and decision making*), P. Pudlák (*On the complexity of propositional calculus*), G. Sacks (The Robin Gandy Lecture: *Recursion theoretic memories 1954–1979*), S. Solecki (*Borel equivalence relations and topology*), K. Stenning (*Learning logic as a cognitive process*), L. van den Dries (*On o-minimal expansions of the field of real numbers*), J. van Oosten ((*Partial*) *combinatory algebras for sequential computation*), and L. Wallen (*Herbrand's theorem and the structure of analytic search spaces*).

The Program Committee consisted of G. Boolos, S. Buss, S. B. Cooper (Chair), W. A. Hodges, M. Hyland, A. H. Lachlan, A. Louveau, Y. N. Moschovakis, L. Pacholski, H. Schwichtenberg, T. A. Slaman, J. K. Truss, and H. Woodin. The local Organizing Committee consisted of S. B. Cooper, J. Derrick, F. R. Drake, B. Herwig, H. D. Macpherson, A. B. Slomson, J. K. Truss, and S. S. Wainer (Chair).

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The abstracts of the short courses, invited talks, talks in special sessions, and contributed talks (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
S. B. COOPER

Abstracts of short courses

► MICHAEL RATHJEN, *The realm of ordinal analysis*.
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A central theme running through all the main areas of Mathematical Logic is the classification of sets, functions or theories, by means of transfinite hierarchies whose ordinal levels measure their 'rank' or 'complexity' in some sense appropriate to the underlying context. In Proof Theory, from the work of Gentzen in the 1930's up to the present time, this is manifest in the assignment of 'proof theoretic ordinals' to theories, gauging their 'consistency strength' and 'computational power'.

These lectures are intended as an introduction to this area of research rather than a survey on recent results.

Chiefly by way of specific examples, the first and second lecture will introduce the main ideas and techniques used in ordinal analysis. Some attempts will be made to explain the role of large cardinals that appear in the definition procedures of so-called *collapsing functions* which then give rise to strong ordinal representation systems.

Ordinal analyses often provide the means to discover deep connections between areas which may on the surface seem quite unrelated. Some examples for this will be presented in the last lecture.

► RICHARD A. SHORE, *Effective model theory: The number of models and their complexity*.
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Effective Model Theory studies model theoretic notions with an eye towards issues of computability and effectiveness. We consider two possible starting points. If the basic objects are taken to be theories, then the appropriate effective version investigates decidable theories (the set of theorems is computable) and structures (ones with decidable theories). If the objects of initial interest are typical mathematical structures, then the starting point is computable structures. We present an introduction to both of these aspects of Effective Model Theory organized roughly around the themes of the number and types of models of theories with particular attention to categoricity (as either a hypothesis or a conclusion) and the analysis of various aspects of the computability structure of families of models.

► A. J. WILKIE, *The model theory of analytic and smooth functions*.
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In his excellent course at Logic Colloquium '93 van den Dries discussed the notion of o-minimality. He showed that collections that can be realised as the definable sets in an o-minimal structure (based, usually, on the ordered domain of real numbers) have many pleasant geometrical and topological properties. Indeed, it seems that almost all such properties previously thought special to, say, the collection of semi-algebraic sets or of sub-analytic sets (which, due to the work of Tarski and Gabrielov/van den Dries-Denef respectively, can be so realised) can be established in the general setting of o-minimality.

In this course I shall briefly review van den Dries' article in the proceedings of Logic Colloquium '93 as motivation for my main theme, which is to present a practical method for deciding whether a given expansion of the ordered domain of real numbers by analytic or, more generally, smooth functions is o-minimal. In particular this method may be applied to the Pfaffian functions, which are analytic functions satisfying certain simple systems of partial differential equations.

Abstracts of invited plenary lectures

► PAUL BENACERRAF, *What mathematical truth could not be–II*[1].
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In a recent paper, "Must we believe in set theory?"[2] George Boolos declares that according to ZFC there exists a cardinal κ, which is equal to $\aleph_\kappa$, and is the limit of the sequence $\langle \aleph_0, \aleph_{\aleph_0}, \aleph_{\aleph_{\aleph_0}}, \dots \rangle$. That is (if it exists) κ is the least ordinal greater than all of the $f(i)$, where $f(0) = \aleph_0$ and $f(i+1) = \aleph_{f(i)}$. To go directly to the source: " … if κ exists, there are at least as many as κ sets. *Are* there so many sets?" Boolos asks. It is, of course, uncontroversial that ZFC is committed to the existence of κ, just as it is committed to the existence of ω. Boolos applauds the latter but seems to regard the former as an *over*commitment—a commitment with which he, personally, cannot go along. Any theory committed to the existence of *as many as* κ things is simply not to be believed, on that ground alone. I take it that such a view is not uncontroversial—indeed that it is not even pellucid what the view *comes to*. I try in this talk to examine the reasons Boolos offers for it and, through them, how we should interpret the view and how defensible it might be to hold it. My focus throughout is, of course, not the belief specifically in κ itself, but how this case illustrates certain tensions in our view of mathematical truth, meaning, and belief.

► ULRICH BERGER, *Continuous functionals of dependent and transfinite type*.
Mathematisches Institut der Universtität München, Theresienstr. 39, München, Germany.

Dag Normann generalized the Kleene-Kreisel continuous functionals [2, 3] to a constructive set theoretic interpretation of dependent types and their basic constructors [4]. For almost all applications of this interpretation the density theorem is crucial [5, 6, 7]. We review Normann's model in a more abstract domain-theoretic setting [8, 1], and prove density theorems for iterated super universe operators.

[1] U. Berger, *Total sets and objects in domain theory*, ***Annals of Pure and Applied Logic***, vol. 60 (1993), pp. 91–117.

[2] S. C. Kleene, *Countable functionals*, ***Constructivity in mathematics*** (A. Heyting, editor), North-Holland, Amsterdam, 1959, pp. 81–100.

[3] G. Kreisel, *Interpretation of analysis by means of constructive functionals of finite types*, ***Constructivity in mathematics*** (A. Heyting, editor), North-Holland, Amsterdam, 1959, pp. 101–128.

[4] L. Kristiansen and D. Normann, *Semantics for some constructors of type theory*, ***Symposia Gaussiana*** (Behara, Fritsch, and Lintz, editors), de Gruyter, 1995, pp. 201–224.

[1]The work from which this talk is drawn is a review of issues arising out of the author's "What numbers could not be" (*Philosophical Review* 1965) and "Mathematical truth," (*Journal of Philosophy* 1973) but principally out of the latter—hence the title (one version of "What mathematical truth could not be–I" appears in *Benacerraf and his critics*, A. Morton and S. Stich, editors, Blackwells, 1996).

[2]Forthcoming in a collection of his papers, *Logic, logic and logic*, J. Burgess and R. C. Jeffrey, editors, Harvard University Press.

[5] D. NORMANN, *Closing the gap between the continuous functionals and recursion in ^3E*, to appear in the ***Archive of Mathematical Logic***, 1995.

[6] ———, *Continuity, proof systems and the theory of transfinite computations*, ***Preprint series 19***, Inst. Math. Univ. Oslo, 1996.

[7] ———, *Representation theorems for transfinite computability and definability*, ***Preprint series 20***, Inst. Math. Univ. Oslo, 1996.

[8] E. PALMGREN and V. STOLTENBERG-HANSEN, *Domain interpretations of Martin-Löf's partial type theory*, ***Annals of Pure and Applied Logic***, vol. 48 (1990), pp. 135–196.

▶ LEO HARRINGTON and ROBERT I. SOARE, *Martin's invariance conjecture and low sets*.
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A class of degrees of computably enumerable (c. e.) sets is *invariant* if it is the set of degrees of some class of c. e. sets invariant under automorphisms of $\mathcal{E}$, the c. e. sets under inclusion. For example, Martin proved that $\mathbf{H}_1$, the set of high$_1$ degrees, is invariant, being the degrees of maximal sets. Lachlan and Shoenfield proved that $\overline{\mathbf{L}}_2$, the complement of the low$_2$ degrees is invariant, being the set of degrees of atomless sets. These results and his work at the time on projective determinacy led Martin to make the following conjecture.

CONJECTURE (MARTIN'S INVARIANCE CONJECTURE). *Among all the jump classes $\mathbf{H}_n$ and $\mathbf{L}_n$ for $n > 0$, and their complements $\overline{\mathbf{H}}_n$ and $\overline{\mathbf{L}}_n$, the invariant classes are exactly $\mathbf{H}_{2n-1}$ and $\overline{\mathbf{L}}_{2n}$.*

After some previous results in favor of Martin's conjecture, researchers looked particularly at $\overline{\mathbf{L}}_1$ because of the important role played by low$_1$ sets in the subject and some attempted unsuccessfully to find a property defining $\overline{\mathbf{L}}_1$ analogous that for to $\overline{\mathbf{L}}_2$. Harrington and Soare now prove noninvariance of $\overline{\mathbf{L}}_1$, and supply other evidence for the Martin conjecture.

THEOREM (HARRINGTON AND SOARE). *There is a low$_2$ nonlow$_1$ c. e. set D such that every c. e. set $A \leq_T D$ is automorphic to a low$_1$ set B. Hence, $\overline{\mathbf{L}}_1$ is noninvariant.*

▶ NEIL D. JONES, *Computability and complexity from a programming perspective*.
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The author's new book proves central results in computability and complexity theory from a programmer-oriented perspective. In addition to giving more natural definitions, proofs, and perspectives on classical theorems by Cook, Hartmanis, Savitch, etc., some new results have come from this approach.

One: for a computation model more natural than the Turing machine, multiplying the available problem-solving time by a constant factor provably increases problem-solving power (in general *not true* for Turing machines).

Another: the class of decision problems solvable by *cons-free* programs on Lisp-like lists, or by Wadler's "treeless" programs, are identical with the well-studied complexity class LOGSPACE.

A third: cons-free programs augmented with recursion can solve *all and only* PTIME problems. Paradoxically, these programs often run in exponential time (not a contradiction, since they can be simulated in polynomial time by memoization). This tradeoff indicates a tension between running time and memory space which seems worth further investigation.

Other new perspectives that result from the programming approach include a series of natural complete problems for PTIME, NPTIME and PSPACE, all based on Boolean programs; a version of Gödel's incompleteness theorem; and a comprehensible proof of Levin's theorem: that for any search problem which is efficiently checkable, one can construct a "witness program" which is optimal up to a constant factor.

► BYUNGHAN KIM, *Simple theories*.
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We recall that a first order theory T is said to be *simple* if, for any complete type $p \in S(B)$, p does not fork over some set A $(\subseteq B)$ with $|A| \leq |T|$. It is well known that every stable theory is simple.

The study of simple theories began with Shelah's paper [5]. In [1], remarkable progress was made. There, it was shown that in simple theories, forking coincides with dividing and nonforking satisfies all of the independence axioms of nonforking in stable theories, except the uniqueness of nonforking extension over a model (uniqueness axiom).

Furthermore, in [4], it was shown that for simple theories, the right substitute for uniqueness axiom is "*the Amalgamation Property over a model*" which is a weaker form of the uniqueness axiom. Also it was shown conversely that any theory equipped with a notion of independence satisfying all the basic axioms together with the Amalgamation Property over a model, should be simple, and that the independence notion is exactly nonforking. Moreover, the correct notion of *strong type* in simple theories appears to be *Lascar strong type*, which is originally introduced by Lascar. So the Amalgamation Property for Lascar strong types is proved. Also in [2], it is proved that for a simple small T, the Amalgamation Property holds over an arbitrary algebraically closed set in $\mathcal{C}^{\text{eq}}$.

The first result, in relation to computing the number of countable models in simple theories context, is proved in [3] recently. There, Lachlan's famous theorem on superstable theories is extended to a full class of supersimple theories.

Recent model theoretic research on specific unstable structures such as smoothly approximable structures, pseudo-finite fields and fields with an automorphism gives rise to another justification for the work on simple theories. These structures turn out to be simple and nonforking supplies a good concept of independence as we mentioned previously. The model theoretic analysis of these structures is of interest both for its own sake, as well as for applications such as to diophantine geometry in the case of fields with an automorphism, or to arithmetic groups in the case of pseudo-finite fields.

In conclusion, it can be said that [1, 4] open up new research areas of simple theories which give one step higher hierarchy to the stability classification. Accordingly many questions can be asked about simple theories trying to generalize results from stable theories. Currently, the notion of canonical base for simple theories is found, and the groups definable in simple theories are being studied.

[1] B. KIM, *Forking in simple unstable theories*, to appear in ***Journal of the London Mathematical Society***.

[2] ———, *A note on Lascar strong types in simple theories*, to appear in ***The Journal of Symbolic Logic***.

[3] ———, *On the number of countable models of a countable supersimple theory*, submitted.

[4] B. KIM and A. PILLAY, *Simple theories*, to appear in ***Annals of Pure and Applied Logic***.

[5] S. SHELAH, *Simple unstable theories*, ***Annals of Mathematical Logic***, vol. 19, pp. 177–203.

► JULIA KNIGHT, *Recursive infinitary formulas*.
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Roughly speaking, *recursive infinitary* formulas are formulas of $L_{\omega_1\omega}$ in which the infinite disjunctions and conjunctions are over r. e. sets. All together, the recursive infinitary formulas have the same expressive power as the *hyperarithmetical* formulas. However, recursive infinitary formulas form natural hierarchies corresponding to certain measures of recursive complexity. Definability by an appropriate recursive infinitary formula explains why, for example, in recursive copies of a given structure, the image of a certain relation always is Σ_3^0 or always 3-r. e., or why, anytime the image of one relation is Σ_3^0, the image of another relation must be Σ_5^0, etc.

► ITAY NEEMAN, *Large cardinals and the determinacy of long games*.
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Transfinite games on integers have been the object of substantial research in Set Theory. Most particularly, there has been a lot of work concerning the *determinacy* of games of length ω; first in establishing mathematical consequences of the determinacy of those games (perhaps most well known—the determinacy of all integer games of length ω implies that all sets of reals are Lebesgue measurable, and have the property of Baire) and more recently establishing close connections between determinacy, and *large cardinal assumptions*. Perhaps the most famous result along these lines is the Martin-Steel theorem, which states that granted the existence of *n* Woodin cardinals and a measurable cardinal above them, all games with a Π_{n+1}^1 payoff are determined.

Thus, the determinacy of length ω games with certain definable payoff was seen to follow from the existence of large cardinals (above, from the existence of Woodin cardinals). It is natural to generalize this, and consider games of length greater than ω. Once we restrict to games with definable payoff (e.g., Π_1^1), the determinacy of our games should also follow from large cardinals.

In my talk, I will present a hierarchy of games of different lengths (all greater than ω), and the large cardinals needed to prove their determinacy. The following theorem is an example of the determinacy results for relatively short games:

THEOREM (NEEMAN). *Let α be a countable ordinal. Assume that there exist α Woodin cardinals and measurable cardinal above them. Then all real games of length α with Π_1^1 payoff are determined.*

The games we consider all have payoff sets which can be naturally coded as sets of reals. Thus the length of the game must be below ω_1. The natural generalization of the above theorem ($\alpha > \omega_1$) seems to be excluded. We may however consider games of *variable* length—where the length of the game, though countable, depends on the moves the two players make. For example,

THEOREM (NEEMAN). *Assume that there exists a cardinal which is strong past a Woodin cardinal, and a measurable cardinal above it. Then all games of continuously coded length, with Π_1^1 payoff, are determined.*

Games of transfinite length begin to occur naturally in the theory of Inner Models, once we try to construct inner models for large cardinals in the region of a Woodin cardinal. The games considered in inner model theory have a certain canonical form—they are all *iteration games*. I will attempt to present an intuitive survey of how those games are used in inner model theory, and how tightly they connect large cardinals with determinacy.

► L. NEWELSKI, *On topological stability conjecture*.
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The subject is related to Vaught's conjecture. Assume T is a complete theory, M is a countable model of T and Q is a 0-definable subset of M. Then $\mathrm{Aut}(Q)$ acts on $S(Q)$, the orbits are called pseudo-types. Notice that this is a Borel action of a Polish group on a Polish space, and the orbits are Borel, too. The topological stability conjecture asserts that if T has $<$ continuum countable models, then all (good) pseudo-types are "topologically nice"

(t-stable). I proved several special cases of this conjecture, mainly for stable and superstable theories. These special cases have interesting consequences, which I will discuss.

► DAVID OVER, *Logic and decision making*.
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There are different and largely separate psychological literatures on logical reasoning and on decision making. This division has limited psychological theories in both areas, and particularly held up the study of ordinary reasoning. People do not ordinarily reason from a restricted set of arbitrary assumptions taken, in effect, to be certainly true. Much more often they try to perform inferences from all their relevant beliefs, holding few of these with absolute confidence. They also often perform inferences from statements made to them, as in two-person arguments, but treat few of these statements as absolutely reliable. Probability judgements are central to both of these ordinary cases, as these clearly affect confidence and perceived relevance. Utility judgements, about both epistemic and non-epistemic benefits and costs, are another major factor affecting relevance. To balance probability and utility in performing an action, including an inference, is to make a decision, if only implicitly. Moreover, to find out how much confidence one should have in a premise one must make decisions, balancing probability and utility, in the search for data or evidence. All these points can be illustrated in experimental work on uncertainty in the premises of a deduction, on so-called belief bias about its conclusion, and on how people investigate conditionals. Suggestions will be made about how psychological theories of deduction can be extended to cover these facts. However, in relating ordinary reasoning and decision making to each other, normative questions arise that are more a matter for logicians and philosophers than for psychologists.

► PAVEL PUDLÁK, On the complexity of propositional calculus.
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There are several reasons for studying the complexity of propositional calculus. Firstly, the set of satisfiable formulas is $\mathcal{NP}$-complete, thus $\mathcal{NP} = \mathrm{co}\mathcal{NP}$ is equivalent to the existence of a proof system (in a general sense) in which every tautology has a proof of at most polynomial length. There are also connections to computational complexity problems from cryptography. Secondly, lower bounds on the lengths of proofs in particular proof systems give independence results for fragments arithmetic. Thirdly, there are quite a few methods in mathematics, mainly in combinatorial optimization, which can be interpreted as particular propositional proof systems. It is interesting to compare their strength and in some cases it is possible to prove that they are not polynomially bounded (e.g., cutting planes proofs in integer linear programming).

The most successful general method of proving lower bounds on the lengths of propositional proofs is based on a property of some systems called *effective interpolation*. This means that the circuit complexity of interpolating boolean functions $f(x)$ of an implication $\phi(x, y) \rightarrow \psi(x, z)$ can be bounded by a polynomial in the length of a proof of the implication. This method has limited applications, apparently it cannot be used for strong systems, but for several proof systems it gives reductions to known lower bounds in boolean complexity.

This research leads to natural problems concerning the lengths of proofs in nonclassical logics, where even less is known than in the classical case. In particular we would like to find more than polynomial lower bounds for monotone and intuitionistic logics.

► GERALD SACKS, *Recursion theoretic memories 1954–1979*.
Department of Mathematics, Harvard University, Cambridge, MA 02138, USA.

Here recursion theory means everything from recursive sets to finite types but with emphasis on theory rather than applications. What happened back then? How does it affect us today?

► SLAWOMIR SOLECKI, *Borel equivalence relations and topology*.
University of California at Los Angeles and Indiana University, USA.

I will present two situations in topology in which techniques and notions developed in the study of Borel equivalence relations turned out to be crucial in proving, and sometimes formulating, theorems concerning topological questions.

I. An indecomposable continuum is a continuum, that is, a connected, compact, metric space, which cannot be represented as the union of two proper subcontinua. Each indecomposable continuum is the disjoint union of its composants. (Two points lie in the same composant if they are contained in a proper subcontinuum.)

One of the problems considered early on by topologists was counting composants in an effective (Borel) way. Mazurkiewicz proved in the 1930's that the "Borel cardinality" of the family of all composants in an indecomposable continuum is at least that of the reals. The question arose if these two Borel cardinalities were the same. I will explain how the study of Borel equivalence relations brought a new understanding to the "counting composants" problem and how work of Kechris and Louveau and of myself allowed to completely classify what Borel cardinalities of the family of composants are possible (and attained).

II. Given a Borel subgroup H of a Polish topological group G, one of the more intriguing questions is whether H carries a Polish group topology which preserves the Borel structure inherited from the inclusion $H \subseteq G$. If there is no such topology, what are the precise obstacles for its existence? If it does exist, how can we recover it? Again I will present answers to these questions (for abelian Π^0_3 H only, for the first question) which involve Borel equivalence relations.

► KEITH STENNING, *Learning logic as a cognitive process*.
Edinburgh University.

This talk will describe some empirical studies of the process of teaching/learning elementary logic by both graphical and sentential methods.

► LOU VAN DEN DRIES, *On o-minimal expansions of the field of real numbers*.
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This is a survey of recent progress on constructing new o-minimal expansions of the real field. I will discuss properties of these expansions, and how these fit into the present picture of all known o-minimal expansions of the real field. (Work by Speissegger and myself.)

► J. VAN OOSTEN, (*Partial*) *combinatory algebras for sequential computation*.
University of Utrecht, The Netherlands.

The language PCF (a typed λ calculus with constants for arithmetic operations and fixed points) is regarded by many as a suitable formalisation of the concept "sequential functional of higher type". (The exact relation with Kleene's formalism appears to be not fully understood, however.) The quest for "full abstraction" (Milner) is the search for mathematically attractive models for PCF. Research has resulted in many, rather complex, categories of domains (Scott domains with extra properties, or extra structure). In my lecture I shall show that some of these categories (at least, the part of them which is generated by the natural numbers) come about in a natural way, as the type structure of (partial) combinatory algebras: defined in the Kleene style, with associates. Apart from a closer analysis of the domains, this gives us these categories as full sub-ccc's of a topos, which is then a model of the statement "all functionals are sequential". I hope to arrive at a similar decomposition of the categories of games, of Hyland-Ong and Abramsky-Jagadeesan-Malacaria.

Abstracts of invited talks in the Special Session in Computability Theory

► PETER A. CHOLAK, CARL G. JOCKUSCH JR., and THEODORE A. SLAMAN, *On Ramsey's theorem for pairs*.
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We study the proof-theoretic strength and effective content of the infinite form of Ramsey's theorem for pairs. The proof-theoretic results are sensitive to whether partitions into two classes or an arbitrary finite number of classes are considered. This work extends that presented at the 1997 annual meeting of the ASL in Cambridge by the authors and is related to previous independent work of Jockusch, Seetapun, and Slaman. The starting point is the following result, which refutes an old conjecture of Jockusch.

THEOREM . *For any computable (recursive) partition of the unordered pairs of natural numbers into finitely many classes, there is an infinite low*$_2$ *homogeneous set* X $(X'' \equiv_T 0'')$.

The proof uses the theorem of Jockusch and Stephan that there is a low$_2$ r-cohesive set as well as a relativization of a new result of the authors that if $A_1, A_2, \dots, A_n$ are Δ^0_2-sets and $\bigcup_{i=1}^n A_i = \mathbb{N}$, then some A_i has an infinite low$_2$ subset. Then, as suggested by a previous conjecture of Slaman, we proved the following two theorems by adapting the argument to produce a notion of forcing for adding infinite homogeneous sets to models of second-order arithmetic while preserving the appropriate level of induction.

THEOREM . $\mathbb{N} \rightarrow [\mathbb{N}]^2_2$ *is* Π^1_1*-conservative over* $RCA_0 + I\Sigma_2$.

THEOREM . $\mathbb{N} \rightarrow [\mathbb{N}]^2_{<\infty}$ *is conservative over* $RCA_0 + I\Sigma_3$ *for* arithmetic *statements.*

Thus any Π^1_1 statement provable from $RCA_0 + I\Sigma_2 + \mathbb{N} \rightarrow [\mathbb{N}]^2_2$ is provable from just $RCA_0 + I\Sigma_2$, and any arithmetic statement provable from $RCA_0 + I\Sigma_3 + \mathbb{N} \rightarrow [\mathbb{N}]^2_{<\infty}$ is provable from just $RCA_0 + I\Sigma_3$. In particular, $RCA_0 + I\Sigma_2 + \mathbb{N} \rightarrow [\mathbb{N}]^2_{<\infty}$ does not prove PA.

In addition, we improve some work of Mytilinaios and Slaman to show:

THEOREM . $RCA_0 + \mathbb{N} \rightarrow [\mathbb{N}]^2_{<\infty} \vdash B\Sigma_3$.

► RICHARD COLES, *The distribution of properly* Σ^0_2 *e-degrees*.
School of Mathematics and Computing Science, P.O. Box 600, Victoria University, Wellington, New Zealand.

(Joint work with Stanislaw Bereznyuk, Novosibirsk State University, and Andrea Sorbi, University of Siena.)

A set $A \subset \omega$ is *enumeration reducible to* (or e-reducible to, written $\leq_e$) a set $B \subset \omega$ if there is an effective procedure, Φ say, for enumerating A given any enumeration of B. Forming equivalence classes under $\leq_e$ we obtain the structure $\langle \mathfrak{D}_e, \leq \rangle$ of enumeration degrees, where $\deg_e(A) = \{ X \mid X \leq_e A \;\&\; A \leq_e X \}$.

Cooper [2] and McEvoy [4] define the jump operator on an e-degree $\mathbf{a}$ as $\mathbf{a}' = \deg_e(\chi_{K^A})$ where $K^A = \{ x \mid x \in \Phi^A_x \}$. Let $\mathfrak{S} = \mathfrak{D}_e(\leq \mathbf{0}'_e)$, i.e., the structure of the enumeration degrees below $\mathbf{0}'_e$; it is known (see [2]) that $\mathfrak{S}$ coincides with the enumeration degrees of the Σ^0_2 sets. Cooper [2] shows also that the structure of the e-degrees below $\mathbf{0}'_e$ is dense. However, the structure of the e-degrees is not dense: Calhoun and Slaman [1] show that there are Π^0_2 e-degrees $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{b}$ is a strong minimal cover of $\mathbf{a}$ in the e-degrees.

We say that a Σ^0_2 e-degree is *properly* Σ^0_2 if and only if it contains no Δ^0_2 set. Cooper and

Copestake [3] show that properly Σ^0_2 e-degrees do exist. In fact they show that there exist Σ^0_2 e-degrees $\mathbf{a}$ that are incomparable with all intermediate Δ^0_2 e-degrees. It follows therefore that the structure $\mathfrak{S}'$, where $\mathfrak{S}' = \mathfrak{S} - \{\mathbf{0}, \mathbf{0}'_e\}$ possesses ideals and filters constituted by properly Σ^0_2 e-degrees.

However not every e-degree in $\mathfrak{S}'$ bounds an ideal of properly Σ^0_2 e-degrees: an e-degree $\mathbf{a}$ is said to be *low* if and only if $\mathbf{a}' = \mathbf{0}'_e$. Cooper and McEvoy [5] characterize the low e-degrees as those degrees such that every set $B \leq_e A$ is Δ^0_2, for $A \in \mathbf{a}$. It follows from Cooper and McEvoy's theorem that there are nontrivial initial segments (those below any low e-degree) containing no properly Σ^0_2 e-degrees.

Are the properly Σ^0_2 e-degrees upwards dense? We show that in fact every e-degree strictly below $\mathbf{0}'_e$ precedes a principal filter consisting of e-degrees that are all properly Σ^0_2 (and in fact Σ^0_2-high), except $\mathbf{0}'_e$.

[1] W. C. CALHOUN and T. A. SLAMAN, *The Π^0_2 e-degrees are not dense*, to appear in the ***The Journal of Symbolic Logic***.

[2] S. B. COOPER, *Partial degrees and the density problem, Part 2: The enumeration degrees of the Σ_2 sets are dense*, ***The Journal of Symbolic Logic***, vol. 49 (1984), pp. 503–513.

[3] S. B. COOPER and C. S. COPESTAKE, *Properly Σ_2 enumeration degrees*, ***Zeitschrift für Mathematische Logik und die Grundlagen der Mathematik***, vol. 34 (1988), pp. 491–522.

[4] K. MCEVOY, *Jumps of quasi-minimal enumeration degrees*, ***The Journal of Symbolic Logic***, vol. 50 (1985), pp. 839–848.

[5] K. MCEVOY and S. B. COOPER, *On minimal pairs of enumeration degrees*, ***The Journal of Symbolic Logic***, vol. 50 (1985), pp. 839–848.

► BILL GASARCH, *Ten years of bounded queries in computability theory*.
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For the last ten years there has been research on the following question: How many queries (to various oracles) are required to solve problems (of various sorts). Here is a sample of the kinds of questions we have asked and (mostly) answered.

(1) Let A be a set. Given n numbers clearly we can determine *which ones* are in A with n queries to A. For which A can we do better? How well can we do?

(2) Let A be a set. Given n numbers clearly we can determine *how many* of them are in A. For which A can we do better? How well can we do?

(3) Let A be a set. Given n numbers clearly we can determine if the number that are in A is *even or odd*. For which A can we do better? How well can we do?

(4) For which sets A are there functions that can be computed with n queries to A that cannot be computed with $n - 1$ queries to A?

(5) Are there sets that can be decided with n queries to A that cannot be computed with $n - 1$ queries?

All of these questions can be varied by considering parallel queries, sequential queries, and queries to sets other than A. Techniques in this field have been a blend of combinatorics and recursion theory.

► MARCIA J. GROSZEK and THEODORE A. SLAMAN, *A solution to Prikry's problem*.

We show that if there is a non-constructible real, then every perfect set of reals contains a non-constructible real. This answers a question of Prikry. (Problem 86 from HARVEY FRIEDMAN, *One hundred and two problems in mathematical logic*, ***The Journal of Symbolic Logic***, vol. 40 (1975), pp. 113–129.)

► HAJIME ISHIHARA and BAKHADYR KHOUSSAINOV, *Computable Kripke models and completeness*.
University of Auckland, Auckland, New Zealand.

By the completeness theorem of intuitionistic predicate logic (IPL), every consistent theory T has an (**adequate**) model $\mathcal{M}$ such that for all ϕ, $\mathcal{M}$ forces ϕ iff $T \vdash \phi$ [4, 7]. If T is computable and finitely axiomatized, then T has a model $\mathcal{M} = (W, \leq, D, V)$ with c.e. forcing of atomic sentences and c.e. base [3]. We improve this in [5]. A model $(W, \leq, D, V)$ is X-**decidable** if the forcing in $\mathcal{M}$ and $(W, \leq)$ are computable in X. Let S be a complete logic. IPL, CPL, $CD = IPL + \forall x(\alpha(x) \vee \beta) \rightarrow \forall x\alpha(x) \vee \beta$, where x is not free in β, $DL = IPL + (\alpha \rightarrow \beta) \vee (\beta \rightarrow \alpha)$, $QJ = IPL + \neg\alpha \vee \neg\neg\alpha$, $KJ = QJ + \forall x \neg\neg\alpha \rightarrow \neg\neg\forall x\alpha$, $KJC = QJ + KJ + CD$ are examples of complete logics [1, 2, 3, 4, 7]. T is **saturated** if $\alpha \vee \beta \in T \implies \alpha \in T \vee \beta \in T$, and if $\exists x\alpha(x) \in T$, then $\alpha(c) \in T$ for a c. T is **complete for a class** K if for every ϕ not deducible from T there is an adequate model of T from K which does not force ϕ.

THEOREM. (1) *Any computable theory in CPL* (IPL, CD) *is complete for the class of decidable Kripke models over antichains* (*trees, constant domain frames*).

(2) *Any computable and saturated theory in DL* (KJ, QJ, KJC) *is complete for the class of decidable* ($\mathbf{0}'$*-decidable*, $\mathbf{0}^{\omega}$*-decidable*, $\mathbf{0}'$*-decidable*) *Kripke models over linearly ordered frames* (*frames with maximum elements, directed frames, constant domain frames with maximum elements*).

The proof of the theorem is obtained by effectivizing proofs from [1, 2, 3, 4, 6].

[1] G. CORSI and S. GHILARDI, *Directed frames*, ***Archive for Mathematical Logic***, vol. 29 (1989), no. 3, pp. 53–67.

[2] G. CORSI, *Completeness theorem for Dummett's LC quantified and some of its extensions*, ***Studia Logica***, vol. 51 (1992), pp. 318–335.

[3] D. GABBAY, ***Semantical investigations in Heyting's intuitionistic logic***, D. Reidel, Dordrecht, 1981.

[4] D. VAN DALEN, *Intuitionistic logic*, ***Handbook of philosophical logic*** (D. Gabbay and F. Guenthner, editors), Kluwer Academic Publishers, 1981.

[5] H. ISHIHARA, B. KHOUSSAINOV, and A. NERODE, *Decidable Kripke models of intuitionistic theories*, to appear.

[6] H. ONO, *Model extension theorem and Craig's interpolation theorem for intermediate predicate logics*, ***Reports on Mathematical Logic***, vol. 15, p. 41–58.

[7] C. A. SMORYNSKI, *Applications of Kripke models*, ***Mathematical investigation of intuitionistic arithmetic and analysis*** (A. S. Troelstra, editor), Lecture Notes in Mathematics, Springer-Verlag, 1983.

► GEOFFREY LAFORTE, *Enumerable sets and quasi-reducibility*.
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(Joint work with R. Downey, Victoria University, and A. Nies, University of Chicago.)

Classical recursion theory first arose in order to study the inherent difficulty of mathematical problems. By far the most deeply studied notion related the difficulty of one problem to another has been that of Turing reducibility; nevertheless, for specific problems, particularly those arising in the study of algebraic structures, other reducibilities are of interest.

For example, in the case of (computably presentable) infinite dimensional vector spaces, it turns out that the inherent difficulty of constructing bases for subspaces coincides exactly with the relation of *wtt* reducibility, rather than Turing reducibility. A similar situation arises in combinatorial group theory, where so-called *quasi-reducibility*, or Q-reducibility, turns out to be a more natural means of comparing enumerable word problems than ordinary T-reducibility. In the case of groups without enumerable word problems, the notion of *Ziegler reducibility* (see [2]) is the natural one. Both of these reducibilities arise as abstractions of the process of effectively using sets of equations true in one structure to check the truth of equations in another.

We study the upper semilattice of enumerable Q-degrees, giving several results comparing this structure with the more familiar structure of the enumerable Turing degrees. We also use coding methods as in [1] to show that the elementary theory of $\langle \mathcal{R}_Q, \leq_Q \rangle$ is undecidable. The proofs of these results have additional technical interest, particularly because of the more subtle methods that have to replace standard permitting techniques for ensuring a constructed set will be computable from some set given in advance. Our constructions illustrate the ideas needed to modify standard finite injury and infinite injury constructions in order to deal with the enumerable Q-degrees, as well as shedding some light on the (easier) proofs of the analogous Turing results, by way of contrast.

[1] A. Nies, R. Shore, and T. Slaman, *Standard models of arithmetic and definability in the enumerable degrees*, this Journal, December 1996.

[2] M. Ziegler, *Algebraisch abgeschlossen gruppen*, ***Word problems II, The Oxford book*** (S. I. Adian, W. W. Boone, and G. Higman, editors), North-Holland, 1980, pp. 449–576.

► MANUEL LERMAN, *Recent results on embedding partial lattices into the computably enumerable degrees*.
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We will discuss recent results which deal with the problem of characterizing the partial lattices which can be embedded into the computably enumerable degrees. In particular, there is such a characterization which is recursive in the degree $0''$ for finite partial lattices which have no critical triples.

► DAG NORMANN, *The superjump as a type constructor*.
The University of Oslo, P.O. Box 1053, Blindern, Norway.

Intuitionistic type theory represents the investigation of complex set-theoretical constructions using constructive means. A general philosophical question is how complex constructions and closure principles we can grasp from an intuitionistic point of view. A special case was discussed at a workshop in Uppsala in 1996: Is it possible to grasp the Mahlo-property intuitionistically?

In this talk we will discuss the kind of type constructors needed in order to obtain a structure of the same classical complexity as the first recursively Mahlo ordinal. Harrington showed in 1972 that this ordinal corresponds to the complexity of computations in the Superjump, a type 3 functional described by Gandy ten years earlier. We will use typed structures of effective domains with totality to simulate computations in the Superjump. Each application of the Superjump will correspond to the application of a universe operator in type theory. The technical result will be the match of the complexity of the constructed typed structure and the computation theory of the superjump.

The construction will indicate which universes we must axiomatise to exist in type theory in order to grasp the concept of Mahloness intuitionistically from a type theory perspective.

► JOSEPH R. SHOENFIELD, *Set recursion*.
Duke University.

This is an expository lecture on the set recursion theory of Normann and Moschovakis, considered primarily as a framework for generalised recursion theory.

Abstracts of invited talks in the Special Session in Model Theory

► ZOE CHATZIDAKIS, *Examples of simple theories*.
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Simple theories have been intensively studied this past year. We described some examples.

► DEIRDRE HASKELL, *One-dimensional p-adic subanalytic sets*.
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In their 1988 paper "p-adic and real subanalytic sets", J. Denef and L. van den Dries use model-theoretic techniques to develop a notion of subanalytic sets over the p-adics, analogous to the notion from algebraic geometry in the reals. They prove quantifier elimination for the p-adics in a language containing restricted analytic functions and a restricted division function. Essentially the same proof can be used for the reals. In particular, subanalytic subsets of the p-adic line are shown to be semialgebraic. In our recent work, L. van den Dries, D. Macpherson and I have extended this last result to arbitrary models. This shows that the theory of the p-adics with restricted analytic functions is P-minimal, analogous to the o-minimality of the reals with restricted analytic functions. In my talk, I will review some of the background material and discuss some of the methods used to prove our theorem.

► WILFRID HODGES, *Relative categoricity in modules*.
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This talk is an attempt to clear the decks of some work begun in the 1980s on spectra of relative categoricity between abelian groups; Anatolii Yakovlev helped me plug the chief gap last year. We consider a first-order theory T with a distinguished 1-ary relation symbol P, such that if M is any model of T then its restriction to P is a substructure M_P. For a pair of cardinals κ, λ we consider the property: if M, N are models of T of cardinality κ and M_P, N_P are isomorphic and of cardinality λ, then the isomorphism extends to an isomorphism from M to N. If T is the theory of an abelian group with subgroup defined by P, the property depends only on whether $\kappa = \lambda$. This is not so for general T.

► PIOTR KOWALSKI, *Model theory of algebraic groups*.
ul. Gogowska 9/50, 53-621 Wrocław, Poland.

Assume G is a stable group and H is its type definable subgroup generated by (the set of realizations of) a strong type p of element of G. L. Newelski described a procedure of finding generic types of H as limit points of some sequences of types related to p, in at most two steps. E. Hrushovski pointed out an example (of infinite Morley rank), showing that in general two steps are necessary.

We conjecture that in case of algebraic groups one step is enough. We show that it is enough to check this conjecture for algebraic types. We rephrase this conjecture in terms of algebraic geometry and verify it for many algebraic groups.

► MIKE PREST, *On the Ziegler spectrum of the first Weyl algebra*.
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Ziegler associated to an arbitrary ring a topological space, now called the Ziegler spectrum, whose closed sets parametrise complete theories of modules over that ring. The first part of the talk will be an attempt to give an overview of what is known about this space in general, what is known about it over particular types of rings and applications to the theory of modules. In the second part of the talk I will present some recent results on the Ziegler spectrum of the first Weyl algebra.

The first Weyl algebra $A_1(k)$ over a field k, which we will take to be of characteristic 0, is the ring of differential operators on the k-algebra $k[X]$ (the algebra of polynomials in a single indeterminate X and with coefficients from k). Explicitly, it is the k-algebra generated by elements x and y subject to the relations $yx - xy = 1$ and a natural action on $k[X]$ is defined by having x act as multiplication by X and y act as differentiation with respect to x. This ring is a simple (i.e., no non-zero proper two-sided ideals) domain all of whose simple (= irreducible) modules are infinite-dimensional over k.

It turns out that the Ziegler spectrum of $A_1(k)$ offers some interesting contrasts to previously investigated Ziegler spectra. The starting point of our investigations is the observation by Bavula and Herzog that results of McConnell and Robson imply that the simple (and, more generally, finitely generated torsion) modules over $A_1(k)$ have rather nice model-theoretic properties.

► PREDRAG TANOVIC, *Nonisolated regular types in superstable theories*.
Matematicki Institut SANU, Knez Mihajlova 35, Beograd.

Let T be a countable, superstable theory and $\mathcal{M}$ its monster model.

DEFINITION . Let $p \in S(A)$ be nonalgeraic.

(a) p is **strongly nonisolated** if for all finite B and all formulas $\varphi(\bar{x})$ over AB consistent with T, there exists $q \in [\varphi]_{AB}$ such that $p \perp^a q$.

(b) p is **eventually strongly nonisolated**, or **esn** for short, if there is a finite set B and a nonforking extension $q \in S(AB)$ which is strongly nonisolated.

We discuss the following:

THEOREM . *Suppose $p \in S(\emptyset)$ is regular, stationary and has limit ordinal U-rank. The following are all equivalent*:

(I) *p is eventually strongly nonisolated*;

(II) *There is an integer n such that for no formula $\varphi(x_1, x_2, \dots, x_n)$ we have*:

$$(\varphi(x_1, x_2, \dots, x_n) \wedge p(x_1) \wedge p(x_2) \wedge \cdots \wedge p(x_n)) \Leftrightarrow p^n(x_1, x_2, \dots, x_n);$$

(III) *There exists a finite set A and a sequence $\{q_n \mid n \in \omega\}$ of types extending p to $S(A)$ such that*

$$\operatorname{bnd}(q_0) < \operatorname{bnd}(q_1) < \cdots < \operatorname{bnd}(q_n) < \operatorname{bnd}(q_{n+1}) < \cdots$$

and

$$\lim_{n\to\infty} \operatorname{bnd}(q_n) = \operatorname{bnd}(p).$$

► KATRIN TENT, *Fields and algebraic groups*.
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(Joint work with L. Kramer.)

We show how the field of definition, k, of an isotropic absolutely almost simple k-group G may be defined in the group $G(k)$ of k-rational points in pure group theoretic language. Our construction is inspired by the famous paper of Borel-Tits [2]. However, since we are not dealing with homomorphisms, we need much less of the heavy machinery used by Borel-Tits. The construction is as follows: inside the center of the unipotent radical of a minimal parabolic k-subgroup we pick a suitable 1-dimensional k-vector space. This defines the additive group structure of k; the action of the center of a Levi k-subgroup L induces the multiplicative structure. Here we make use of a result of Azad-Barry-Seitz [1] concerning the action of L.

This answers a question by Borovik-Nesin [3, p. 367] (under the additional assumption that G is k-isotropic). Easy examples also show that if G is not absolutely simple, then $G(k)$ might be a simple group of finite Morley rank without k being algebraically closed since k might be the 'wrong' field of definition. In most such cases one can replace G by an absolutely simple group H with $H(l) \cong G(k)$ for some suitable field extension l of k—thus l has to be algebraically closed.

[1] H. AZAD, M. BARRY, and G. SEITZ, *On the structure of parabolic subgroups*, ***Communications in Algebra***, vol. 18 (1990), pp. 551–562.

[2] A. BOREL and J. TITS, *Homomorphismes "abtraits" de groupes algébriques simples*, ***Annals of Mathematics***, vol. 97 (1973), pp. 499–571.

[3] A. BOROVIK and A. NESIN, ***Groups of finite Morley rank***, Oxford Science Publications, 1994.

► MARKUS TRESSL, *On Dedekind cuts in polynomially bounded o-minimal expansions of real closed fields*.
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For example a convex valuation ring A of a polynomially bounded, o-minimal expansion M of a real closed field, can be identified with the cut (L, R), where $R = \{ a \in M \mid a > A \}$ and $L = M \setminus R$. The Cherlin-Dickmann theorem says that the theory of (M, A) is model complete if M is only a real closed field. The talk is on a generalization of this result for all cuts of M:

THEOREM. *Let $p = (L, R)$ be an arbitrary cut of a polynomially bounded, o-minimal expansion M of a real closed field. Let G be the convex subgroup $\{ a \in M \mid a + p = p \}$ and let $A(G)$ be the convex valuation ring $\{ b \in M \mid b \cdot G \subseteq G \}$.*

Then the theory of $(M, L, G, A(G))$ is model complete (relative $\mathrm{Th}(M)$) in the language of M augmented by three unary predicates $\mathcal{L}$, $\mathcal{G}$ and $\mathcal{A}$ for the subsets L, G and $A(G)$ of M.

A consequence for real closed fields is for example:

If M is a real closed field (in the language of ordered fields) and G is a convex subgroup of M, not of the form $a \cdot A(G)$ or $a \cdot \mathfrak{m}_{A(G)}$ $(a, b \in M)$ (this case is treated in the Cherlin-Dickmann theorem), then $\mathrm{Th}(M, G, A(G))$ has quantifier elimination.

Furthermore we give an axiomatization of all the theories $\mathrm{Th}(M, D)$ relative to the theory of M, where D runs through the convex unbounded subsets of M.

The proofs use basic notions from stability theory, such as heirs or Morley sequences of types.

► FRANK WAGNER, *Groups in simple theories*.
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Recent work of Kim and Pillay has led to the generalization of many stability-theoretic techniques (those related to "forking" rather than "multiplicity") to a wider class of theories, called simple. Traces of this had been noticed before by Hrushovski and Pillay in particular algebraic cases; most notable is the work of Hrushovski and Chatzidakis on difference fields, and Hrushovski and Cherlin on smoothly approximated structures.

However, while a lot of the theory transfers smoothly, other parts don't, and there are a surprising number of relatively basic open problems. In this talk I shall concentrate on groups definable in a simple theory, and discuss the relevant chain conditions, existence of quotients or subgroups internal in some given quotient, and type-definability.

Abstracts of invited talks in the Special Session in Proof Theory

► TOSHIYASU ARAI, *Introducing the hardline*.
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G. Gentzen [2] published his new version of consistency proof for first order number theory in 1938. He had already had two consistency proofs [3] and [1]. The first used *constructive but rather abstract notion of functions*. In the second he had first introduced *transfinite ordinals* in proof theory. Although he formulated the result as a consistency proof, his interest seems to involve a taking off from Hilbert's program. As to this turning G. Kreisel [4, p. 262] wrote:

> by introducing a *quantitative ordinal measure* he (= Gentzen) forces us to pay attention to combinatorial complexity and thereby makes it at least more difficult for us to slip into an abstract reading.

It seems that the purpose of the third "Neue Fassung" is to make a lucid exposure of this *combinatorial complexity* which Gentzen discovered in *finite proof figures* of number theory. G. Takeuti [5] followed this idea and developed a proof theory of systems of second order arithmetic including Π^1_1-Comprehension Axiom, $\Pi^1_1 - CA$. We follow in the wake of Gentzen and Takeuti.

In this talk we survey the results in proof theory for theories of recursively large ordinals.

[1] G. GENTZEN, *Die Widerspruchsfreiheit der reinen Zahlentheorie*, ***Mathematische Annalen***, vol. 112 (1936), pp. 493–565.

[2] ———, *Neue Fassung des Widerspruchsfreiheitsbeweises für die reine Zahlentheorie*, ***Forschungen zur Logik und zur Grundlegung der exakter Wissenschaften, Neue Folge***, vol. 4 (1938), pp. 19–44.

[3] ———, *Der erste Widerspruchsfreiheitsbeweis für die klassische Zahlentheorie*, ***Archiv für mathematische Logik und Grundlagenforschung***, vol. 16 (1974), pp. 97–118.

[4] G. KREISEL, *Review of the book 'The collected papers of Gerhard Gentzen'* (edited and translated by M. E. Szabo), ***Journal of Philosophy***, vol. 68 (1971), pp. 238–265.

[5] G. TAKEUTI, ***Proof theory***, second ed., North-Holland, Amsterdam, 1987.

► ADAM CICHON, *Ordinal terms as a tool for the classification of computational complexity*.
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Proving termination of a rewrite system means establishing the well-foundedness of the induced rewrite ordering. This problem is undecidable in general and so one cannot hope to characterise this class of orderings completely. We are thus led naturally to investigations which attempt to characterise "large" subclasses of these orderings. A syntactic approach based on Kruskal's theorem has given rise to mechanisable methods with the definition of recursive path orderings. But it is now known that the Multiset Path Ordering and Lexicographic Path Ordering, the main examples of recursive path orderings, are restrained to primitive recursive and multiply recursive derivation lengths respectively. An alternative method of proving termination of rewrite systems is by means of interpretations. This consists in the assignment, to each symbol occurring in the rewrite system, of a monotonic interpretation on a well-founded structure. In our case, we shall be concerned with interpretations on natural numbers. We shall see that this gives information on the computational complexity of the system in bounding derivation lengths. The problem is, however, that there is no general deterministic method for defining suitable interpretations. Most of the time one has to guess. A natural question arises: how can we rationalise this task? We describe here how one can exploit subrecursive hierarchies of functions indexed by ordinals for the termination problem. The main examples of such hierarchies are the *slow-growing hierarchy*, the *fast-growing hierarchy* and the *Hardy hierarchy*. Here we concentrate on the use of the Hardy hierarchy where, in order to increase the applicability of standard hierarchy results, we are led to introducing a flexible approach to developing ordinal notations.

[1] E. A. CICHON and H. TOUZET, *An ordinal calculus for proving termination in term rewriting*, ***21st international colloquium on trees in algebra and programming—CAAP '96***, Lecture Notes in Computer Science 1059, Springer-Verlag, pp. 226–241.

▶ ROY DYCKHOFF, LUIS PINTO, and JACOD HOWE, *Proof search in constructive logics*.

We present an overview of Gentzen calculi organised not for "theorem-proving" but for proof search, where the proofs themselves (and the avoidance of known proofs on back-tracking) are objects of interest. The main calculus discussed is that of Herbelin [1994] for intuitionistic logic, which extends methods used in hereditary Harrop logic programming; we give a similar discussion of calculi for linear logic.

▶ HUGO HERBELIN, *Sequent calculus and computation*.
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Gentzen's LJ and LK sequent calculi enjoy the subformula property. For this reason, they are interesting both from a theoretical point of view (consistency proofs, ...) and from a practical one (automated deduction, ...). However, in contrast with intuitionistic natural deduction (NJ) which is in correspondence with lambda-calculus, there is no "clear" computational aspects for these calculi.

Of course, at least on the intuitionistic side, genuine Gentzen's proof of cut elimination implicitly provides with a computational contents of sequent calculi proofs. But how to justify it with respect to other cut elimination proofs? There are also Tait's proof based on an inversion lemma, Zucker's and Pottinger's cut-elimination inherited from Prawitz's translation of LJ into NJ, ... and even other variants of the Hauptsatz by Gentzen?

Moreover, Prawitz's translation maps different proofs of LJ to the same NJ proof and then to the same lambda-term. How to understand it?

We show that, through Curry-Howard isomorphism, the cut rule can be seen as an explicit operation of substitution. This leads to a functional interpretation of LJ proofs based on a variant of lambda-calculus temptatively called lambda-bar-calculus.

This functional notation suggests to put the emphasis not on LJ but rather on a computationally better-behaved restriction of it call LJT.

The μ construction of Parigot can be added to lambda-bar-calculus in order to extend the functional interpretation to LK and LKT. Strong normalisation results can be established.

Besides this functional interpretation of LJ and LK proofs, an "interactive" interpretation of proofs of some various sequent calculi as winning strategies in a two-players game (following Lorenzen and Coquand) can be made.

The talk will cover these computational interpretations.

▶ JÖRG HUDELMAIER, *Bicomplete calculi for intuitionistic propositional logic*.
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Semantical methods have been successfully applied for automated theorem proving in classical logic. This is due to the fact that for classical logic there is a very perspicuous relation between semantical definitions of tautologies and some proof theoretical approaches. In fact for certain calculi such as the well known tableau calculi we immediately obtain for any nonprovable formula a semantical counterexample. In contrast to this the situation for intuitionistic logic is much different. Already A. Heyting himself complained in conversation with W. E. Beth, that there is no straightforward way to extract such semantical counterexamples from failing deduction attempts. In general nonprovable formulae of intuitionistic propositional logic do not even have failing deduction attempts. Usually information regarding nonprovability of a sequent has therefore to be extracted by a kind of metaargumentation from an unending sequence of inference steps. (For instance by considering loops in this sequence.) This situation also shows in the usual semantical completeness proofs for intuitionistic logic as may be found in the monographs. Such completeness proofs are much more complicated than similar proofs for classical logic. We are going to show that these problems may be solved by considering so called bicomplete sequent calculi for intuitionistic propositional logic. By this we mean calculi by which any sequent not provable in intuitionistic logic is refutable in a sense to be made precise. We are going to show that failing deduction attempts in such calculi provide a very perspicuous means for obtaining semantical counterexamples for arbitrary nonprovable formulae of intuitionistic logic. Moreover these calculi also yield a thorough analysis of the concept of a semantics itself: To any such calculus there is a canonical semantics based on it, and the relation between any bicomplete calculus and its semantics exactly mirrors the relation between classical semantics and the usual tableaux calculi for classical provability. Therefore in a sense these calculi serve to embed semantical approaches into proof theory and thus provide a means to make semantical approaches fruitful for automated deduction in intuitionistic logic.

► ANTON SETZER, *Why large cardinals are not needed for denoting small ordinals*.
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In the article "Well-ordering proofs in Martin-Löf's Type Theory" (submitted) we introduced the notion of ordinal notation systems "from below". These were systems of terms OT built from one ordinal function f (which codes several ordinal functions usually needed) together with an ordering $<$ such that for some additional ordering $<'$ on the arguments of f essentially the following conditions hold:

- f is inflating, i.e., $\beta_i < f(\vec{\beta})$.
- The well-foundedness of $<'$ reduces in an elementary way to the well-foundedness of $<$.
- If $\alpha < f(\vec{\beta})$, then $\alpha < \beta_i$ for some i or $\alpha = f(\vec{\alpha})$ for some $\vec{\alpha} <' \vec{\beta}$.

We could show that the Bachmann-Howard ordinal is an upper bound for the order type of such systems and recently as well that this bound is sharp.

In this talk we will explore the result of replacing the set of arguments of f by more complicated structures and of using stronger principles in the reduction of $<'$ to $<$.

In a first step, this set of arguments will be essentially a set of terms OT_2 based on the ordinal notations OT to be defined and the reduction of $<'$ to $<$ follows by $(\mathrm{OT}_2, <')$ being an ordinal notation system from below in the original sense, but based on $(\mathrm{OT}, <)$. This construction will reach $\psi(\varepsilon_{\Omega_2+1})$ and can be regarded as "from below": the function f will still be inflating with respect to OT.

The next construction will be to add a third level $(\mathrm{OT}_3, <'')$ and by adding even more levels indexed by ordinals we will eventually reach the strength of KPI or $(\Delta^1_2 - \mathrm{CA}) + (\mathrm{BI})$.

These constructions are just another way of looking at ordinary ordinal notation systems which use collapsing functions and large cardinals or their recursive analogues. (We will only consider the levels up to one inaccessible cardinal, but it is only a matter of bureaucracy to extend this to systems using larger cardinals.) In this sense large cardinals are not needed for denoting these ordinals but are only used to define in a short way ordinal notation systems.

► THOMAS STRAHM, *Aspects of metapredicativity*.
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We address proof-theoretic aspects of various kinds of transfinitely iterated fixed point theories, thereby providing examples of so-called *metapredicative* theories, i.e., theories which have proof-theoretic ordinal beyond the Feferman-Schütte ordinal Γ_0, but can still be treated by methods of predicative proof theory.

The theories $\widehat{ID}_n$ for finitely iterated fixed points of positive arithmetic operators have been shown by Feferman [3] to provide a framework for predicative mathematics, i.e., the limit of the proof-theoretic ordinals of the theories $\widehat{ID}_n$ for $n < \omega$ is exactly Γ_0. Reporting joint work with Jäger, Kahle, and Setzer [4], we provide an ordinal analysis of the theories $\widehat{ID}_\alpha$ of α times iterated fixed points. The proof-theoretic ordinal of these theories can be characterized by means of a *ternary* Veblen function. Interesting connections between transfinitely iterated fixed point theories and extensions of Friedman's ATR_0 with dependent choice are addressed (cf. Jäger and Strahm [5]). In particular, we have that $ATR_0 + (\Sigma^1_1\text{-}DC)$ and $ATR + (\Sigma^1_1\text{-}DC)$ are proof-theoretically equivalent to $\widehat{ID}_{<\omega^\omega}$ and $\widehat{ID}_{<\varepsilon_0}$, respectively.

It is natural to ask about the proof-theoretic strength of the intuitionistic counterparts $\widehat{ID}^i_\alpha$ of $\widehat{ID}_\alpha$. Recently, Buchholz [2] has shown that $\widehat{ID}^i_1$ is not stronger than Heyting arithmetic HA, and subsequently Arai [1] has strengthened this result to $\widehat{ID}^i_n$ for arbitrary n. In Rüede and Strahm [6] a proof-theoretic analysis of the theories $\widehat{ID}^i_\alpha$ for larger α's is provided. A general relationship between these theories and systems for iterated arithmetic comprehension without set parameters is given. For example, $\widehat{ID}^i_\omega$ is proof-theoretically equivalent to $(\Pi^0_\infty\text{-}CA)$.

Finally, we provide a general discussion of autonomous fixed point processes with classical and intuitionistic logic, and we study the so-called principle of *transfinite fixed point recursion* (FTR), thereby giving a proof-theoretic treatment of the associated theories $\mathrm{Aut}(\widehat{ID})$, FTR_0 as well as FTR, cf. Strahm [7].

[1] T. Arai, *From the attic*, to appear in ***Annals of Pure and Applied Logic***.

[2] W. Buchholz, *An intuitionistic fixed point theory*, to appear in ***Archive for Mathematical Logic***.

[3] S. Feferman, *Iterated inductive fixed-point theories: applications to Hancock's conjecture*, ***The Patras symposium*** (G. Metakides, editor), North-Holland, Amsterdam, 1982.

[4] G. Jäger, R. Kahle, A. Setzer, and T. Strahm, *The proof-theoretic analysis of transfinitely iterated fixed point theories*, submitted for publication.

[5] G. Jäger and T. Strahm, *Fixed point theories and dependent choice*, submitted for publication.

[6] C. Rüede and T. Strahm, *Transfinitely iterated fixed point theories with intuitionistic logic*, in preparation.

[7] T. Strahm, *Autonomous fixed point processes and transfinite fixed point recursion*, in preparation.

► JACO VAN DE POL, *Termination of higher-order rewrite systems*.
Eindhoven University of Technology, The Netherlands.

Higher-order rewrite system (HRS) extend usual term rewriting systems (TRS) with bound variables. In a HRS, the objects are simply-typed lambda terms, containing typed function symbols. These function symbols are defined by providing rewrite rules.

We have only one binding operator, λ. Other bindings can be mimicked, using higher-order function symbols, e.g., $\forall : (\mathtt{term} \to \mathtt{prop}) \to \mathtt{prop}$. Moreover, we use β-reduction as a mechanism to perform (possibly nested) substitutions. To match a given term with the rules, β-expansion is required sometimes. The β-steps are not seen as separate steps, but are performed "under water". Technically, the rewrite relation is defined *modulo* β-equality.

Termination (or strong normalization) of higher-order rewrite systems is the property that every rewrite sequence eventually terminates.

Termination of a TRS can be shown by giving a monotone algebra. A monotone algebra is an interpretation of the function symbols as monotone functions over some well-founded partial order, such that each rewrite rule denotes a descending pair. The existence of a monotone algebra guarantees termination of a rewrite system.

This approach is extended by interpreting an HRS in the model of functionals of finite type, based on a well-founded partial order. This ordering is pointwise lifted to functionals. Note that β-equal terms denote the same functional. If a model can be found where the constants denote strict functionals, and the rules denote pairs of the lifted order, then termination is again guaranteed.

Applications of HRSs can be found in proof theory: proof systems in natural deduction style can be modeled as HRSs, by mapping proofs to lambda terms, and normalization steps to rewrite steps.

Using the method sketched above, termination can be proved of the usual conversion rules on natural deduction rules, including proper reductions and permutative conversions. Also termination of Gödel's T can be proved by this method.

Acknowledgment. The talk is based on my thesis, written at Utrecht University. There are connections with the work of Gandy, Klop, Nipkow, de Vrijer, Zantema. Moreover, part of the research is joint work with H. Schwichtenberg.

► ANDREAS WEIERMANN, *How is it that infinitary methods can be applied to finitary mathematics?*.
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We outline a vision of how to apply a certain type of infinitary methods to questions of finitary mathematics and we are going to survey the following topics.

1. Applications of infinitary proof theory to the proof theory of first order Peano arithmetic. We describe how Pohlers' method of local predicativity and Buchholz' method of operator controlled derivations yield genuine characterizations of the provably recursive functions of PA.

2. Applications of methods from infinitary proof theory to primitive recursion (in higher types). Using Schütte's 1977 exposition of Howard's 1970 proof in combination with an appropriate Mostowski collapse operation we are going to define by constructive means (of a lowest possible proof-theoretic complexity) a function I from the terms of Gödel's T into the set of natural numbers such that if a term a reduces to a term b then $I(a) > I(b)$. This recent result (to appear in the ***The Journal of Symbolic Logic***) strengthens considerably—in a condensed and compact form—a variety of known results on questions of (strong) normalization for Gödel's T.

3. Applications of concepts from infinitary proof theory to subrecursion theory and the rewrite system for the Ackermann function. We show that there exist at least three non-pathological but genuinely different slow growing hierarchies $(G_\alpha)_{\alpha<\varepsilon_0}$. These investigations indicate that the notion of natural assignment of fundamental sequences for the limits less than ε_0 is deeply connection to Lipshitz-continuity properties of the binary ordinal addition function. If time is left we shall discuss the merits of a 1977 paper by Vogel on the slow growing hierarchy and we discuss some interesting applications of Vogel's paper to the rewrite system for the Ackermann function.

Abstracts of invited talks in the Special Session in Set Theory

► SY D. FRIEDMAN, *Products of κ-Suslin trees*.
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We extend Jensen's work on κ-Suslin trees in L to the study of sequences of such trees. Assume $V = L$. Suppose that λ is a limit of limit cardinals and that for each limit cardinal $\lambda_0 < \lambda$, T_{λ_0} is a λ_0^+-Suslin tree. The *reduced product* of the T_{λ_0}, $\lambda_0 < \lambda$ is the product of the T_{λ_0}, $\lambda_0 < \lambda$, factored by the equivalence relation which identifies two elements of this product when they agree on a final segment of the λ_0's.

THEOREM. *Assume $V = L$. There exist $\langle T_\lambda \mid \lambda$ a limit cardinal$\rangle$ such that T_λ is a λ^+-Suslin tree for each λ and when λ is a limit of limit cardinals, T_λ is embeddable into the reduced product of the T_{λ_0}, $\lambda_0 < \lambda$.*

One can extract from the proof a combinatorial principle which relates the $\diamondsuit$ and $\square$ sequences at different limit cardinals to each other, via a nice scale.

► MARTIN GOLDSTERN, *Order-polynomially complete lattices must be inaccessible*.
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(Joint work with Saharon Shelah.)

Let us call a lattice $(L, \wedge, \vee)$ 'order-polynomially complete' (o.p.c.) if every weakly monotone function from L^n to L is induced by a lattice-theoretic polynomial.

The question whether infinite o.p.c. lattices exist is open. The existence of such lattices is not provable in ZFC, since the cardinality of an infinite o.p.c. lattice must be a strongly inaccessible cardinal.

Although the problem originates in algebra, the proof is purely set-theoretical. The main tools are partition and canonisation theorems.

The result will appear in *Algebra Universalis*. A preprint is available from either of the following:

- http://www.math.fu-berlin.de/~goldstrn/large.dvi
- http://math.rutgers.edu/~shelah/all/633.dvi.

► BILL MITCHELL, *Simplified fine structure?*
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Jensen's discovery of fine structure has given us the covering lemma, the square principles, and many other combinatorial principles. It works in L on up through models with Woodin cardinals.

In the intervening time, a number of people have worked on simplifications of some of these results. I will argue that, in fact, Jensen got it right the first time.

► ERNEST SCHIMMERLING, *Covering properties of core models*.
Department of Mathematics, University of California, Irvine, USA.

Jensen showed that if 0-sharp does not exist, then L has the "covering property": every uncountable set of ordinals is contained in a constructible set of the same cardinality. The covering property implies the "weak covering property": if $\alpha \geq \omega_2$ is a successor cardinal of L, then the cofinality of α is equal to the cardinality of α. The weak covering property implies that L computes successors of singular cardinals correctly.

Kunen found that if 0-sharp does not exist, then L computes successors of weakly compact cardinals correctly.

Extensions to these results must respect the limitations implied by Prikry forcing. One kind of extension is illustrated by:

THEOREM 1 (MITCHELL AND SCHIMMERLING). *Assume that every set has a sharp and that there is no model with a Woodin cardinal. Then K has the weak covering property.*

THEOREM 2 (SCHIMMERLING AND STEEL). *Assume that every set has a sharp and that there is no model with a Woodin cardinal. Then K computes successors of weakly compact cardinals correctly.*

One sees a different approach in:

THEOREM 3 (SCHIMMERLING AND WOODIN). *If W is an iterable weasel without measurable cardinals, and W-sharp does not exist, then W has the covering property.*

The terminology, history, references, main ideas, and applications will be discussed in the talk.

▶ MACK STANLEY, *Closed sets and class forcing*.
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Assume the GCH. If S is a set of ordinals, say that C is a *club subset* of S when C is closed and $\sup(C) = \sup(S)$. It is well known that if S is an unbounded subset of $\aleph_1$, then S has a club subset in an $\aleph_1$-preserving outer model iff S is stationary in $\aleph_1$ iff S has a club subset in a cardinal preserving set generic extension. If S is an unbounded subset of $\aleph_2$, then S may have a club subset in an $\aleph_1$ and $\aleph_2$ preserving class generic extension, but none in any $\aleph_1$ and $\aleph_2$-preserving set generic extension.

Suppose that $\kappa \geq \aleph_{\alpha+1}$ is regular and that S is unbounded in κ and has a club subset in an outer model in which $\kappa \geq \aleph_{\alpha+1}$. Depending on α and the "pattern width" of S, class forcing may be required to produce an outer model in which S has a club subset and $\kappa \geq \aleph_{\alpha+1}$.

▶ PHILIP WELCH, *Minimal upper bounds in the theory of Δ^1_3-degrees*.
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We consider the interaction between core model theory, Δ^1_2-determinacy and the theory of the $\boldsymbol{\Delta}^1_3$-degrees, (where a degree is an equivalence class in the reducibility ordering: $r \leq_{\Delta^1_3} s \iff r$ (as a set of integers) is a Δ^1_3-relation in the predicate s), establishing a result on minimal upper bounds for countable collections of such degrees.

We prove:

> $ZFC + \boldsymbol{\Pi}^1_1$-determinacy $\vdash$ Either $\boldsymbol{\Delta}^1_2$-determinacy holds, or there is a real d_0 so that if $D = \{\mathbf{d}_n\}_{n\in\omega}$ is a countable collection of Δ^1_3-degrees above that of d_0, then D has a minimal upper bound.

Abstracts of contributed talks

▶ YOSHIHIRO ABE, *Distributivity and Π^1_1-indescribability in $P_\kappa\lambda$*.
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There are several generalizations of weak compactness to $P_\kappa\lambda$. Baumgartner defined Π^1_1-indescribability in $P_\kappa\lambda$ as follows:

Let $\langle V_\alpha(\kappa,\lambda) \mid \alpha \leq \kappa \rangle$ be recursively defined by $V_0(\kappa,\lambda) = \lambda$,

$$V_{\alpha+1}(\kappa,\lambda) = P_\kappa(V_\alpha(\kappa,\lambda)) \cup V_\alpha(\kappa,\lambda)$$

and $V_\gamma(\kappa,\lambda) = \bigcup_{\beta<\gamma} V_\beta(\kappa,\lambda)$ if γ is limit.

$X \subset P_\kappa\lambda$ is Π^1_1-*indescribable* if for every $R \subset V_\kappa(\kappa, \lambda)$ and Π^1_1 sentence φ with $(V_\kappa(\kappa, \lambda), \in, R) \vDash \varphi$, there is $x \in X$ such that $x \cap \kappa$ is a cardinal and $(V_{x\cap\kappa}(x \cap \kappa, x), \in, R \cap V_{x\cap\kappa}(x \cap \kappa, x)) \vDash \varphi$.

We present a combinatorial characterization of Π^1_1-indescribability and restate it from the point of view of distributivity of ideals.

We also remark that Π^1_1-indescribability has rather strong reflection than expected.

These facts extend the investigation of Shelah property by Carr and Johnson.

► GABRIEL AGUILERA, INMA P. DE GUZMÁN, and MANUEL OJEDA-ACIEGO, *DP-distributions: a new efficiency strategy for the TAS reduction method.*
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We present a new distribution strategy for the TAS reduction method. This methodology allows to get parallel and computationally efficient non-clausal automated theorem provers; besides, it has proven to be a useful tool in both classical [1] and non-classical logics [2].

The TAS methodology uses efficient criteria to reduce the size of the formula being analyzed to decrease the number of required distributions (or even to avoid them all); its distribution module follows a lazy distribution strategy. The DP distribution is based on the Davis and Putnam rule but it has three important advantages:

1. It is applicable to non-clausal formulas.
2. It can be shifted to non-classical logics.
3. Its interaction with the reduction strategies turns out to be extremely efficient.

The definition of the DP distribution on the classical propositional case is as follows:

Given a Boolean formula A if $A = \bigwedge_{i=1}^n A_i$ and there exists a propositional symbol $p \in Q$ and $i_1, i_2 \in \{1, \dots, n\}$ such that $p \models A_{i_1}$ and $\neg p \models (A_{i_2}$, then [3]

$$A \quad \text{and} \quad \left(\bigwedge_{i \neq i_1} A_i[p/\top]\right) \vee \left(\bigwedge_{i \neq i_2} A_i[p/\bot]\right) \text{ are equisatisfiable.}$$

The previous result can be straightforwardly shifted to 3-valued logic obtaining an equi-unsatisfiable formula with three disjuncts where propositional symbol p has been substituted by constants, as in the classical case.

Let $A = \bigwedge_{i=1}^n A_i$ if there exist A_{i_1}, A_{i_2} such that $|\{p0, p\frac{1}{2}, p1\} \cap \Delta_1(A_{i_1})| = 2$, $|\{p0, p\frac{1}{2}, p1\} \cap \Delta_1(A_{i_2})| = 2$ and $\{p0, p\frac{1}{2}, p1\} \subseteq \Delta_1(A_{i_1}) \cup \Delta_1(A_{i_2})$, then

$$A \text{ and } \left(\bigwedge_{i \neq i_2} A_i[p/\odot_1]\right) \vee \left(\bigwedge_{i \neq i_1} A_i[p/\odot_2]\right) \vee \left(\bigwedge_{i_1 \neq i \neq i_2} A_i[p/\odot_3]\right) \text{ are equi-unsatisfiable,}$$

where $\odot_i$ are constant symbols depending on the elements in the Δ sets.

[1] G. Aguilera, I. P. de Guzmán, and M. Ojeda, *Increasing the efficiency of automated theorem proving*, ***Journal of Applied Non-Classical Logics***, vol. 5 (1995), no. 1, pp. 9–29.

[2] ———, *A reduction-based theorem prover for 3-valued logic*, to appear in ***Mathware & Soft Computing***, special issue on deduction methods on many-valued logics, 1997.

► KIWAMU AOYAMA and KENJI FUKUZAKI, *Direct proof of equivalence of the induction schema and the least number principle for open formulas.*
Department of Mathematics, Faculty of Science, Kagoshima University, 1-21-35 Korimoto,

[3] The key tools of TAS methodology are $\Delta_b(A)$ labels, containing unitary partial assignments assuring the truth value b for a given formula A.

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Let L be the usual language for arithmetic, and PA^- be the theory of nonnegative part of the discretely ordered rings. IOpen and LOpen will denote the theory PA^- with the induction schema for open L-formulas and the theory PA^- with the least number principle for open formulas, respectively.

Shepherdson proved in [1] and [2] that if M is a model of PA and $R(M)$ is the real closure of the ordered ring corresponding to M, then M is a model of IOpen if and only if for every positive r in $R(M)$ there exists a s in M such that $r = s$ or $0 < r - s < 1$. From that proof one can deduce model theoretically that IOpen proves the least number principle for open formulas ([3]).

We give a syntactical proof of that fact. For that purpose we need some properties of atomic formulas in L.

PROPOSITION . *Let $\varphi(x)$ be an atomic equality (not identical) of degree n with respect to x. Then PA^- proves that $\varphi(x)$ has at most n solutions.*

PROPOSITION . *Let $\varphi(x)$ be an atomic inequality of degree n with respect to x. Then PA^- proves that $\varphi(x)$ does not have more than n sign alternations.*

PROPOSITION . *Let $\varphi(x)$ be an atomic inequality of degree n with respect to x. Then IOpen proves that there exist at most n points such that $\varphi(x)$ is equivalent to a disjunction of at most $[n/2] + 1$ intervals endpoints which are those points.*

THEOREM . *Let $\varphi(x)$ be any open formula in L. Then there is an $m \in \mathbb{N}$ such that IOpen proves that there exist at most $[m/2]$ intervals endpoints of which are those points.*

From the above theorem we easily deduce that IOpen proves LOpen. Then we get equivalence of IOpen and LOpen, since it is easy to see that LOpen proves IOpen.

[1] J. C. SHEPHERDSON, *A non-standard model for a free-variable fragment of number theory*, ***Bull. Acad. Sci. Ser.***, vol. 12 (1964), pp. 79–86.
[2] ———, *Non-standard models for fragments of number theory*, ***Symposium on the theory of models***, North-Holland, 1965, pp. 342–358.
[3] L. VAN DEN DRIES, *Some model theory and number theory for models of weak systems of arithmetic*, ***Model theory of algebra and arithmetic***, Lecture Notes in Mathematics, no. 834, Springer-Verlag, Berlin, 1980, pp. 346–362.

► M. M. ARSLANOV and I. SH. KALIMULLIN, *Weak presentations of partial orderings*.
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DEFINITION (C. G. Jockusch, Jr., and A. Shlapentokh [1]). An algebraic structure $\mathcal{N}$ with a universe $N \subseteq \omega$ is a weak recursive presentation of a structure $\mathcal{M}$ if $\mathcal{M}$ isomorphic $\mathcal{N}$ and functions and predicates of $\mathcal{N}$ extendible to total recursive functions and predicates. In this case we say also that the set N supports the structure $\mathcal{M}$.
(Below we consider weak recursive presentations with r. e. universe.)

Clearly, a structure is recursive if and only if it is supported by ω. It is obvious also that if infinite r. e. set supports a structure $\mathcal{M}$ then $\mathcal{M}$ is recursive. Studying weak recursive presentation of countable partial orderings we obtained the following results:

THEOREM . *There is a recursive lattice that no simple set supports it as a lattice.*

THEOREM . *Every infinite r. e. set supports any recursive linear ordering.*

THEOREM . *There is a recursive partial ordering $\mathcal{L}$ such that every nonzero r. e. degree contains an r. e. set which does not support $\mathcal{L}$.*

THEOREM . *Let $\mathcal{L} = \langle \omega, <_L \rangle$ be a recursive partial ordering such that*

$$\mathcal{L} \vDash (\forall x)(\forall y)(\exists z)[x <_L z \;\&\; y <_L z],$$

then any r. e. degree contains an r. e. set supporting $\mathcal{L}$.

[1] C. JOCKUSCH, JR. and A. SHLAPENTOKH, *Weak presentation of computable fields*, ***Journal of Symbolic Logic***, vol. 60 (1995), pp. 199–208.

► S. A. BADAEV, *Effectively minimal enumerations*.
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We refer to [2] for undefined notions and notations. Minimal enumerations are interesting from the following two points of view. There is close connection between algorithmic dimension of recursive models and a number of minimal enumerations [3], and there is a long-standing problem of Yu. L. Ershov on a spectrum of the minimal enumerations of an arbitrary family of r. e. sets [2].

Let θ_α stand for enumeration equivalence $\{ \langle n, m \rangle \mid \alpha n = \alpha m \}$ of enumeration α. Let ε be computable principal enumeration of the class of recursively enumerable equivalences on the set N. It follows from [1] that enumeration α is minimal if and only if

$$\forall n \left([W_n]_{\theta_\alpha} = N \rightarrow \exists k \left(\varepsilon_k \subseteq \theta_\alpha \wedge [W_n]_{\varepsilon_k} = N \right) \right). \tag{1}$$

Enumeration α is called *effectively minimal* if for given n in (1) one can find k by uniformly effective procedure and α is said to be *strong effectively minimal* when k may be chosen the same for all n, [1].

THEOREM 1. *There exists positively computable discrete class $\mathcal{A}$ of r. e. sets possessing computable effectively minimal enumeration which is not strong effectively minimal.*

Note that the class $\mathcal{A}$ in Theorem 1 has infinite number of non-equivalent computable minimal enumerations.

THEOREM 2. *There exists discrete class of r. e. sets all computable enumerations of which are effectively minimal but are not strong effectively minimal.*

[1] S. A. BADAEV, *On minimal enumerations*, ***Siberian Advances in Mathematics***, vol. 2 (1992), no. 1, pp. 1–30.
[2] S. A. BADAEV and S. S. GONCHAROV, *On computable minimal enumerations*, ***Proceedings of the third international conference on algebra***, Walter de Gruyter, Berlin-New York, 1995, Krasnoyarsk, August 23–28, 1993, pp. 21–32.
[3] S. S. GONCHAROV, *The problem of the number of non-self equivalent constructivizations*, ***Sov. Math. Dokl.***, vol. 21 (1980), pp. 411–414.

► ARNOLD BECKMANN, *How to separate fragments of bounded arithmetic by methods from ordinal analysis*.
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In the talk we define the *dynamic ordinal* of a theory which is a suitable analogue of the usual proof-theoretic ordinal for weak theories of arithmetic. We compute the dynamic ordinal of certain fragments of bounded arithmetic by adapting methods from the Schütte-style ordinal analysis of the fragments $I\Sigma_n^0$ of Peano-arithmetic. Then different dynamic ordinals immediately imply the separation of the associated fragments of bounded arithmetic.

In particular we obtain the following results: $(m > 0)$

- $T_2^1(X)$ is not included in $s\Sigma_m^b(X) - L^m\mathrm{Ind}$
- $s\Sigma_{m+1}^b(X) - L^{m+1}\mathrm{Ind}$ is not included in $s\Sigma_m^b(X) - L^m\mathrm{Ind}$.

This includes the following result of Krajíček et al.

- $T_2^1(X)$ is a proper extension of $S_2^1(X)$

and the new results

- $sR_2^2(X)$ is a proper extension of $S_2^1(X)$,
- $S_2^2(X)$ is a proper extension of $sR_2^2(X)$ and
- $T_2^1(X)$ is not included in $sR_2^2(X)$.

Here we use the following notations and definitions: Let $\Sigma_n^b(X)$ be the bounded analogue of the level $\Sigma_n^0(X)$ of arithmetical formulas with set variables. The prenex or strict version of $\Sigma_n^b(X)$ is denoted by $s\Sigma_n^b(X)$. With $\Psi - L^m\mathrm{Ind}$ we indicate the theory of bounded arithmetic which, besides the usual axioms, consists of the following induction schema:

$$F(0) \wedge \forall x < |t|_m\,(F(x) \rightarrow F(x+1)) \rightarrow F(|t|_m)$$

with $F \in \Psi$ and $|t|_m$ the m-fold iteration of the binary length function applied to some term t. The famous theories, where $S_2^n(X)$ and $T_2^n(X)$ were firstly introduced by S. Buss, are given by

- $T_2^n(X) = \Sigma_n^b(X) - L^0\mathrm{Ind} = s\Sigma_n^b(X) - L^0\mathrm{Ind}$
- $S_2^n(X) = \Sigma_n^b(X) - L^1\mathrm{Ind} = s\Sigma_n^b(X) - L^1\mathrm{Ind}$
- $R_2^n(X) = \Sigma_n^b(X) - L^2\mathrm{Ind}$
- $sR_2^n(X) = s\Sigma_n^b(X) - L^2\mathrm{Ind}$.

▶ ANATOLY P. BELTIUKOV, *Hierarchy of small subrecursive operator classes based on bounded recursion.*
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In this work the author transfers his results concerning coincidence of small subrecursive predicate classes, based on different modes of bounded recursion (by A. Grzegorczyk), to the corresponding classes of predicate operators. The results are obtained both for numeric and for word predicates, both for ordinary and for simultaneous recursion.

Let $\mathcal{E}(f, F)$ denote the class based on bounded recursion (by A. Grzegorczyk [4]) with the class $\{f\} \cup F$ added to the standard initial functions of the Grzegorczyk classes.

Let $\mathcal{E}(f)_{**}$ denote the class of operators over predicates (0-1-valued functions) defined by the following way. The class $\mathcal{E}(f)_{**}$ consists of all maps that put a predicate Q into correspondence to a predicate P by some definition scheme from the class $\mathcal{E}(f, \pi P)$ where πP is the class of all predicates obtained by substitution of polynomials into the arguments of P.

Theorem . $\mathcal{E}(\lambda x.x)_{**} = \mathcal{E}(\lambda x.x + [x^{1-1/p}])_{**}$ *for any natural* $p > 1$.

This theorem generalizes the result [1]. It is proved by using stack register machines (see [1]) with oracles.

The theorem is transferred to the classes of operators based on word bounded recursion as in [2] thus generalizing the work [2]:

$$\mathcal{E}_d(\lambda x.x)_{**} = \mathcal{E}_d(\lambda x.xa^{\langle \mathrm{length}(x)^{1-1/p}\rangle})_{**},$$

where $\mathcal{E}_d(f)_{**}$ is the corresponding word analog of the class $\mathcal{E}(f)_{**}$, $a^{\langle i\rangle}$ is a letter a repeated i times.

The theorem is also transferred to the cases of simultaneous recursions (both ordinary and word ones) thus generalizing the work [3].

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► ELIAS TAHHAN BITTAR, *Strong normalization proofs for cut elimination in Gentzen's sequent calculi*.

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The purpose of this research is to give a presentation of several techniques of strong normalization proofs for sequent calculi like Gentzen's calculi LJ and LK. We give an equivalent variant LK_{sp} of the calculus LK, in LK_{sp} weakenings or contractions can be done in *parallel*. We define several cut elimination systems for LK and LK_{sp}; strong normalization proofs for these cut elimination systems are investigated. The use of parallel structural rules and symmetrical mix elimination systems allow us to give simple proofs of strong normalization for sequent calculi. We give hereafter the main goals achieved in this research.

• We adapt Huet's Parallel move lemma and O'Donnell's lemma from orthogonal rewriting systems theory to orthogonal restrictions of mix elimination systems for the calculi LK and LK_{sp}.

• We expose classical proofs of the well foundation of lexicographical path orderings (via Kruskal's Tree Theorem); and we give a direct proof, by transfinite induction, of the well foundation of lexicographical path orderings (for the case of well founded total signatures with bounded arities). This well foundation proof technique by transfinite induction can also be applied to other recursive path orderings. We apply lexicographical path ordering criteria to prove and machine check the termination of a *finite* rewriting system interpreting a mix elimination system for LK_{sp}.

• Gentzen's innermost normalization property proof is performed by structural induction on a rank associated to derivations; we define a rank of derivation which allows us to prove strong normalization by structural induction. We gave a direct proof (via König lemma) of strong normalization of mix elimination systems, by structural induction on ranks of proofs we defined.

• Finally we give strong normalization proofs by interpretation of the inferences by monotone subrecursive functions on natural number; this proof is constructive and finitary.

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A *Hilbert algebra* (Ha) is a triple $(A, \rightarrow, 1)$, where A is a non-empty set, $\rightarrow$ is a binary operation on A, $1 \in A$ is an element such that the next axioms are verified for every $x, y, z \in A$:

1. $x \rightarrow (y \rightarrow x) = 1$.
2. $(x \rightarrow (y \rightarrow z)) \rightarrow ((x \rightarrow y) \rightarrow (x \rightarrow z)) = 1$.
3. If $x \rightarrow y = y \rightarrow x = 1$, then $x = y$.

EXAMPLES . (1) Classical propositional calculus, where $\rightarrow$ denotes the logical implication.

(2) If $(A, \leq)$ is a poset with 1, then $(A, \rightarrow, 1)$ is a Ha, where for $x, y \in A$ $x \rightarrow y = 1$ if $x \leq y$ and $x \rightarrow y = y$ if $x \not\leq y$.

(3) A Heyting algebra is a lattice A with 0 such that for every $x, y \in A$ there exists an element $x \rightarrow y \in A$ with the property that for $z \in A$, $x \wedge z \leq y$ if and only if $z \leq x \rightarrow y$.

If $(A, \vee, \wedge, 0)$ is a Heyting algebra, then $(A, \rightarrow, 1)$ is a Ha, where $1 = a \rightarrow a$ for an $a \in A$.

(4) If $(A, \vee, \wedge, \neg, 0, 1)$ is a Boolean lattice, then $(A, \rightarrow, 1)$ is a Ha, where for $x, y \in A$, $x \rightarrow y = (\neg x) \vee y$.

The origin of Hilbert algebras is in Mathematical Logic.

In 1923, Hilbert was the first who remarked the possibility of study a very interesting part of the classical propositional calculus, taking as axioms only the Hilbert ones concerning only for logical implication (this field is known as "positive implicative propositional calculus").

If $(A, \rightarrow, 1)$ is a Ha, we define *pseudo-valuation* on A as a real-function v on A satisfying $v(1) = 0$ and $v(x \rightarrow y) \geq v(y) - v(x)$ for any $x, y \in A$ (v is called a *valuation* if $x = 1$ whenever $v(x) = 0$).

The aim of my communication is to prove that every pseudo-valuation (valuation) v induces a pseudo-metric (metric) d_v on A defined by

$$d_v(x, y) = v(x \rightarrow y) + v(y \rightarrow x)$$

under which $\rightarrow$ is uniformly continuous.

Also, is proved many theorems of extension for pseudo-valuations.

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Computable Set Theory studies the decision problem for fragments of naive set theory. A complete account of the state-of-the-art up to 1989 can be found in the book [2].

The satisfiability problem for The so-called Multi-level Syllogistic (**MLS**, for short) and several of its extensions with set operators and predicates, such as $\{\cdot\}$ (singleton), $\mathcal{P}$ (power-set), $\bigcup$ (unionset), *Finite* (finiteness predicate), etc., was studied very thoroughly (we recall that **MLS** is the propositional closure of atoms of the following types: $x = y \cup z$, $x = y \cap z$, $x = y$, $x \subseteq y$, $x \in y$). It is to be noticed that for each satisfiability problem and *ad hoc* technique was used in the past.

We introduce in this work a unifying approach to the satisfiability problem for fragments of naive set theory, based on the novel notion of *trace* of a set collection. The trace is intended to formalize in a precise fashion the "history" of the construction of the collection itself in the form of a transfinite sequence. We have shown that under suitable conditions, given a formula φ and a set assignment M for it, from any trace $\{A_\mu\}_{\mu<\xi}$ of the collection $\mathcal{C} = \{Mx : x \text{ occurring in } \varphi\}$, one can extract a *representative* k-subsequence $\{A_{\mu_i}\}_{i<k}$, with $k \in \omega$ depending solely on the cardinality of $\mathcal{C}$. The representative subsequence can be used to constuct satisfying assignments for a suitable collection $\{\varphi_i\}_{I\in\omega}$ of approximating

formulae, which is equisatisfiable with φ. The trace approach has been used to give a decision procedure for the extension of **MLS** with singleton, finiteness, powerset and unionset, thus combining and extending the decision results contained in [1] and [3].

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► ENRIQUE CASANOVAS, *A test for expandability*.
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Let M be a model in countable language L. We call M *expandable* if for every extension L' of L with $|L'| \leq |M|$, every set Σ of sentences of L' which is consistent with $\mathrm{Th}(M)$ is satisfiable in M, i.e., it is satisfied in some expansion of M. M is *compactly expandable* if for every extension L' of L with $|L'| \leq |M|$, every set Σ of sentences of L' which is finitely satisfiable in M is satisfiable in M.

Expandable models implicitly appear in the work of Morley and Shelah on categoricity of PC-classes (see [2] and [3]). Compactly expandable models were introduced in [1]. Saturated models are expandable and expandable models are compactly expandable. Special models are expandable too. Using iterated ultrapowers we can obtain expandable models which are neither saturated nor special. A compactly expandable model is universal and ω-saturated. Thus, the notions of saturation, expandability and compactness for expandability agree in the countable case. As shown in [1], these notions also agree for models of cardinality $\geq 2^{\omega}$ of a superstable theory without the finite cover property and for arbitrary models of any ω-stable theory without the finite cover property.

The question of whether there are compactly expandable models which are not expandable is left open in [1]. There are several examples of superstable theories with the finite cover property in which all compactly expandable models are expandable but not necessarily saturated. Here we give a sufficient condition for a compactly expandable model M of power $\geq 2^{\omega}$ of a superstable theory being expandable in terms of the existence of an elementary submodel $U \prec M$ which satisfies certain conditions. The conditions we place on the submodel $U \prec M$ partly concern the theory of the pair (M, U). This theory should eliminate all Ramsey quantifiers outside U, which is a weak form of the finite cover property.

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The effort in providing constructive and predicative meaning to non-constructive modes of reasoning has almost without exception been applied to theories with full classical logic [1]. In this talk we show how to combine the unrestricted countable choice, induction on infinite well-founded trees and restricted classical logic in a constructively given model. For those familiar with intuitionistic systems [3], we may describe the theory we model as follows. Expand the extensional version of HA^{ω} with a new basic type for the countably branching well-founded trees. Add countable and dependent choice together with the axiom of unique choice at all types. The equality relations on the two basic types are supposed to be decidable. Moreover we assume a schema which states that decidable formulae are closed under quantification over natural numbers.

A theorem, well known in topos theory, states that the axiom of choice (and hence the principle of excluded middle) is valid in a topos of sheaves over a complete boolean algebra. This theorem is non-constructive and relies on Zorn's lemma. We constructivise this topos to give a model of the above mentioned theory. This is done by considering boolean algebras that are merely σ-complete and whose topologies are generated by finite or countable covering relations. By a judicious choice of the boolean algebra we can directly extract effective content from Π_2-statements true in the model. For some related examples of the use of boolean models, see [2, Section 4].

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In 1918 Weyl suggested an "elementary mathematics" in order to obtain a more adequate mathematics to theoretical physics [5]. When its formalisation was accomplished [3] it resulted to enjoy the following features, all lacking in constructive mathematics: (i) one quantifier only upon a set of rational numbers is allowed; (ii) the objects perceived geometric intuition (e.g., extremants of a function) are represented; (iii) least upper bound is represented, provided that one is able to extract a convergent sequence of rational numbers.

Along two centuries a very problem was to recognise the formal content of the crucial word "omnes" (all) in Cavalieri's method of indivisibles; never it was interpreted in logical terms [4, 1]. I show that by equating the above word to the quantifier, one obtains a consistent interpretation of Cavalieri's method as pertaining to Weyl's mathematics [2], to which Cavalieri was close just for giving confidence to his geometric intuition.

Cavalieri's method was then followed by Torricelli, that introduced the inertia principle by means of a geometrical intuition of the same kind.

The above interpretation proves that the historical birth of the quantifier has to be anticipated almost two centuries before the rigorous calculus. Moreover, it proves that the birth of modern science—since the times of Calilei's school—was tied to the use of mathematical techniques including the notion of actual infinity. Moreover, it gives reason of the curious disappearence of geometry in the foundations of some subsequent scientific theories—e.g., chemistry, thermodynamics, computer science—all being bounded to use a less powerful mathematics than Weyl's one, that is at most the constructive mathematics [2].

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The purpose of this paper is to show that the automorphism groups of many of the 'cycle-free' partial orders studied in [3] and [1] are simple. This contrasts strongly the situation for trees, of which they form a natural generalization. It was shown in [2] that the automorphism group of any sufficiently transitive tree has at least $2^{2^{\aleph_0}}$ normal subgroups. All the infinite chain cycle-free partial orders studied in [1] have simple automorphism groups. The finite chain case is more involved; where the ordering on chains of the Dedekind-MacNeille completion can be expressed as a lexicographic product by a non-trivial discrete (transitive) ordering (respected by the group), the automorphism group is not simple. For both finite and infinite chain cases the simple automorphism groups split into two classes: those where there is a bound (≤ 12) on the number of conjugates required to express one non-identity element in terms of another, and those in which there is no such bound.

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► BIRGIT ELBL, *Cut elimination for a calculus with context-dependent rules*.
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Context-dependent rules arise when considering logics which do not satisfy certain laws of distributivity as $p \wedge p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$ or $p \wedge (q \vee r) \rightarrow q \vee (p \wedge r)$. A system like this has been used in [1] for a semantics for depth-first programs. There a sequent calculus is presented where structural rules are not permissibile and, furthermore, some rules are subject to conditions on the context. Here we focus on a propositional fragment of this calculus, however, sufficient to exhibit the problems caused by context-dependent rules. As they differ strongly from standard connectives, we adopt the notations $\otimes$ and $*$ for the conjunction and disjunction discussed below. In that calculus, e.g., $(q \implies \Gamma), (r \implies \Delta) \vdash (q * r \implies \Gamma, \Delta)$ is an inference, but $(p, q \implies \Gamma), (p, r \implies \Delta) \vdash (p, p, q * r \implies \Gamma, \Delta)$ is not. The cut rule $(\Gamma \implies q), (q \implies \Delta) \vdash (\Gamma \implies \Delta)$ is an inference, in contrast to the variant $(q * r \implies q, r), (p, r \implies p \otimes r) \vdash (p, (q * r) \implies q, (p \otimes r))$.

Conditions on the context restrict the permutability of rules. So the usual cut elimination procedure fails. Indeed, the possibility to arrange inferences using cut moderates the application conditions a bit. It can be shown that there are derivable sequents without a cut-free derivation. But a cut-free formulation would be useful, e.g., for proof search and in showing non-derivability.

The problem is solved by modifying the calculus. We retain context-dependent rules, but deviate from typical sequent calculus formulations. In contrast to constructing formulas only on the top-level indicated by the sequent symbol $\implies$, we introduce connectives also at deeper occurrences of (generalized) implication. Concerning these "deep" inferences, the new calculus resembles Schütte's calculi (see [2]). The notion of *sequent form* is introduced to give a precise definition of the context in which inferences may take place. Using the method of *deep inferences*, we arrive at a calculus equivalent to the original one but permitting cut elimination.

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► ALI ENAYAT, *Conservative elementary extensions of models of set theory*.
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A model N is said to be a conservative extension of a model M if the intersection of every parametrically definable subset of N with M is again parametrically definable within M (this is a local version of "N is first order definable within M"). We plan to discuss old and new results concerning conservative elementary extensions of models of ZF. Among the new results we discuss are the following:

Theorem A. *Every model of ZF+global choice (of any cardinality) has a conservative elementary extension in which it is not cofinal. However, if N is a conservative elementary extension of a model M of ZFC with the same set of natural numbers, then M is cofinal in N.*

Theorem B. *Suppose M is a model of ZF (of any cardinality). M has a conservative elementary extension if and only if M believes "there exists a nonprincipal ultrafilter".*

Note that if M satisfies the axiom of choice a simple ultrapower will provide a conservative elementary extension, so the right to left implication in Theorem B is nontrivial for models in which AC fails. We use M. Spector's work on *extended ultrapowers* (a forcing construction) to overcome the absence of AC.

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Evidence Logic (EL) goes beyond Classical Logic (CL) in its primitive expressivity by including both confirmatory and refutatory predications, additionally equipped with evidence level annotations. Previous work has characterized the Boolean Sentence Algebras (BSAs) of the monadic, functional, and undecidable varieties of EL (see ***The Journal of Symbolic Logic***, vol. 59 (1994), pp. 347–348). From the perspective that our knowledge of the world is often less-than-certain, that is to say "evidential", applicationwise EL is conceptually antecedent to CL and provides a broad foundational framework wherein axiomatizable extensions reach out to a number of the more domain-specific recent constructions of logics for the representation and processing of uncertainty in Artificial Intelligence (AI). In this paper we analyse EL from this point of view.

For any $n > 1$ let $E_n = \{ e_i = i/(n-1) : i = 1, \dots, n-1 \}$, with evidence increment $\varepsilon = e_1$, be a space of evidence annotations. Focusing on notions related to the degree of evidential conflict a theory may permit, we construct three hierarchies of axiomatizable extensions of EL and characterize the BSAs of the theories they entail. Let d, $d \in \{1, \dots, n\}$ denote the "degree of conflict toleration of confirmatory and refutatory evidence". In the paper we characterize the monadic, functional, and undecidable BSAs across all three hierarchies. Here, let us informally state the results for monadic similarity types μ stipulating p proposition symbols, k constant symbols, and u unary predicate symbols. Then each such theory, it turns out, has ordered basis of order type $\omega^{m^u} \cdot m^p \cdot \sum_{i=1}^{k} s_{ki} \cdot m^{u_i}$ where m depends on which of the three conflict toleration schemes is chosen, the size $n-1$ of the evidence space E_n, and the degree d of conflict toleration permitted in the theory.

Roughly, the dth logic in the first hierarchy will tolerate (and hence represent and process) conflict of evidence only so long as the minimum of the confirmatory and refutatory evidence is at most $(d-1)\varepsilon$. This hierarchy of increasingly permissive logics begins with a logic allowing no conflict at all and ends with the maximally permissive logic EL. In fact, these logics can be viewed as logics generalizing ideas of Aristotle concerning the nature of privatives. The dth monadic logic here has BSA with ordered basis as above with $m = d(2n-d)$.

The dth logic in the second hierarchy will tolerate conflict only so long as the sum of the confirmatory and refutatory evidence is at most $1 + (d-1)\varepsilon$. This hierarchy is one of increasingly permissive Dempster-Shafer type logics, beginning with a logic modeling part of the Dempster-Shafer logic itself and ending with EL. The dth monadic logic here has BSA with ordered basis as above with $m = (n(n+1) + (d-1)(2n-d))/2$.

The dth logic in the third hierarchy will tolerate conflict only so long as the sum of the confirmatory and refutatory evidence is at most $(d-1)\varepsilon$. This is a hierarchy of increasingly permissive sub-Dempster-Shafer type logics, beginning with a trivialized logic effectively containing only equality and constants and ending with the logic modeling part of the Dempster-Shafer logic. Here the dth monadic logic has BSA with ordered basis as above with $m = d(d+1)/2$.

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Let Ha (Ha_{fp}, Ha_{fin}) denote the category of (finitely presented, finite) Heyting algebras, J_0 the canonical topology on $Ha_{\text{fin}}^{\text{op}}$. In [2] we have proved that the functor

$$Ha_{\text{fp}}^{\text{op}} \to \mathrm{Sh}(Ha_{\text{fin}}^{\text{op}}, J_0)$$

sending a finitely presented Heyting algebras H to the functors $Ha(H, -)\colon Ha_{\text{fin}} \to Set$ is a conservative r-Heyting category morphisms. This may serve as a hint on how to verify that $Ha_{\text{fp}}^{\text{op}}$ is indeed a r-Heyting category. The question whether the dual of the category of finitely presented algebras of an equational theory T is an r-Heyting category is interesting since, as we have shown in [1], it implies and for large class of theories is equivalent to the existence of the model completion of the theory T. In this paper we address the question of *characterizing the geometric theory in* $L_{\omega_1,\omega}$ *classified by the topos* $\mathrm{Sh}(Ha_{\text{fin}}^{\text{op}}, J_0)$. Such theory will turn out to be the theory of algebraically closed, locally finite and prime Heyting algebras; we prove also that this theory has only two countable models, differing each other by the existence of a penultimate element (a condition which is elementary but not coherent).

Most proofs are quite general, so we isolated a larger class of equational theories to which they apply and called them nice equational theories (NET).

Finally, we study existentially closed models (where a classification is impossible) and prove the existence of a prime model.

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► EVAN GRIFFITHS, *Completely mitotic degrees and jump classes*.
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A computably enumerable (c. e.) set is A is mitotic if it can be split into a pair of disjoint c. e. sets each of which has the same Turing degree as A. Ladner showed that this condition is equivalent to the set A being autoreducible, and that there exists a nonzero c. e. degree such that all c. e. sets in the degree are mitotic; this is called a completely mitotic degree. Later it was shown that the degrees containing nonmitotic c. e. sets are dense in the c. e. degrees, though there exist low, low$_2$-low, and high completely mitotic degrees.

We show (1) there is a low c. e. degree above which there is no completely mitotic degree, (2) the completely mitotic degrees are nowhere dense, and (3) there is a degree computably enumerable in and above $0'$ (REA $0'$) which is not the jump of any completely mitotic c. e. degree. The proof of (3) uses techniques from Cooper's construction of a jump class of noncappable degrees, and confirms a conjecture of Downey and Slaman.

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The question we are going to consider is for which filters in Boolean algebras it is possible to construct the independent set of generators from given one by means of boolean operators only. The answer depends on the minimal cardinality of the sets of generators of the filter (but in some sense, doesn't depend on the Boolean algebra). It can be proved that in the countable case, for any filter F there is a set of generators of F, from which no free set of generators of F can be constructed. On the other hand, if a filter is freely and regularly generated, then its independent sets of generators can be constructed from any one. The results have consequences concerning algebraic independent axiomatizations in classical logic.

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Gödel logic is the many valued logic with the real interval $[0, 1]$ of truth values, connectives $\wedge, \vee, \rightarrow$ interpreted by minimum, maximum and Gödel's implication respectively ($x \rightarrow y = 1$ for $x \leq y$ and $= y$ otherwise) and $\neg$ interpreted by Gödel's negation ($\neg 0 = 1$, $\neg x = 0$ for $x > 0$). (See [3].) A model of a predicate language (with predicates and constants) is a structure $\mathbf{M} = \langle M, (r_P)_{P \text{ predicate}}, (M_c)_{c \text{ constant}} \rangle$ where $M \neq \emptyset$, $r_P \colon M^{\mathrm{arity}(P)} \rightarrow [0, 1]$, $m_c \in M$. Tarskian definition of satisfaction obviously generalizes using the above truth functions and interpreting $\forall$ as inf and $\exists$ as sup. The propositional part is axiomatized by axioms of intuitionistic logic plus the prelinearity axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$ (by Dummett's result [2]), i.e., a propositional formula is a 1-tautology if and only if it is provable from these axioms using modus ponens. Strong completeness of the predicate logic with respect to a (recursive but cumbersome) Gentzen style system with an additional (non-monotonic) inference rule was proved by Takeuti and Titani [6]. Their system was successively simplified by the present author [4] and Baaz and Zach (unpublished); the last authors have announced the redundance of the additional deduction rule. We offer a surprisingly simple strong completeness proof with respect to the above axiom system for propositional part, five natural quantifier axioms and deduction rules modus ponens and generalization.

The meaning of the result for metamathematics of fuzzy logic [5] will be shortly discussed. For related results on Gödel logic see [1].

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► VALENTINA HARIZANOV, *Intrinsically Δ^0_2 relations in computable structures*.

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One of the important and interesting questions in computable model theory is how a specific aspect of a computable model may change if the model is isomorphically transformed so that it remains computable. A model $\mathcal{B}$ isomorphic to a computable model $\mathcal{A}$ is not necessarily computable. However, even if $\mathcal{B}$ is computable, it can still lose many of the computable properties of $\mathcal{A}$. A computable property of $\mathcal{A}$ which we consider is an additional computable relation R on the domain of $\mathcal{A}$ (that is, R is not named in the language of $\mathcal{A}$). Let $\mathcal{P}$ be a certain class of relations. R is called intrinsically $\mathcal{P}$ on $\mathcal{A}$ if the image of R under every isomorphism from $\mathcal{A}$ to a computable model belongs to $\mathcal{P}$. We also consider Turing degrees of the images of R under all isomorphisms from $\mathcal{A}$ to other computable models, in symbols, $\mathrm{DgSp}_{\mathcal{A}}(R)$. It is known that if every Δ^0_3 degree can be realized in $\mathrm{DgSp}_{\mathcal{A}}(R)$ via an isomorphism of the same degree as its image of R, then $\mathrm{DgSp}_{\mathcal{A}}(R)$ contains all Turing degrees. We will focus on intrinsically Δ^0_2 relations R for which $\mathrm{DgSp}_{\mathcal{A}}(R)$ contains all Δ^0_2 degrees. Examples of such relations include the ω-part of a computable linear ordering of order type $\omega + \omega^*$, and the elements of a computable reduced abelian p-group $\bigoplus_{n\geq 1} Z_{p^n}$ which have even height.

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► SACHIO HIROKAWA, YUICHI KOMORI, and MISAO NAGAYAMA, *A simple proof of $P - W$ problem*.

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This paper gives new proofs to the following theorems of the relevant logic $P - W$. The logic is defined by axiom schemes $B = (b \to c) \to (a \to b) \to a \to c,\ B' = (a \to b) \to (b \to c) \to a \to c,\ I = a \to a$ and modus ponens rule.

(A) *If $\alpha \to \beta$ and $\beta \to \alpha$ are provable in $P - W$, then $\alpha = \beta$.*

(B) *No formula with form $\alpha \to \alpha$ is provable in $P - W$ without using the axiom I.*

(C) *$\alpha \to \beta$ is provable in BB' if $\alpha \to \beta$ is provable in $BB'I$ and $\alpha \neq \beta$.*

In [Anderson and Belnap 75], (A) was raised as a problem and had been known as the $P - W$ problem. In [Martin and Meyer 82], (A), (B) and (C) were proved using a semantic method. But simple syntactic proofs have been sought [Kron 85, Nagayama 93, Komori 93, Fine and Martin 9?]. (B) means that the logic defined by B and B' is "progressive" in the sense that any deduction from an assumption α does not return to the start point itself. The deduction creates something new and does not yield the tautology $\alpha \to \alpha$, which would be a "logical nonsense".

In [Hirokawa 96], the set HRML of hereditary right-maximal linear λ-terms was introduced as an extension of combinatory abstraction algorithm of [Trig, Hindley and Bunder 94]. It is a natural deduction formulation of the logic $P - W$. It was proved in [Hirokawa 96], that following (D) is equivalent to (B).

(D) *If a HRML λ-term has a type $\alpha \to \alpha$, then M is $\beta\eta$-reducible $\lambda x.x$.*

This paper gives a direct proof of (D). The simplicity of our proof is due to an observation that sub-term of the form $P(QR)$ remains in the result of any β-reduction of $B = \lambda xyz.x(yz)$ and $B' = \lambda xyz.y(xz)$. Applying this observation to HRML λ-terms, we obtain simple proofs to (A), (B) and (C) as well.

► ALEXANDRE IVANOV, *Strongly minimal theories of bounded valency and finite axiomatisability*.

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Let W be an infinite structure of a finite relational language L. We say that W is of bounded valency if there is a natural number n such that for any $r(\bar{x}) \in L$ and $a \in W$ the number of all realisations of $r(\bar{x})$ containing a is bounded by n. It is shown in [2] that the problem of the existence of a finitely axiomatisable, strongly minimal theory with disintegrated acl is reduced to the problem of the existence of a finitely axiomatisable, transitive, strongly minimal structure of bounded valency, where the algebraic closure of a point is infinite.

Let W be a transitive structure of bounded valency and $\pi\colon M \to W$ be a finite cover of W (see [1]), where M is a transitive strongly minimal structure of a finite language. We show that M can be considered as a structure of bounded valency.

The following observation suggests the questions (below) that we study. Assume that the kernel of the cover π is finite. Let $a \in M,\ \Gamma = \mathrm{acl}_M(a)$ and $G = \mathrm{Aut}(\Gamma)$. The cover π induces a surjective homomorphism $\tau\colon G \to H = \mathrm{Aut}(\mathrm{acl}_W(\pi(a)))$ with a finite kernel. It can be verified that if H is a finitely presented group containing a finite set of non-trivial conjugacy classes such that any cyclic subgroup has a non-trivial intersection with one of them, then G also has these properties and M is a finitely axiomatisable structure. It is not known if finite axiomatisability of W implies that H satisfies these conditions. We have some partial results concerning this.

The main question that we consider is as follows. *Is M finitely axiomatisable over W?* This is confirmed in several natural cases. We define a variant of an AZ-enumeration, which is applicable in the case of structures of bounded valency. The main result shows that if W is enumerable in this way then M is finitely axiomatisable over W.

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Systems of explicit mathematics were introduced by Feferman in [1] to formalize Bishop-style constructive mathematics. They turned out to be very important for proof-theoretic studies of subsystems of second order arithmetic as well as of set theory.

Starting from the basic theory EET of explicit mathematics various type existence axioms allow to define a broad spectrum of theories of mathematical interest. One of these concepts is the notion of *universes* as known from Martin-Löf type theory. It has been first introduced in explicit mathematics by Feferman in [2] and extensively studied in Marzetta's thesis [5]. In his approach Marzetta guarantees the existence of finitely many universes by use of a *limit axiom* which is closely related to the corresponding one in the Kripke-Platek set theory KPi^0. In the presence of type induction the resulting theory UTN has the proof-theoretic strength Γ_0, the so-called *limit of predicativity*.

Marzetta's formalization of the limit axiom is *non-uniform*, i.e., there is no operation giving a witness for the existential quantifier used in the axiom. Since uniformity is a very natural requirement in explicit mathematics, we show that UTN plus type induction is not strengthened by use of a *uniform* version of the limit axiom [4]. For the proof we modify the model construction for UTN of Marzetta and Strahm in [6].

Replacing type induction by formula induction we get a first step to *metapredicativity*, which we address in the context of so-called *Frege structures* [3]. These are truth theories over applicative theories closely related to UTN.

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mathematics with universes and iterated fixed point theories with ordinals, to appear in ***Archive for Mathematical Logic***, 199x.

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THEOREM 1. *Let k be an uncountable cardinal. Every k-Souslin graph G on reals satisfies one of the following two requirements*:

(i) *G admits a $k+1$-Borel colouring by ordinals below k*;
(ii) *there exists a continuous homomorphism* (*in some cases embedding*) *of a certain locally countable Borel graph G_0 into G*.

THEOREM 2. *In the Solovay model, every OD graph G on reals satisfies one of the following two requirements*:

(i) *G admits an OD colouring by countable ordinals*;
(ii) *as above.*

► ADAM KOLANY, *Hypergraphs in logic—current state of the theory*.
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The notion of hypergraph satisfiability—a generalization of usual satisfiability of families of clauses—was defined by R. H. Cowen and improved by A. Kolany. Cowen also proposed a resolution like method for checking hypergraph satisfiability. Soundness and completeness of the method have been proved. Many set-theoretical existential statements can be expressed in terms of hypergraph satisfiability. We have here: n-colorability, existence of R-consistent choices, Systems of Distinct Representatives, Independent Transversals, and others. The resolution on hypergraphs yields a uniform method of solving those problems.

Four natural possibilities of extending the usual consequence operation to the case of hypergraphs $\mathrm{Cn}_1, \dots, \mathrm{Cn}_4$, can be defined. Operations Cn_2 and Cn_4 coincide in the class of edge-disjoint hypergraphs and the operations Cn_3 and Cn_4 coincide in the class of graphs. In the case of edge-disjoint graphs, which corresponds to the usual propositional satisfiability, all three operations Cn_2, Cn_3 and Cn_4 coincide, which shows that they all are possible extensions of the usual consequence operation.

Every locally finite and compact hypergraph determines a Heyting Algebra. The set of all propositional formulae which are valid in all such algebras is the set of all propositional formulae provable in the Intuitionistic Propositional Calculus. There is the question: given a class of hypergraphs, what is the set of all propositional formulae which are valid in all Heyting algebras determined by a hypergraph from the given class? Naturally, it must be an Intermediate Logic. Since not every intermediate logic can be obtained in this way (Dummet Linear Logic is one of them), the question is which intermediate logics are determined by classes of hypergraphs?

Given an inconsistent set of propositional formulae, one can consider its inconsistency hypergraph. Then, the satisfiability with respect to that hypergraph is the same as the usual satisfiability. Every compact minimal hypergraph with countably many vertices is isomorphic to an inconsistency hypergraph of a certain set of propositional formulae. Since every uncountable set of consistent propositional formulae contains an infinite consistent subset, this cannot be generalized to the uncountable case. The problem is which hypergraphs are inconsistency hypergraphs of sets of propositional formulae?

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An honest function is, roughly speaking, a unary, recursive, and strictly increasing function with a very simple graph. The honest subrecursive degrees are the degree structure induced on the honest functions by the reducibility relation "being (Kalmár) elementary in". A recursive function turns out to be total if and only if it is elementary in some honest function. Thus, since the set of functions elementary in a particular honest function constitutes a complexity class, the structure of honest subrecursive degrees will provide a measure for the computational complexity of any total recursive function f. If f is not elementary in an honest function of degree $\mathbf{a}$, it is because f is too hard to compute, i.e., it requires more resources to compute f than the honest degree $\mathbf{a}$ allows.

The most popular current subrecursive degree theory is *not* a theory of honest degrees, but a theory that is concerned with reducibility between recursive sets. In our discussion it is convenient to use the reducibility relation "being (Kalmár) elementary in", and we use $\leq_E$ to denote this relation. Thus the degrees in the set-degree theory are the equivalence classes induced on the recursive sets by the $\leq_E$-relation. This set-degree theory is in many respects different from the honest degree theory induced by the same reducibility relation. The proof methods are different; the degree structures are different. Further, it is *not* natural to view the set-degrees as degrees of computational complexity. Let B be a recursive non-elementary set, and let $\mathcal{B} = \{\, A \mid A \leq_E B$ and A is a set$\,\}$. Then $\mathcal{B}$ is not a complexity class in the sense that every set computable within a certain amount of resources belongs to $\mathcal{B}$. No, $A \leq_E B$ if and only if B contains enough information to decide membership in A within elementary time. So $\mathcal{B}$ is the class of sets that is computable in elementary time if we have access to the information in B, i.e., if we do not have to compute B. It is reasonable to view the set-degrees as degrees of information content and not as degrees of computational complexity. If a set A is not subrecursively reducible to a set in the set-degree $\mathbf{a}$, it is because A contains too much information, i.e., more information than the degree $\mathbf{a}$ permits.

The objective of my research is to obtain relationships between the computational complexity and information content of sets. Here are some technical results from [1]:

Let $\cdot'$ be the jump operator on the honest degrees. Let $\mathbf{a} \ll \mathbf{b}$ denote that the honest degree $\mathbf{a}$ lies *way below* the honest degree $\mathbf{b}$. When $\mathbf{c}$ is the elementary degree of the honest function f, we define then the cone $\leq_E (\mathbf{c})_* \stackrel{\text{def}}{=} \{\, A \mid A$ is elementary in f and A is a set$\,\}$.

• The cone $\leq_E (\mathbf{b})_*$ contains an effectively $\leq_E (\mathbf{a})_*$-hard set if and only if $\mathbf{a}' \leq \mathbf{b}$.

• Assume $\mathbf{0} \ll \mathbf{b}$. The cone $\leq_E (\mathbf{b})_*$ contains a $\leq_E (\mathbf{a})_*$-hard set if and only if $\mathbf{a} \ll \mathbf{b}$.

• $\leq_E(\mathbf{a})_* \subseteq \leq_E(\mathbf{b})_*$ if and only if $\mathbf{a} \leq \mathbf{b}$.

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Recently, some new algorithmical tools for SAT decision have been developed, resulting in new worst case upper bounds for SAT decision. For example:

• a two-stage construction for "good" branchings, using a generalization (called "Generalized Sign Principle" (GSP)) of the "positive clause criterion," which, together with various refinements of the (original) Davis-Putnam method, results in the bound $2^{0.177 \cdot n}$ for SAT-3 (every variable occurs at most three times) in [2], where n is the number of variables, improving the bound $2^{1/3 \cdot n}$ from [3];

• a generalization of GSP in [1] (by combination with "Autarkness"), yielding the bound

$2^{0.309 \cdot k}$ for SAT, where k is the number of clauses, improving the bound $2^{1/3 \cdot k}$ from [3] (see also [2] for a very short algorithm for that bound (by GSP)).

We discuss these and related ideas to point out some future directions for more "intelligent" SAT algorithms.

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Some theorems of Hajnal about partition relations on $\omega_1 \cdot \omega$ and $(\omega_1)^2$ are strengthened by replacing the Continuum Hypothesis with the assumption that there is a scale of length ω_1 under eventual domination in functions from ω into ω.

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A question from the folklore of set theory is, "how many normal measures can a cardinal host?" Kunen [4] showed that starting wtih one measurable cardinal κ, there is an inner model $L[U]$ such that κ hosts exactly one normal measure. Later, Kunen-Paris [5] showed that under the same initial assumption, there is a forcing extension where κ hosts 2^{2^κ}, the maximum possible number of measures.

DEFINITION (Mitchell). Given two normal measures U and U' both on the same cardinal κ, say $U \lhd U'$ if and only if U is in the transitive collapse of $\mathrm{Ult}(V, U')$. The ordering $\lhd$ is well-founded, and hence assigns a rank to all normal measures on κ. The order of κ, $o(\kappa)$, is the set of these ranks.

Employing these stronger hypotheses, Stuart Baldwin [1] showed using inner model theory that if there is a model of $o(\kappa) = \lambda < \kappa$, then there is a model where κ is the only measurable cardinal and κ hosts λ normal measures. Recently, Cummings [2, 3] and Witzany [7] have explored Mitchell's $\lhd$-ordering in Cohen generic extensions of the core model. We use techniques similar to these (a blend of inner model theory and forcing) to prove results extending Baldwin's.

THEOREM . *Suppose there is a model of ZFC with a measurable cardinal* κ *of order* $\lambda < \kappa^{++}$. *Then there is a model of ZFC such that* κ *is the only measurable cardinal, and such that* κ *hosts exactly* λ *normal measures.*

To see this, we force over the core model for sequences of measures with Magidor's [6] iterated Prikry notion.

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► JUI-LIN LEE, *Branching program, commutator, and icosahedron.*

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Given a finite function $f : B \times B \to B$ with $|B| = k$, a constant. For inputs $\vec{b} = b_1 b_2 \dots b_n \in B^*$ and $n = 2^m$, we may consider the following computations: $f(\dots f(f(b_1, b_2), b_3)\dots)$ (denoted by $LR(f)[\vec{b}]$), $f(b_1, f(b_2, \dots f(b_{n-1}, b_n)\dots))$ (denoted by $RL(f)[\vec{b}]$), and divide-and-conquer via f on $\vec{b}$ (denoted by $DC(f)[\vec{b}]$). $LR(f)$ and $RL(f)$ are of the same type, iteration with depth n and value bounded by k. $DC(f)$ is tree-like with depth $\log n$.

In this paper first we give a direct proof (as in [2]) of a known result: k-BRN ($k \geq 4$) is equivalent to $\mathrm{tree}(x)$ under AC^0 reduction, where k-BRN is the recursion on notation with value bounded by k and recursion $|x|$ many times, and $\mathrm{tree}(x)$ is the OR-AND complete binary tree applying to the first 4^m bits of x such that $4^m < |x| \leq 4^{m+1}$. The crucial part of this proof is the existence of three 5-cycles σ, δ, τ (in [1]) with $\sigma\delta\sigma^{-1}\delta^{-1} = \tau$, by which the Boolean evaluation $(c_1 \wedge c_2)$ can be simulated. This phenomenon catches our attention. Let $\alpha * \beta = \alpha\beta\alpha^{-1}\beta^{-1}$, the commutator operation. By calculating $*$ over 5-cycles in S_5, we find an interesting structure: There are 2 disjoint subsets of 5-cycles (in S_5), say, A, A', which satisfy the following conditions: (1) $|A| = |A'| = 12$. (2) $\alpha \in A$ (or A') implies $\alpha^{-1} \in A$ (or A'). (3) For any $\alpha, \beta \in A$ (or A'), if $\alpha * \beta \neq e$, the identity element in S_5, then $\alpha * \beta \in A$ (or A'). By $*$ and such α, β, $A \cup \{e\}$ (or $A' \cup \{e\}$) can be generated.

By conjugate operation it is clear that A, A' are isomorphic. (Note that $A' = \{\alpha^2 : \alpha \in A\}$.) We may restrict $*$ on $A \cup \{e\}$. Now we investigate the computing power of LR, RL, DC according to the properties of f and the size of B. We have the following results: (1) If $|B| \leq 4$, then $AC^0(LR(f)) \subseteq AC^0(6)$. (2) If f is associative and commutative, then $AC^0(LR(f)) \subseteq ACC$. (3) If f is associative and $|B| < 60$, then $AC^0(LR(f)) \subseteq ACC$. (4) There is a nonassociative, commutative f with $|B| = 2$ such that $AC^0(DC(f)) = NC^1$. (5) There is a nonassociative, commutative f with $|B| = 5$ such that $AC^0(LR(f)) = NC^1$. (6) $AC^0(DC(*)) = NC^1$ and $AC^0(LR(*)) = AC^0(10)$ for $B = A \cup \{e\}$. Note that the results are true for the RL case.

Our main contribution in this paper is (6). To prove the second part in (6), we need to visualize $*$ geometrically, by assigning the twelve 5-cycles in A to the vertices of icosahedron so that $\alpha * \beta = \gamma$ is geometrically invariant, i.e., for any rotation R on the icosahedron, $R(\alpha) * R(\beta) = R(\gamma)$.

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► KERKKO LUOSTO, *Resumptions of cardinality quantifiers.*

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The simplest instance of a resumption or a vectorization occurs when we are given a cardinality quantifier: Let S be a class of cardinals and C_S the corresponding *cardinality quantifier*, i.e., for every $\{U\}$-structure $\mathfrak{M}$ we have

$$\mathfrak{M} \models C_S x U(x) \iff |U^{\mathfrak{M}}| \in S.$$

Here C_S binds a single variable x. Intuitively, we get the nth *resumption* $C_S^{(n)}$ by replacing x with a n-tuple $\bar{x}$, so that if R is an n-ary relation symbol, then for every $\{R\}$-structure $\mathfrak{M}$ it holds that

$$\mathfrak{M} \models C_S^{(n)} \bar{x} R(x) \iff |R^{\mathfrak{M}}| \in S.$$

Resumptions are of considerable interest in finite model theory, most notably due to the result of Anuj Dawar [1] to the effect that if there is a logic for PTIME queries, then there is one generated by a sequence of resumptions of a single quantifier Q, i.e., one of the form $\mathcal{L} = \mathcal{L}_{\omega\omega}(\{ Q^{(n)} \mid n \in \omega \})$. The expressive power of $\mathcal{L}_{\omega\omega}(Q^{(n)})$ for an arbitrary quantifier Q is not generally known. Concentrating on second resumptions of cardinality quantifiers, I prove that if $S \subset \omega$ satisfies a reasonable non-periodicity condition named strong non-periodicity, then

$$\mathcal{L}_{\omega\omega}(C_S^{(2)}) \text{ is monadically hierarchical,}$$

and consequently $C_S^{(2)}$ is not definable by means of any monadic quantifier (or quantifier binding variables in one or many formulas, but only one in each). The proof uses methods of Ramsey theory and of topological dynamics, and is an elaboration of my result [2] that $\mathcal{L}_{\omega\omega}(I^{(2)})$ is monadically hierarchical where I is the Härtig quantifier.

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[2] Kerkko Luosto, *Hierarchies of monadic generalized quantifiers*, ***Reports of the Department of Mathematics 125***, University of Helsinki, 1996, 24 pages.

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We study some natural problems in the theory of ordinary differential equations in $\mathbb{R}^n$ from the point of view of descriptive set theory.

A Cauchy problem of dimension $n > 0$ is a triple (F, x, y) where $F : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous, $x \in \mathbb{R}$ and $y \in \mathbb{R}^n$. A solution of the Cauchy problem (F, x, y) is a differentiable $\varphi : I \to \mathbb{R}^n$ where $I \subseteq \mathbb{R}$ is an open interval, $x \in I$, $\varphi(x) = y$ and $\varphi'(t) = F(t, \varphi(t))$, for all $t \in I$. Let $\mathcal{E}^n = C(\mathbb{R}^{n+1}, \mathbb{R}^n)$ be the space of all differential equations of dimension n and $\mathcal{C}^n = \mathcal{E}^n \times \mathbb{R} \times \mathbb{R}^n$ be the space of all Cauchy problems of dimension n. If we endow $C(\mathbb{R}^{n+1}, \mathbb{R}^n)$ with the compact-open topology, then $\mathcal{E}^n$ and $\mathcal{C}^n$ become Polish spaces. We are interested in various "natural" subsets of these spaces.

Let $\mathcal{U}^n \subset \mathcal{C}^n$ be the set of Cauchy problems of dimension n which have a unique solution and let $\mathcal{U}^n_\forall \subset \mathcal{E}^n$ be the set of differential equations of dimension n such that for every $(x, y) \in \mathbb{R} \times \mathbb{R}^n$, $(F, x, y) \in \mathcal{U}^n$. Let $\mathcal{LU}^n \subset \mathcal{C}^n$ be the set of Cauchy problems of dimension n which have a locally unique solution. Let $\mathcal{G}^n \subset \mathcal{C}^n$ be the set of Cauchy problems of dimension n which have a global (i.e., defined on $\mathbb{R}$) solution and let $\mathcal{G}^n_\forall \subset \mathcal{E}^n$ be the set of differential equations of dimension n such that for every $(x, y) \in \mathbb{R} \times \mathbb{R}^n$, $(F, x, y) \in \mathcal{G}^n$.

We classify these sets within the Borel and projective hierarchies by proving: for every $n > 0$, $\mathcal{U}^n$ and $\mathcal{U}^n_\forall$ are $\mathbf{\Pi}^0_2$-complete, $\mathcal{LU}^n$ is $\mathbf{\Sigma}^0_3$-complete; $\mathcal{G}^1$ is $\mathbf{\Sigma}^0_4$-complete, $\mathcal{G}^1_\forall$ is $\mathbf{\Pi}^0_3$-complete, and for every $n > 1$, $\mathcal{G}^n$ is $\mathbf{\Sigma}^1_1$-complete and $\mathcal{G}^n_\forall$ is $\mathbf{\Pi}^1_1$-complete. Therefore the complexity of the uniqueness problem is independent from the dimension of the problem and is Borel, while the complexity of the globality problem is greater and varies with dimension, being non-Borel if $n > 1$. Notice that $\mathcal{G}^1$ is a very rare example of a $\mathbf{\Sigma}^0_4$-complete set which arises naturally in mathematics.

If $n > 1$ we can specialize to differential equations of dimension n which arise from (one-dimensional) differential equations of order n; for these restricted classes of Cauchy problems and ordinary differential equations we obtain results similar to the ones listed above.

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Say $\vec{E}$ is a coherent sequence of extenders on κ. We introduce the notion of an extender sequence, a generalization of measure sequence. Using it we'll present a forcing notion, generalizing Gitik-Magidor forcing, which blows 2^κ by adding many Radin sequences to κ while keeping the nice properties of both Radin and Gitik-Magidor forcings. The wider the extenders the bigger is 2^κ. If the coherent sequence length is κ^+ then κ remains inaccessible in the generic extension. If the length is κ^{+3} then κ remains measurable.

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We show that many cardinal invariants of the reals can be changed without changing cardinals or the reals, and we show some results on the consistency strength of such a procedure. We also show that there are some settings in which cardinal invariants are really invariant.

You may conclude that in *ZFC* the terminology "invariant" is not too questionable, whereas believing in large cardinals some objection against the terminology may be justified in the light of our results.

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In the axiomatic systems of probability of Popper [2, Appendices $*$iv and $*$v] and in the weaker systems of Popper and Miller [3, Section 1] it is demonstrable that the operation represented by concatenation satisfies the conditions of idempotence $p(xx|w) = p(x|w)$, commutation $p(xz|w) = p(zx|w)$, and associativity $p(x(yz)|w) = p((xy)z|w)$ in the first argument. It may be shown, in contrast, that substitutivity of lattice-theoretically identical terms holds in the second argument of p if and only if some purpose-built axiom such as

(sub2) $$\forall y\,[p(x|y) = p(z|y)] \implies \forall y\,[p(y|x) = p(y|z)]$$

is introduced. Popper himself noted that it would suffice in his systems to postulate, in place of (sub2), the law of commutativity for the second argument of p; that is, that

(comm2) $$p(y|xz) = p(y|zx).$$

In [1] Leblanc showed that (sub2) is a consequence of (comm2) when we have the premises:

(bounds) $$0 \leq p(x|z) \leq 1$$

(unity) $$p(x|x) = 1$$

(comm1) $$p(xz|y) = p(zx|y)$$

(product) $$p(xz|y) = p(x|zy)p(z|y).$$

He was at pains to emphasize the weakness of these premises, and conjectured that each of them is necessary to the proof. In this paper we show that the premises are stronger than Leblanc supposed, and we offer two alternative sets of premises, each set strictly weaker than Leblanc's, for the derivation of (sub2) from (comm2). The first is obtained by replacing (bounds) and (unity) by the two premises

(nonzero) $p(y|y) \neq 0$

(id2) $p(x|z) = p(x|zz)$

and the second is the set

(upbound) $p(x|z) \leq p(y|y)$

(monotony) $p(xz|y) \leq p(x|y)$

(product) $p(xz|y) = p(x|zy)p(z|y)$.

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▶ ANGELO MONTANARI, ADRIANO PERON, and ALBERT POLICRITI, *Exploiting systolic and Rabin tree automata to decide time granularity*.
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In this work, we focus on decidability problems for metric and layered temporal logic (MLTL for short). MLTL has been proposed by Montanari in [3] as a formal tool for specififying granular real-time systems, and it can be viewed as the combination of a number of differently-grained metric temporal logics. The decidability problem for the pure metric (non-granular) fragment has been addressed by Alur and Henzinger in [1], where they showed that, under suitable assumptions about the temporal domain and the associated operations, the validity and satisfiability problems for real-time logics extending propositional temporal logics with metric features can be reduced to the corresponding decidability problems for the decidable theory $S1S$. A first extension of their results, that allows one to treat situations in which a **finite** number of coarsenings/refinements of the temporal domain is sufficient, has been presented in [2]. The basic tool for proving decidability is still the theory $S1S$, and the basic engine is Büchi theorem on the decidability of regular ω-languages. In the present work, we deal with the more general case of temporal structure consisting of **infinitely many** temporal layers. We first consider temporal structures in which there is a finest temporal domain together with an infinite number of coarser and coarser domains. To deal with such structures, we use the more expressive theory, called $S1S^k$, which is a proper extension of $S1S$. The decidability of $S1S^k$ is shown by using a more powerful basic engine, namely the decidability of ω-languages recognized by k-ary Systolic Tree Automata [4]. Successively, we deal with the problem of deciding infinitely refinable structures. We prove that the decidability of the satisfiability problem for the theory of such structures can be reduced to the decidability of the same problem for the well-known monadic second-order decidable theory of k successors SkS. In this case the basic decidability engine in Rabin's Tree Theorem.

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In constructive theories a notion of apartness is often taken as basic, and equality appears as a defined concept, the negation of apartness. The same can be done with order relations, and weak linear order can be defined as the negation of a strict linear order. In the case of the intuitionistic continuum, it is the positive relations of apartness and strict order that have a direct computational (observable) meaning, whereas the verification of equality and partial order can involve a nonterminating computation. The process of "positivization" of order relations can be pushed further so as to include partial orders, lattices and Heyting algebras (as in the work by Jan von Plato). We take up here the task of investigating into the proof theory of these apartness and order relations.

We establish for these theories sequent calculi which enjoy cut elimination and the disjunction property. This is achieved by adding to the logical calculus G3ip for intuitionistic propositional logic the axioms of the theories under consideration in the form of sequent calculus axiom and rules. The rules are put in a form that guarantees cut eliminations to be maintained also for these extensions of the calculus.

We pose the question whether the positive theories, based on apartness and positive order relations, are conservative over those based on equality and weak order when these are defined as the negations of the positive relations. This is not just a question of application of proof theory to the study of mathematical structures but was originally prompted by the application of positive Heyting algebras for a semantics of refutation for intuitionistic logic.

As expected, the answer is positive: By techniques of manipulation of proofs, we show how to replace the axioms for the theories based on apartness and order with the corresponding axioms based on equality and partial order. All the results are presented in a modular way, i.e., the same proof goes through for all the theories considered.

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A set A is *enumeration reducible* to a set B ($A \leq_e B$) if there exists a computably enumerable set W such that

$$A = \{\, x : (\exists u)\,[\langle x, u\rangle \in W \;\&\; D_u \subseteq B]\,\},$$

where D_u denotes the finite set with canonical index u. Let $A \equiv_e B$ if and only if $A \leq_e B$ and $B \leq_e A$, and, for any set X, let $\deg_e(X)$ be the *e-degree* of X, i.e., the equivalence class of X under $\equiv_e$. The e-degrees constitute an uppersemilattice $\langle \mathfrak{D}_e, \mathbf{0}_e, \leq\rangle$ with zero, where $\leq$ is the partial ordering originated by $\leq_e$, and the least element $\mathbf{0}_e$ is the e-degree of the computably enumerable sets; $\mathfrak{D}_e$ can be equipped with a suitable jump operation $'$. It is known that $\mathbf{0}'_e = \deg_e(\bar{K})$ (where $\bar{K}$ is the complement of the halting set) and, for every set A, $\deg_e(A) \leq \mathbf{0}'_e$ if and only if $A \in \Sigma^0_2$. Let $\mathfrak{S} = \mathfrak{D}_e(\leq \mathbf{0}'_e)$. We prove that in $\mathfrak{S}$ every element different from $\mathbf{0}'_e$ is branching:

THEOREM . *If* $\mathbf{a} < \mathbf{0}'_e$ *then there exist* $\mathbf{b}, \mathbf{c} \in \mathfrak{S}$ *such that* $\mathbf{a} < \mathbf{b}, \mathbf{a} < \mathbf{c}$ *and* $\mathbf{a} = \mathbf{b} \cap \mathbf{c}$.

The theorem shows one more elementary difference between $\mathfrak{S}$ and the structure of the Turing degrees of the computably enumerable sets, in which nonbranching incomplete elements are known to exist.

► KARL-HEINZ NIGGL, *The* μ*-measure as a tool for classifying computational complexity*.
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Consider the following simply typed term systems both for representing primitive recursive functions ($\mathcal{PR}$), with variables of *ground type* ι only:

$$PR_1 \ := \ x \mid (\lambda x.r^\tau)^{\iota\to\tau} \mid (r^{\iota\to\tau}s^\iota)^\tau \mid 0^\iota, S^{\iota\to\iota} \mid (R\,r^\iota s^{\iota,\iota\to\iota})^{\iota\to\iota}$$
$$PR_2 \ := \ x \mid (\lambda x.r^\tau)^{\iota\to\tau} \mid (r^{\iota\to\tau}s^\iota)^\tau \mid 0^\iota, s_0^{\iota\to\iota}, s_1^{\iota\to\iota} \mid (RN\,r^\iota S_0^{\iota,\iota\to\iota}S_1^{\iota,\iota\to\iota})^{\iota\to\iota}.$$

So PR_1 is based on *primitive recursion* and the successor function, and PR_2 on *recursion on notation* and the binary successor functions. Terms of PR_i represent *algorithms* via a strongly normalising reduction relation $\to_i$ with uniquely determined normal forms. To determine the computational complexity of an algorithm $t \in PR_i$ we use the μ*-measure* of [4]. The latter operates on a simply typed term system $\mathcal{PR}^\omega$ for defining higher type *subrecursive functionals* on Scott domains, supporting *parallelism* in the sense of Plotkin.

The μ-measure provides a notion of *modified recursion lying on top of each other* based on both a *global* and a *local* view of the computational complexity of an algorithm t. This entails that μ is invariant on $\alpha\beta\eta$ representations of t (cf. [4]). Furthermore, μ uniformly integrates traditional results in subrecursion theory with resource-free characterisations of low complexity classes. To set the stage, we define *modified Heinermann classes*

$$\mathcal{R}_i^n := \{f \in \mathcal{PR} \mid f \text{ has a representation } t_f \in PR_i \text{ with } \mu(t_f) \leq n\}.$$

Extending the Schwichtenberg/Müller result [5, 3], we show $\forall n \geq 1.\ \mathcal{E}_{n+1} = \mathcal{R}_1^n$. Thus by Ritchie's result, $\mathcal{R}_1^1$ characterises the set FLINSPACE of linear-space computable functions. Accordingly, $\mathcal{R}_2^1$ characterises the set FPTIME of polynomial time computable functions. This streamlines the result and proof of [1]. Furthermore, the classes $\mathcal{R}_2^n$ and $\mathcal{R}_1^n$ coincide at and above level 2. Remarkably every definition of a function in $\mathcal{E}_{n+2}$ can be *directly* transformed into a definition in $\mathcal{R}_1^{n+1}$, unlike the Schwichtenberg and Müller proofs. Thus there is no reference to any machine-based computation model.

Recently Bellantoni [2] proposed similar results with respect to classes $\mathcal{PR}_i^n$ based on a ranking ρ for *definitions* D_f of primitive recursive functions f. But the proof of the so-called Bounding Lemma, the pivot of all these results, contains a severe proof gab that cannot be closed in the way the lemma is set up. The present paper establishes particularly a modified Bounding Lemma that can take the rôle of Bellantoni's Bounding Lemma. In contrast to ρ, the μ-measure recognises subterms having no *computational meaning* [4] and *improper recursions*, i.e., recursions where the *critical* input position(s) of the step term(s) is (are) redundant. Thus μ does not require any initial functions other than zero and successor.

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Thanks to J. Etchemendy's incisive criticism of Tarskian model theoretic definition of logical consequence, we can witness the resurgence of interest in some of the most fundamental notions in logic. In this paper, we propose to examine the logical status of the axiom of infinity via defending Tarski against Etchemendy.

What is notable in Etchemendy's criticism is his claim that in order for Tarskian position to be definsible we need to adopt what Etchemendy calls "the reduction principle", which is counterintuitive. The problem is that we cannot find any textual evidence in Tarskian corpus for that claim. Further, there is no need for the reduction principle as long as we adopt the axiom of infinity. So, everything boils down to the logical status of the axiom of infinity.

Indeed, Etchemendy uncritically follows Wittgenstein's step in his criticism of the axiom of infinity, thereby downgrading it as extralogical. In this paper, I will show that by exploiting Tarski's discussion of the distinction between what is logical and what is extralogical we may understand the axiom of infinity as logical as any other typical axioms of set theory. Also, I shall present an argument reminiscent of Leibniz to nullify Wittgensteinian criticism. Unlike Etchemendy, who assumes that we can infer from the imcompossibility of the axiom of infinity and its negation the contingency of both, I shall prove that either one must be a logical truth.

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This paper originated while solving the problem of the structure and the cardinality of one-variable fragments of propositional classical linear logic (without modalities) extended with weakening and n-contraction. In particular, n-contraction ($n \geq 2$) is a version of the contraction rule where $(n+1)$ occurrences of a formula may be contracted to n occurrences. However, the linear systems discussed do not enjoy the cut-elimination property. Thus, the standard proof-theoretic methods to examine provable equivalence of formulas are not available. Therefore, we took a model-theoretic approach constructing free algebras (on one generator) that correspond to the linear (one-variable) fragments considered. In particular, we introduced free algebras for multiplicative classical linear logic and its affine extensions with n-contraction (witness: ***Notre Dame Journal of Formal Logic***, vol. 37, 1996). In the same paper, we also proved the existence of two infinite chains in the free algebra on one generator ($\langle\star,+\rangle - CPL_2^a$-algebra), corresponding to the one-variable affine linear system with 2-contraction; solving along the cardinality problem of some weaker fragments as well. In the present piece of work, we give the solutions to the open problems stated in the earlier paper. Briefly, we construct free algebras (on one generator) corresponding to the affine multiplicative classical linear logic with constants and negation, as well as for their extensions with n-contraction. The respective embeddings of the infinite free $\langle\star,+\rangle - CPL_2^a$-algebra into the free algebras just discussed are furnished with. And finally, we prove that the one-variable fragment of the full affine classical linear logic with n-contraction is infinite: we first construct a free lattice with top and bottom generated by a suitable free partially ordered double commutative monoid with involution. This infinite free structure is not (yet) a Lindenbaum algebra of the one-variable fragment considered but can be turned into its infinite algebraic model with a surjective valuation.

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Vaught's conjecture is that, for any countable theory T, the number of nonisomorphic countable models of T is either countable or continuum. Vaught's conjecture for ω-stable theories of modules over a ring was proved by Garavaglia [3]. Ziegler [4] has shown that Vaught's conjecture is true for any complete theory of modules over a (countable) commutative Dedekind domain.

An R-module M is called *uniserial* if the lattice of submodules of M is a chain. A module that is a direct sum of uniserial modules is called *serial*. A ring R is called *serial* if both R_R and ${}_R R$ are serial modules.

It is proved

THEOREM . *Vaught's conjecture is true for an arbitrary complete theory of modules over a countable serial ring.*

There is no doubt that for proving Vaught's conjecture for modules over a fixed ring we need to have the well developed model theory of modules, i.e., some classification of pure-injective modules over this ring. The main technical tools we make use of are the Eklof-Herzog's classification of pure-injective modules over a serial ring [1] and the Facchini-Puninski's machinery of localization in a serial ring with respect to an Ore set corresponding to an ω-stable indecomposable module [2]. One of the main points is to prove that every indecomposable pure-injective module with few types over a serial ring is ω-stable.

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[4] M. ZIEGLER, *Model theory of modules*, ***Annals of Pure and Applied Logic***, vol. 26 (1984), pp. 149–213.

► REINERT A. RINVOLD, *Categories of sets with structure*.
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Girard introduced the concepts of dilator and ptykes in proof theory. He also used similar ideas in domain theory. Stable functions in qualitative domains and normal functors in quantitative domains have much in common with dilators. In my thesis from 1992 I developed a generalized concept of denotation system including all these cases in a common theory. The dilators are functors from ordinals to ordinals. In the general theory I replaced ordinals with what I call categories of sets with structure, CSS. This talk with introduce CSS.

CSS is an axiomatization of what a set with structure is. In the language of model theory a set with structure is a model of a theory in a (first order) language. In a CSS however, the structure of a CSS object is given by the morphisms of the category and a closure operator generating subobjects.

As in universal algebra the subobjects of an object is a lattice. The morphisms in a CSS are maps between the underlying sets which respects the closure operator. It is also required that two morphisms are equal if they are equal on a generating set. In universal algebra, the morphisms are given by the operations of the algebra. The morphisms of a CSS however, are not uniquely given by the closure operation of the CSS.

Every CSS has an internal language of relations and operators. The arities of the internal relations and operations can be any ordinals. One of the results is that the morphisms are uniquely given by the internal relations and that the closure operator is generated by the internal operations of the CSS.

A CSS has its internal theory in its internal language. Actually there is one theory in $L_{\lambda\lambda}$ for each cardinal λ in the internal relations of arity less than λ. As λ increases the category of models for the theories get better and better approximations to the CSS.

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Our main result concerns the expressive power of fragments of first-order logic defined in terms of quantifier prefixes. If p, q are quantifier prefixes, that is, finite words over $\exists, \forall$, we write $p \leq q$ if and only if p can be monotonically embedded in q. (Equivalently, p can be obtained from q by removing some letters from the latter.) For p a prefix, let $\mathrm{FO}(p)$ be the set of first-order sentences in prefix normal form with prefix p. We prove:

THEOREM . *Let p be a quantifier prefix. Then there is a sentence $\theta \in \mathrm{FO}(p)$, over a single binary relation, such that for each prefix q, if $p \not\leq q$, then θ is not equivalent to any sentence in* $\mathrm{FO}(q)$.

This solves a conjecture due to Grädel and McColm [1], and strengthens a result of Walkoe [2]. The proof uses a modified Ehrenfeucht-Fraïssé game and ultrahomogeneous structures.

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► MATTHEW C. SALTS, *On c. e. predecessors of d. c. e. degrees*.
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Cooper and Yi [2] prove that some d. c. e. degrees (the isolated degrees) have immediate c. e. predecessors—i.e., there exists a c. e. degree $\mathbf{a}$, and a d. c. e. degree $\mathbf{d} > \mathbf{a}$ such that any c. e. degree $\mathbf{b} < \mathbf{d}$ is also below $\mathbf{a}$. Further results concerning the relationship of c. e. and d. c. e. degrees were provided by Cooper and Yi [2], who showed that there exist non-isolated d. c. e. degrees, LaForte [4], and, independently Ding and Qian [3], who proved that the isolated d. c. e. degrees (and the c. e. degrees which isolate them) are dense in the upper semi-lattice of c. e. degrees, $\mathcal{C}$, Arslanov, Lempp and Shore [1], who construct a c. e. degree which fails to isolate a d. c. e. degree, and show that these non-isolating degrees are downwards dense in $\mathcal{C}$, and Salts [5], who constructs an interval of isolating c. e. degrees. These results and further questions concerning the distribution of isolating and non-isolating degrees in $\mathcal{C}$ will be discussed.

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Our aim is to prove that for example if $(\delta_n : n \in \omega)$ is a strictly increasing sequence of weakly compact cardinals and $\delta_{2n+2} = \delta_{2n+1}^+$ for all $n \in \omega$ then there is an inner model with ω many Woodin cardinals.

► VLADIMIR ST. SOSKOV, *A characterization of the next admissible set via enumerations*.
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Let $\mathfrak{A} = (B; R_1, R_2, \dots, R_k)$ be a countable abstract structure, where each R_j is an a_j-ary predicate on B.

A one to one mapping f of the set of the natural numbers N onto B is called *enumeration* of $\mathfrak{A}$.

Every enumeration f of $\mathfrak{A}$ determines a unique structure

$$\mathfrak{B}_f = (N; R_1^{\mathfrak{B}_f}, R_2^{\mathfrak{B}_f}, \dots, R_k^{\mathfrak{B}_f}),$$

where for all $x_1, \dots, x_{a_j} \in N$, $R_j^{\mathfrak{B}_f}(x_1, \dots, x_{a_j}) = R_j(f(x_1), \dots, f(x_{a_j}))$.

By $D(\mathfrak{B}_f)$ we shall denote the set of all Gödel numbers of the elements of the diagram of $\mathfrak{B}_f$.

For $X \subseteq N$ by Ind^X we shall denote the set of the indices of the hyperarithmetical relative to X sets, see [1].

By J_e^X we denote the *hyperarithmetical* relative to X set have index e.

We define the set of the *generalized indices* relative to X by means of the following inductive definition:

Definition .
(i) $\langle 0, e\rangle \in \mathrm{GInd}^X$ for all e.
(ii) If $e \in \mathrm{Ind}^X$ and $J_e^X \subseteq \mathrm{GInd}^X$ then $\langle 1, e\rangle \in \mathrm{GInd}^X$.

Let f be an enumeration of $\mathfrak{A}$. We define a mapping f^* of $\mathrm{GInd}^{D(\mathfrak{B}_f)}$ into the class $\mathbb{V}_B$ of all sets with urelements from B by means of the following inductive definition:

Definition .
(i) $f^*(\langle 0, e\rangle) = f(e)$;
(ii) $f^*(\langle 1, e\rangle) = \{ f^*(x) \mid x \in J_e^{D(\mathfrak{B}_f)} \}$.

Let $HYP_{\mathfrak{A}}$ be the least admissible set above $\mathfrak{A}$.

Theorem . *Let $\mathfrak{A}$ be such that $o(HYP_{\mathfrak{A}}) > \omega$ and $X \in \mathbb{V}_B$. Then the following assertions are true*:

(1) $X \in HYP_{\mathfrak{A}} \iff (\forall f)(\exists e \in \mathrm{GInd}^{D(\mathfrak{B}_f)})(f^*(e) = X)$.

(2) *X is Σ on $HYP_{\mathfrak{A}}$ if and only if for all enumerations f there exists a Π_1^1 in $D(\mathfrak{B}_f)$ subset A of* $\mathrm{GInd}^{D(\mathfrak{B}_f)}$ *such that $f^*[A] = X$.*

This theorem has some interesting corollaries. For example, combining Ivan Soskov's results from [2] with this theorem we can obtain *The next admissibile set*-theorem of Barwise, Gandy and Moschovakis for countable abstract structures.

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► KAZUYUKI TANAKA and TAKESHI YAMAZAKI, *A non-standard construction of Haar measure and* $\mathbf{WKL}_0$.
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This paper is a contribution to *Reverse Mathematics*, an ongoing program to determine which set existence axioms are needed to prove particular theorems of ordinary mathematics.

In [1], we have proved that every countable non-standard model of $\mathbf{WKL}_0$ has a proper initial part isomorphic to itself. This is a generalization of the self-embedding theorem for first-order arithmetic $\mathbf{I}\Sigma_1$ due to H. Friedman, Ressayre and others. By our theorem, any countable non-standard model $V = (M, S)$ of $\mathbf{WKL}_0$ has also an isomorphic extension ${}^*V = ({}^*M, {}^*S)$. Hence, taking ${}^*V$ as a non-standard universe in view of V, one can carry out many popular arguments in non-standard analysis within $\mathbf{WKL}_0$.

In this paper, we show

THEOREM $(\mathbf{RCA}_0)$. *The following are equivalent*:
(1) $\mathbf{WKL}_0$.
(2) *Any compact group has a Haar measure.*

Recall some definitions. A *complete separable metric space* $\hat{A}$ is coded by its dense set $A \subset \mathbb{N}$ together with a pseudo-metric d on it. Then $\hat{A}$ is said to be *compact* if there is a sequence of finite subsets of A, $\langle A_n : n \in \mathbb{N}\rangle$, such that A_n is a 2^{-n}-net of $\hat{A}$ for each n. Let $\hat{G}$ be a compact group, i.e., a compact separable metric space with continuous group-operations. A *Haar measure* on $\hat{G}$ is a σ-additive left-invariant positive linear operator μ on the space $C(\hat{G})$ of continuous functionals on $\hat{G}$ such that $\mu(1) = 1$.

Our proof for (1) $\rightarrow$ (2) of the theorem is not a straightforward formalization of the usual non-standard argument. Since the *infimum* or *supremum* may not exist without arithmetical comprehension, more sophisticated arguments of approximation are involved in the proof, some of which are inspired by the intuitionistic construction of Haar measure due to E. Bishop.

[1] K. TANAKA, *The self-embedding theorem of* $\mathbf{WKL}_0$ *and a non-standard method*, ***Annals of Pure and Applied Logic***, vol. 84 (1997), pp. 41–49.

► SERGEI TUPAILO, *Epsilon-substitution method for* Σ^1_1*-axiom of choice*.
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In [3, 2] Hilbert's epsilon-substitution method was extended to ramified language and two predicatively reducible subsystems, Arithmetical Comprehension $\mathbf{EA}$ and Δ^1_1-Comprehension Rule $\mathbf{\Delta}^1_1 - \mathbf{CR}$. The present paper is devoted to its extension to a stronger predicative theory, Σ^1_1-Axiom of Choice $\mathbf{\Sigma}^1_1 - \mathbf{AC}$. Extension to this theory presents additional difficulties, since its embedding into Ramified Analysis in standard treatments begins with elimination of cuts on formulas of complexity higher than Σ^1_1. This requires passage to an ω-derivation, raises its height up to ε_0, and makes an original finite system of critical formulas essentially infinite.

In [2] we dealt with infiniteness by introducing ordinal variables, whose values were to be computed at each step of epsilon-substitution process. This allowed to deal always with a finite (and the same) system of critical formulas. We show that our method applies to cut-free derivations in $\mathbf{\Sigma}^1_1 - \mathbf{AC}$ (still having $\Sigma^1_1 - AC$ as an axiom). Translation to a finite system of critical formulas now uses *Predicative Axiom of Choice*

$$\forall x\, \exists X^\alpha\, F[x, X] \rightarrow \exists Y^\beta\, \forall x\, F[x, Y_x]$$

where $\max(\alpha, \mathrm{level}(F[x, X])) < \beta$, which was introduced in [4]. We prove that epsilon-substitution algorithm terminates.

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► JAN VON PLATO, *Algebraic semantics of refutation for intuitionistic logic*.
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Recent systems of proof search for intuitionistic propositional logic produce in case of failed search Kripke models as counterexamples. We will here give a constructive algebraic semantics of refutation avoiding Kripke models. It reduces to the usual semantics of intuitionistic logic based on valuations to a Heyting algebra in case all possible refutations fail. The new semantics is based on the notion of a *positive* Heyting algebra that has a basic relation $a \succ b$, read as "a exceeds b". The order axioms for this relation are

(1) $-a \succ a$.
(2) $a \succ b \rightarrow a > c \vee c \succ b$.

The usual (weak) partial order is defined as negation of excess, and it follows from (1) and (2) that it is reflexive and transitive. Lattice and Heyting algebra operations are axiomatized using excess, which gives a constructively stronger system than a usual Heyting algebra. We define formula B to be *invalid* under formula A if there is a valuation v to a positive Heyting algebra such that $v(A) \succ v(B)$. It follows from our definition that B is a consequence of A if and only if for all valuations v, $v(A) \leq v(B)$. This is the standard definition of logical consequence for intuitionistic logic in terms of Heyting algebras.

Intuitionistic validity is here a universal closure of a negative property, therefore itself negative. The possibility of such a representation is due to the conservativity of the positive structures over the usual ones for a certain class of formulas.

The syntactic notion of underivability that corresponds to our definition of invalidity, is obtained in a straightforward manner from the axioms for positive Heyting algebras. It follows that the Lindenbaum algebras of intuitionistic propositional logic have the structure of positive Heyting algebras. A concrete example is give by the free Heyting algebra with one generator, or in other words, by the equivalence classes of propositions in just on atomic formula.

It is essential for our definition of constructive validity that a formula $a \leq b$ provable from the axioms of positive Heyting algebras, be also provable from those of a usual Heyting algebra and from the representation of weak partial order as a negative relation. A direct syntactic proof of this conservativity result through a sequent calculus formulation of positive Heyting algebras is given in a forthcoming paper by Sara Negri.

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The well known theory of algebraizable deductive systems cannot handle in a satisfactory way the algebraization of logics in which signature symbols may vary. For example, to algebraize equational or first-order logic one must first transform them to propositional logics in a rather artificial way. The model theoretic concept of institution and its possible algebraization through the use of categorical algebraic techniques seem to provide an appropriate context for handling these logics. Here we explore this possibility together with the relationship of this new algebraization process with the traditional one.

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In computability theory there are a number of theorems that state that certain *effective operations* are *continuous*. An effective operation is (roughly) a function between numbered sets that has an associated recursive function; the point is that the behaviour of the effective operation on objects is given by the behaviour of the associated recursive function on the indices of the objects. For instance, the Myhill-Sheperdson Theorem states that an effective operation on the partial recursive functions (with the standard numbering) is continuous, in the sense that the value for a given function is decided by a finite part of the function.

Another basic theorem, the Kreisel-Lacombe-Shoenfield Theorem, states that an effective operation on the total recursive funtions is also continuous. This result has been generalized in several directions; the most interesting for our purposes is the generalizations by Ershov and Berger. Berger shows that there is a correspondence between the hereditarily effective operations of finite type and the effective continuous functionals of finite type.

The goal of this paper is to generalize this and show a correspondence between the hereditarily effective operations of transfinite type (HEOT) and the effective continuous functionals of transfinite type (ECFT). HEOT have been defined by Beeson, whereas ECFT were first defined by Normann. Both of these constructions have been used to give semantics for Martin-Löf's Type Theory: HEOT by Beeson and ECFT by Waagbø.

The correspondence holds for a hierarchy of functionals with the natural numbers at the bottom and closed under Π- and Σ-type constructors. However, to get a model for Martin-Löf's Type Theory, one needs to include in the hierarchy an empty type and a type constructor for the identity type, and in this case the correspondence fails. So Beeson's model and the model based on Normann's constructions are in fact different.

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Kleene, Post and Spector proved that if $\mathbf{b}$ is an upper bound (u. b.) for an ascending sequence $\{\mathbf{a}_n\}_{n\in\omega}$ of degrees, then there exists a degree $\mathbf{c}$ such that $\mathbf{b}$ and $\mathbf{c}$ are an exact pair for $\{\mathbf{a}_n\}_{n\in\omega}$ (that is, $\mathbf{b}$ and $\mathbf{c}$ are u. b.s for $\{\mathbf{a}_n\}_{n\in\omega}$ such that $(\forall\mathbf{e})[[\mathbf{e}\leq\mathbf{b}\wedge\mathbf{e}\leq\mathbf{c}]\implies(\exists n)[\mathbf{e}\leq\mathbf{a}_n]]$. Owings raised the question of whether, for a given degree $\mathbf{d}>\mathbf{b}$, one can in general find such a $\mathbf{c}<\mathbf{d}$; Slaman has shown, by a cardinality argument, that this is not the case. However, it can be shown (using an argument based on 1-generic sets in the sense of Jockusch) that when $\mathbf{b}$ is a *uniform* upper bound for $\{\mathbf{a}_n\}_{n\in\omega}$, then for any $\mathbf{d}>\mathbf{b}$ which is computably enumerable (c. e.) relative to $\mathbf{b}$, there exists a degree $\mathbf{c}<\mathbf{d}$ such that $\mathbf{b}$ and $\mathbf{c}$ are an exact pair for $\{\mathbf{a}_n\}_{n\in\omega}$.

Slaman and Soare raised the question of whether the condition that $\mathbf{d}$ be c. e. relative to $\mathbf{b}$ is necessary. It turns out that if we weaken this condition so that $\mathbf{d}$ is only required to be Δ_2 relative to $\mathbf{b}$, the result no longer holds. In fact, $\{\mathbf{a}_n\}_{n\in\omega}$, $\mathbf{b}$, and $\mathbf{d}$ can be constructed (using a full approximation construction in the sense of Cooper) such that $\mathbf{b}$ is a uniform upper bound for $\{\mathbf{a}_n\}_{n\in\omega}$, $\mathbf{b}<\mathbf{d}<\mathbf{O}'$, and for any $\mathbf{c}<\mathbf{d}$, if $\mathbf{c}$ is an upper bound for $\{\mathbf{a}_n\}_{n\in\omega}$ then $\mathbf{b}<\mathbf{c}$. (After this second result was proved, it was discovered that it follows as an immediate corollary of the uniformity inherent in Lerman [1981].)

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Let Prov_Π represent in arithmetic the predicate of provability with a given logical form Π

(e.g., k-provability). Reflection principle $\mathrm{Prov}_\Pi(A) \to A$ was used by Parikh [4] to settle Kreisel's conjecture for the monadic version PA^* of Peano Arithmetic. Since $\mathrm{Prov}_\Pi(A)$ can be described in the monadic case by use of linear diophantine equations, the truth of $\mathrm{Prov}_\Pi(A)$ yields its derivability in PA^*. This suffices for deducing Kreisel's conjecture as the reflection principle is derivable in PA and its variants like PA^*. Kreisel [3] noted the possibility of using Parikh's argument for a sharpened version of the old conjecture.

THEOREM . *If $A[S^n0]$ is derivable with a proof data Π and n is sufficiently large relatively (some parameters determined by) A and Π, then the following formula is derivable for some r and m which are not large*:

$$a \equiv_r S^n0 \wedge a \geq S^m0 \to A(a).$$

The above holds for a wide class of extensions and variants of PA^*, see [1]. Extensions with a finite number of additional axioms make no problem in contrast to extensions with schemata. It is significant how one formulates rules of proofs as the identification of proof systems with the sets of their theorems is not relevant in the area. In any of the systems considered one can show the finitary version of Kreisel's conjecture in which the general statement is deduced from derivability of a finite number of its instances, comp [2]. We can also extend Kreisel's conjecture on formulae in several variables to get, see [1].

THEOREM . *Suppose that our proof system is complete with respect to Presburger arithmetic. Then for every formula $A(a_1 \dots a_p)$ and every k one can find a number m such that $\forall_{x_1 \dots x_p} A(x_1 \dots x_p)$ is derivable in our system if $A(S^{m_1}0 \dots S^{m_p}0)$ is derivable in $\leq k$ steps for each $m_1, \dots, m_p \leq m$ there.*

No easy reduction of the above to its subcase with $p = 1$ is possible. In particular, one should not expect to pass (without additional assumptions) from a proof of $A(S^{n_1}0, \dots, S^{n_p}0)$ with $n_1, \dots, n_p$ large to:

$$n_1 \equiv_{r_1} x_1 \wedge \dots \wedge n_p \equiv_{r_p} x_p \to A(x_1 \dots x_p).$$

[1] M. BAAZ and P. WOJTYLAK, *Generalizing proofs in monadic languages*.

[2] J. KRAJÍČEK and P. PUDLÁK, *The number of proof lines and size of proofs in first order logic*, ***Archive for Mathematical Logic***, vol. 27 (1988), pp. 69–84.

[3] G. KREISEL, *Proof theory: some personal recollections*, appendix to G. TAKEUTI, **Proof theory**, 2nd ed., North-Holland, 1987, pp. 397–407.

[4] R. J. PARIKH, *Some results on the length of proofs*, ***Transactions of the American Mathematical Society***, vol. 117 (1973), pp. 29–36.

► A. D. YASHIN, *A modified neighborhood semantics for strong future tense operator in intuitionistic propositional logic*.
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The strong future tense operator φ is defined in the class of tree-like intuitionistic Kripke models by

$a \vDash \varphi(A) \iff$ for every branch l starting from a there is a node $b \in l$ such that $b \vDash A$.

Properties of this operator may be described by the following four axiom schemes: 1°: $A \to \varphi(A)$, 2°: $\varphi(A) \to \neg\neg A$, 3°: $\varphi(A \to B) \to (\varphi(A) \to \varphi(B))$, 4°: $\varphi\varphi(A) \to \varphi(A)$ (see [1]). The logic *Kam* obtained from the intuitionistic propositional logic *Int* by extending of the language and adding 1°–4° will be called the *Kaminski logic*. *Kam* is sound and complete in the class of all tree frames of the height $\leq \omega$. However *Kam* does not have the finite model property with respect to above-mentioned interpretation: for example, the formula $\neg\neg p \to \varphi(p)$ is valid in the class of all finite frames but does not belong to *Kam*. So we need a suitable modification of semantics for interpreting φ.

We define *a frame with marked nodes* as a pair of the form $(W, \mathbf{D})$, where W is an ordinary

Kripke frame and $\mathbf{D} \subseteq W$ is a fixed subset (the set of *marked nodes*). Let us interpret φ in $(W, \mathbf{D})$ by

$$a \vDash \varphi(A) \iff \forall b \geq a\, (b \in \mathbf{D} \implies b \vDash A).$$

LEMMA . $(W, \mathbf{D}) \vDash 1^\circ \wedge 3^\circ \wedge 4^\circ$; $(W, \mathbf{D}) \vDash 2^\circ \iff \mathbf{D}$ *is cofinal in* W.

THEOREM . *If Kam* $\nvDash A$ *then there exists a finite frame* W *and a cofinal set* $\mathbf{D} \subseteq W$ *of marked nodes such that* $(W, \mathbf{D}) \nvDash A$.

EXAMPLE . *Kam* $+ 5^\circ$: $\varphi(A \vee B) \rightarrow (\varphi(A) \vee \varphi(B))$ is conservative over *Int*. Note that 5° is not valid with respect to the "branch" interpretation.

[1] M. KAMINSKI, *Nonstandard connectives for intuitionistic propositional logic*, ***Notre Dame Journal on Formal Logic***, vol. 29 (1988), no. 3, pp. 309–331.

Abstracts presented by title

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The notion of weak presentation of a model in a degree has been introduced by Jockusch and Shlapentokh. A model $\mathcal{M} = \langle \mathcal{M}, \{_\infty, \{_\in, \dots, \mathcal{R}_\infty, \mathcal{R}_\in, \dots \rangle$ is said to have weak presentation in a degree $\boldsymbol{a}$ if and only if there exists $A \in \boldsymbol{a}$ which is the universe of a model $\mathcal{A} = \langle \mathcal{A}, \{_\infty^\mathcal{A}, \{_\in^\mathcal{A}, \dots, \mathcal{R}_\infty^\mathcal{A}, \mathcal{R}_\in^\mathcal{A}, \dots \rangle$ where functions and relations of $\mathcal{A}$ have recursive extensions, and $\mathcal{A} \cong \mathcal{M}$.

We show that any recursive dense linear ordering without end points has weak presentations in any r. e. turing degree. More specifically we prove that any r. e. degree contains an ordering with the order type of rational numbers whose ordering is induced from a recursive ordering on ω. We further show that for any nonzero r. e. degree $\boldsymbol{c}$ there is a linear ordering $L \leq_T \boldsymbol{c}$ on ω having no weak presentation in any r. e. set, i.e., $\langle \omega, L \rangle \cong \langle B, R \rangle$ for an r. e. set B implies that R has no recursive extensions to whole ω.

► MOHAMMAD ARDESHIR, *Robinson's consistency theorem in basic predicate logic*.
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We prove a weak form of Robinson's consistency theorem in Basic Predicate Logic, BQC (see [1, 2, 4, 5]), a subsystem of Intuitionistic Predicate Logic, IQC. This weak form of Robinson's theorem is strong enough to be equivalent to the Craig interpolation theorem in BQC.

Combining the Craig interpolation theorem in BQC and a translation of IQC into BQC [1], results the Craig interpolation in IQC [3].

[1] M. ARDESHIR, *Aspects of basic logic*, ***Ph.D. thesis***, Marquette University, Milwaukee, 1995.

[2] M. ARDESHIR and W. RUITENBURG, *Basic propositional calculus I*, to appear, 1995.

[3] D. GABBAY, *Semantic proof of the Craig interpolation theorem for intuitionistic logic and extensions, Parts I and II*, ***Logic colloquium '69***, North-Holland, pp. 391–410.

[4] W. RUITENBURG, *Constructive logic and the paradoxes*, ***Modern Logic***, vol. 1 (1991), no. 4, pp. 271–301.

[5] A. VISSER, *A propositional logic with explicit fixed points*, ***Studia Logica***, vol. 40 (1981), pp. 155–175.

► OLGA N. BUSHMAKINA, *Idea of negative ontology*.
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Knowledge system is directed on the building of definition. But these definitions are negative. The limit of positiveness is statement of negativeness. The essence of limit or border has double nature that is involved paradoxal existence. The border is transformed in the object with infinite character. It is eliminated out of the limits of knowledge system and becomes transcendental. Existence of negative ontology gives us a possibility to define positive fundamental positions within the limits of this system. The positiveness as a basic position of system exists as self-identity or tautology. Tautology is represented as existence of uncertainty and representationlessness. The representationlessness as the positiveness or the putlessness are negation of position. But this negation is impossible so as the putless is before-putness that exists before negation. The positionlessness as before-putness is understood as pre-position or sub-position. But supposition is position or assumption. Supposition as position is selfistated as border between "before"-position and "after"-position. Positionlessness is supposed as supposition that supposes itself into its positiveness and is revealed itself as outline (as line draws itself) or suppliness. The outline of border exists as process of representation of representationless or as a process of putting in that is appeared relation of putting of putness or relation of self-suppose.

► ALEXANDER DENISOV, *On symbolism of double structural diagrams*.
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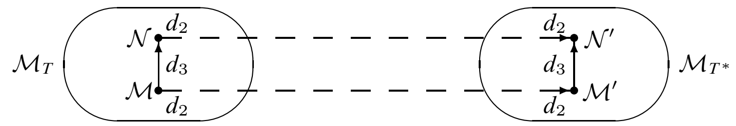
Let T be a theory of the first order language, M_T be the set of all (nonisomorphic) countable models of T. It is convenient to represent elements of $\mathcal{M}_T = \langle M_T, \leq \rangle$ by points of the plane with signs for $d_1(\mathcal{M})$—(Turing) degree of recursiveness of corresponding model, $d_2(\mathcal{M})$—degree of decidability of $\mathcal{M}$, $d_3(\mathcal{M}, \mathcal{N})$—degree of decidability of a map between two models in $\mathcal{M}_T$, $n(\mathcal{M})$—number of nonautoequivalent recursive numerations of $\mathcal{M}$. For example, if model $\mathcal{M} \in M_T$ is decidable, then its point carries symbol O.

DEFINITION ([1]). The set $\mathcal{M}_T^{(d_1, d_2, d_3, n)}$ will be called a structural semiordering of T.

THEOREM. *For every structural semiordering $\mathcal{M}_T^{(d_2, d_3)}$ there exists theory T^* equivalent to T by Turing reducability with an effective elimination of quantifiers such that $\mathcal{M}_T^{(d_2, d_3)} \cong \mathcal{M}_{T^*}^{(d_2, d_3)}$.*

In terms of structural diagrams [2]



Proof follows from the next structural diagram

[1] A. S. DENISOV, *Homogeneous O'-elements in structural semiorderings*, ***Algebra i logica***, vol. 28 (1989), no. 6, pp. 619–642, in Russian.

[2] ———, *Structual diagrams in mathematics*, ***Abstracts of short communications of ICM94***, 1994, Zurich, Switzerland, p. 1.

► FARZAD DIDEHVAR, *On the information content of integers with respect to a given acceptable enumeration of r. e. sets*.
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We define the notion of *Information content* in a standard enumeration of r. e. sets by induction on natural numbers as follows:

$$\Sigma^1(n) = W_n;$$
$$\Sigma^{k+1}(n) = \{\, x : \exists y \in \Sigma^k(n) \ \&\ x \in W_y \,\};$$
$$\Sigma(n) = \bigcup_{k=0}^{\infty} \Sigma^k(n).$$

For some $n \in \omega$ if $m \in \Sigma^k(n)$, we say that m is in *k-stage information of n* and if $m \in \Sigma(n)$ we say that m belongs to the *cumulative information of n*. We define a relation $\vdash$ on ω as follows:

$x \vdash y$ if for some natural number k, $\exists y_1, \dots, y_k\ (x \in W_{y_y}, y_z \in W_{y_3}, \dots, y_{k-1} \in W_k)$.

Let $\mathcal{L} = \langle \{\Sigma(n)\}_{n\in\omega}, \cup, \cap, \subseteq \rangle$. We show that $\mathcal{L}$ is a lattice. Furthermore, we mainly prove the following results:

(i) Let $\varphi = \{\varphi_i\}_{i\in I}$ and $\psi = \{\psi_i\}_{i\in I}$ be two acceptable enumerations of all partial-recursive functions, then the corresponding lattices $\mathcal{L}_\varphi$ and $\mathcal{L}_\psi$ are recursively isomorphic. (The independency of $\mathcal{L}$ from acceptable enumerations of partial recursive function.)

(ii) For each r. e. binary reation $R \subseteq \omega \times \omega$, there is a natural number n such that $\langle w, R \rangle$ is recursively isomorphic to $\langle \Sigma(n), \vdash \rangle$ (the classification of cumulative informations).

► J. M. HENLE, *Non-nonstandard analysis: Real infinitesimals*.
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We offer an alternative to Robinson's nonstandard analysis. Like nonstandard analysis, non-nonstandard analysis uses infinitesimals, but they are "standard," constructed from sequences of reals. No choice is needed.

Theorems with easy nonstandard proofs have easy non-nonstandard proofs, in fact, the proofs are almost identical. Many theorems with difficult or nonexistent nonstandard proofs (e.g., the Baire category theorem) have easy non-nonstandard proofs. Applications appear to include all of elementary calculus, analysis, topology, measure and operator theory.

The system, in some sense, predates nonstandard analysis. It is especially appropriate for a calculus student's first encounter with rigor.

► SHAMIL ISHMUKHAMETOV, *On enumerability of d. r. e. degrees in lesser ones*.
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The recursively enumerable (r. e.) degrees play the important role in Mathematical Logic, since they are exactly the degrees in which axiomatisible theories lie. Relativising this class to some oracle A, we obtain so called A-REA degrees. Since each A-REA degree is also B-REA degree for each B, $A \leq_T B$, so the problem arises to find for a degree $\mathbf{d}$ the minimum degree $\mathbf{a}$ such that $\mathbf{d}$ is A-REA for $A \in \mathbf{a}$. If such $\mathbf{a}$ exists, it plays for the degree $\mathbf{d}$ the same role as $\mathbf{0}$ for r. e. degrees, namely any set A from $\mathbf{a}$ contains the minimum of information necessary to enumerate some set from $\mathbf{d}$. We study this problem for the d. r. e. degrees.

A set D is d. r. e. if $D = W_x \setminus W_y$ for some r. e. W_x and W_y. Turing degree $\mathbf{d}$ is d. r. e. if it contains some d. r. e. set. $R[d]$ denotes the class of r. e. degrees in which d is enumerable.

THEOREM 1. *Given d. r. e. set D, there is a r. e. degree b, b is the least degree in the class of all degrees, in which D is recursively enumerable.*

This theorem fails if we replace d. r. e. set D by d. r. e. degree $\mathbf{d}$.

THEOREM 2. *For some d. r. e. d, $R[d]$ has no minimum element.*

THEOREM 3. *For some d. r. e. d, $R[d]$ has the minimum element.*

THEOREM 4. *For any d. r. e. $d > 0$ and r. e. a such that $0 < a < d$ and $a \in R[d]$, there is an ω-r. e. degree $b < d$ such that d is r. e. in b, but $a \nleq b$.*

THEOREM 5. *For some d. r. e. d, there is an ω-r. e. a, such that a is the least degree in the class of all degrees, in which d is enumerable.*

[1] SH. T. ISHMUKHAMETOV, *On r. e. predecessors of d. r. e. degrees*, to appear in ***Archives for Mathematical Logic***.

[2] C. JOCKUSCH and R. SHORE, *Pseudojump operators II: transfinite iterations, hierarchies, and minimal covers*, ***The Journal of Symbolic Logic***, vol. 49, pp. 1205–1236.

► OLIVIER LESSMANN, *Abstract decomposition theorem and applications*.
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We show that if a class of models $\mathcal{K}$ admits a dependence relation on the subsets of its models, as well as prime models satisfying a prescribed set of axioms, then under the parellel of NDOP every model of $\mathcal{K}$ is prime and minimal over an independent tree of models. This axiomatization is different from that of Shelah in the Main Gap in two crucial ways: the dependence relation satisfies weaker axioms than those of forking and the axiomatization does not depend on compactness. In fact the theorem is in the vein of combinatorial geometry; the logical ingredient being hidden in checking the particular axioms for the classes of models into consideration.

This result covers all the known cases: the various classical first-order cases as well as the non-elementary cases (excellent classes). In addition, it yields a new decomposition theorem for "Finite Diagrams", namely classes of models of a theory omitting a prescribed set of types. In this context I recently introduced ranks, pregeometries and a natural dependence relation satisfying those axioms. This provides additional evidence that one could hope to understand models of nonelementary classes by looking at their relations with their pregeometries, in a similar way to the first-order case.

We can also obtain the parallel nonstructure results.

This work is joint with Rami Grossberg.

► PAOLO MANCOSU, *Explanatory proofs: An exposition and criticism of the "classical" theory*.
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A great deal of contemporary philosophy of science has been concerned with the problem of scientific explanation. Only very recently have philosophers of mathematics begun asking whether an analogous phenomenon is at all present in mathematics. Central to this investigation is the question of whether or not there are mathematical proofs which, in addition to proving that a certain result holds, also show why the result holds. In a recent paper ([4]) I have shown that the distinction between explanatory and non-explanatory proofs in mathematics has a long history, going back at least to Aristotle's distinctions between dioti and hoti proofs. As Kitcher [3] points out, the most elaborate version of the "classical" notion of explanatory proof in the realm of mathematics was provided in Bolzano's *Wissenschaftslehere*.

I shall give a presentation and criticism of Bolzano's position on explanatory proofs. In particular, I will show that there are two different types of proof which Bolzano recognizes as explanatory, that is (1) proofs which show how a certain result was found, and (2) proofs which give the ground of the result. An account of the notion of explanatoriness involved in the second case relies on the Bolzanian distinction in the realm of truths between grounds and consequences. In the second part of the talk, drawing in part on Detlefsen [2], I shall point out the shortcomings of Bolzano's account for a satisfactory theory of mathematical explanation.

[1] B. Bolzano, ***Wissenschaftslehre***, Sulzbach, 1837.

[2] M. Detlefsen, *Fregean hierarchies and mathematical explanation*, ***International Studies in Philosophy of Science***, vol. 3 (1988), pp. 97–116.

[3] P. Kitcher, *Bolzano's ideal of algebraic analysis*, ***Studies in History and Philosophy of Science***, vol. 6 (1975), pp. 229–269.

[4] P. Mancosu, *On mathematical explanation*, 1996, forthcoming in ***History and philosophy of mathematics*** (H. Breger and E. Grosholz, editors).

▶ CYRUS F. NOURANI, *Functorial models, admissible sets and generic rudimentary fragments*.
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Admissible computable sets, functors and models were defined applying generic diagrams [5, 6, 7, 8] and the language category $L\omega1, B$ defined on the [2] fragment.

Theorem 1. *Admissible models are obtained by taking a reduct from the admissible hull to the Skolem hull definable by a generic diagram.*

Define an ordering for the initial structures [9].

Definition 1. For functions f and g in a structure for a language L, define the morphic *preorder* $f \ll g$ iff there are formulas φ and ψ from L such that the formulas define f and g respectively, and φ is a subformula of ψ in the sense of definition of the fragments for infinitary logic [2].

Apply our generic diagrams and sets [4] to the Σ formulas [1] and a theorem from Feferman-Kreisel (1966).

Theorem 2. *If a formula θ is preserved under end extensions then it is equivalent to a Σ formula.*

Apply our *generic rudimentary sets with Theorem* 2, *with* positive forcing [4] and fragment consistency models [7] to obtain Theorem 3. See Kiesler [2] and Barwise [1].

Theorem 3. *The morphic preorders on initial structures are definable by formulas which are preserved by end extensions on fragment consistent models.*

Theorem 4. *The morphic preorder functor defines initial models for $L\omega1, B$. The model has the morphic preorder property.*

[1] J. Barwise, ***Infinitary languages***, Lecture Notes in Mathematics, no. 72, Springer-Verlag, Berlin, 1968.

[2] H. J. Keisler, ***Forcing and the omitting types theorem***, MAA, 1973.

[3] C. F. Nourani, Lecture Notes in Computer Science, no. 80, Springer-Verlag, Berlin, 1978.

[4] ———, *Forcing with universal sentences*, vol. 49, ASL, Boston, 1983.

[5] ———, *Functorial model theory and infinite language categories*, September 1994, ASL, January 1995, San Francisco.
[6] ———, *Computable functors*, December 1996.
[7] ———, *Functorial model theory and generic fragment consistency models*, October 1996, AMS-ASL, San Diego, January 1997.
[8] ———, *Functorial admissible models*, November 1996, ASL, MIT, Cambridge, March 1997.
[9] J. W. Thather, E. G. Wagner, and J. B. Wright, ***Notes on algebraic fundamentals for theoretical computer science***, ***Foundations of Computer Science III***, CWI Mathematical Center Tracts, no. 109, IBM T. J. Watson, New York, 1979.

► D. E. PAL'CHUNOV, *Homogeneous Boolean algebras with distinguished ideals*.
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We consider Boolean algebras with $\lambda \in \omega$ distinguished ideals, we call them *I-algebras*. Let $I_1, \dots, I_\lambda$ be the symbols for the distinguished ideals. One can find definitions related to I-algebras in [1]. In this paper we study homogeneous Boolean algebras with distinguished ideals. Countable homogeneous Boolean algebras are described in [2].

Present some definitions from [1]. A formula $V(x)$ is called *pointwise* if

$$(\forall x)(V(x) \to (\forall y, z \leq x)(y \cap z = 0 \to (\neg V(y) \vee \neg V(z))));$$

it is said to be *continuous* if

$$(\forall x)(V(x) \to (\exists y \leq x)(V(y) \,\&\, V(x \smallsetminus y))).$$

For formulas $V(x)$ and $W(x)$ we denote $V < W$ if $(\forall x)(W(x) \to (\exists y \leq x)(V(y)))$. In [1] the sequence of formulas $V_n(x)$, $n \in \omega$, together with related characteristic $r_x(n)$ are introduced. For an element $a \in \mathcal{A}$ we denote $M(a) \iff \{\, n \in \omega \mid r_n(a) \neq 0 \,\}$, $N(a) \iff \{\, n \in M(a) \mid \text{for any } k \in M(a), k \neq n, \text{ we have } V_n \not< V_k \,\}$. An element $a \in \mathcal{A}$ is said to be *local* if $M(a)$ is finite; a is called *n-simple* if it is local and $N(a) = \{n\}$. We denote $r_{\mathcal{A}} \iff r_{1^{\mathcal{A}}}$, $M(\mathcal{A}) \iff M(1^{\mathcal{A}})$, $N(\mathcal{A}) \iff N(1^{\mathcal{A}})$. An I-algebra $\mathcal{A}$ is called *local* (*n-simple*) if $1^{\mathcal{A}}$ is local (n-simple).

Denote $P_n = P_n(\mathcal{A}) \iff \{\, a \in \mathcal{A} \mid r_n(a) < \infty \,\}$, $P = P(\mathcal{A} \iff \{\, a \in \mathcal{A} \mid a \text{ is local} \,\}$; $P_n(\mathcal{A})$ and $P(\mathcal{A})$ are ideals of the I-algebra $\mathcal{A}$.

Theorem 1. *A finitely axiomatizable I-algebra $\mathcal{A}$ is homogeneous if and only if the following statement holds*:

$(*)$ *for any $n \in M(\mathcal{A}) \smallsetminus N(\mathcal{A})$ if V_n is pointwise then*:

(a) *if there exists an n-simple $a \in \mathcal{A}$ such that a/P_n is not atomless than a/P_n is an atom and there is the unique $k \in M(\mathcal{A})$ with $k \neq n$ and $V_n < V_k$. For this number k, the formula V_k is pointwise and $r_{\mathcal{A}}(k) = 1$;*

(b) *if there exists an n-simple $a \in \mathcal{A}$ such that $a/P_n \neq 0$ and a/P_n is atomless then for any $b \in \mathcal{A}$ for each $k \in M(\mathcal{A})$ if $V_n < V_k$, $n \neq k$ and $r_b(k) \neq 0$ then there exists an n-simple $c \leq b$ such that $c/P_n \neq 0$ and c/P_n is atomless.*

Theorem 2. *A local I-algebra $\mathcal{A}$ is homogeneous if and only if the statement $(*)$ and the following statement hold*:

$(**)$ *for any $n \in N(\mathcal{A})$ if V_n is pointwise then either $1/P_n$ is atomless or it is atomic and contains at most two atoms.*

Theorem 3. *A countable superatomic Boolean algebra with one distinguished ideal $\mathcal{A}$ is homogeneous if and only if the statements $(*)$, $(**)$, and the following statement hold*:

$(***)$ *either $1/P$ is atomic and contains at most two atoms or $1 \in P$.*

[1] D. E. PAL'CHUNOV, *Countably-categorical Boolean algebras with distinguished ideals*, ***Studia Logica***, vol. 46 (1987), no. 2, pp. 121–135.

[2] A. S. MOROZOV, *Countable homogeneous Boolean algebras*, ***Algebra and Logic***, vol. 21 (1982), no. 3, pp. 269–282.

► LANCE RIPS, *Two-person argument: Formal and cognitive implications*.
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Two-person arguments take place when individuals exchange views on whether it is worthwhile to believe some claim or take some action. This talk presents a theory of the structure of these arguments in terms of the conversational moves participants make within them—for example, asking for a justification, giving a reason, offering an objection, or conceding a point. The structure determines which of the argument's claims each participant is committed to publicly at any stage of the argument, and it identifies changes in commitment as the argument unfolds. The analysis of argument also illuminates concepts like burden of proof that are difficult to capture in current theories.

► V. V. RYBAKOV, *Quasi-characteristic inference rules for modal logic $S4$*.
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The aim of this research is to give a complete description for all quasi-characteristic rules which are admissible in modal logic $S4$. For any finite and subdirectly irreducible modal algebra $\mathcal{A}$, we can in a canonical way define a quasi-characterizing inference rule $r(\mathcal{A})$. This rule characterizes algebra $\mathcal{A}$ as follows: for any modal algebra $\mathcal{B}$, $\mathcal{B} \vDash r(\mathcal{A})$, if and only if the algebra $\mathcal{A}$ cannot be isomorphically embedded in $\mathcal{B}$. This naturally generalizes the notion and properties of Yankov's formulas. To describe our results we need the following rules:

$$\begin{aligned}
Tr_M &:= [\Box(\Box x \to \Box y) \to \Box x \vee \Box z]/\\
&\qquad [\Box(\Box(\Box x \to \Box y) \to \Box x) \vee \Box(\Box(\Box x \to \Box y) \to \Box z)],\\
r_{1,1} &:= \Diamond x \wedge \Diamond \neg x / y,\\
r_{2,2} &:= \alpha_1 \wedge \alpha_2 \to \Box(\alpha_1 \wedge \alpha_2 \to \Diamond(\alpha_1 \wedge \alpha_2 \wedge y) \wedge \Diamond(\alpha_1 \wedge \alpha_2 \wedge \neg y))/\neg(\alpha_1 \wedge \alpha_2),
\end{aligned}$$

where $\alpha_1 := \Diamond(z_1 \wedge \Box\neg z_2)$, $\alpha_2 := \Diamond(z_2 \wedge \Box\neg z_1)$.

THEOREM 4. *Let $\mathcal{A}$ be a finite subdirectly irreducible modal algebra from the variety* $\mathrm{Var}(S4)$. *The following statements are equivalent*:
(i) *$\mathcal{A}$ is embeddable in the free modal algebra $\mathcal{F}_{S4}(\omega)$*;
(ii) *the rules Tr_M, $r_{1,1}$ and $r_{2,2}$ are valid on $\mathcal{A}$.*

COROLLARY 1. *For any finite subdirectly irreducible modal algebra $\mathcal{A}$, the rule $r(\mathcal{A})$ is admissible in the logic $S4$ if and only if $r(\mathcal{A})$ is the corollary of rules Tr_M, $r_{1,1}$ and $r_{2,2}$. I.e., the rules Tr_m, $r_{1,1}$ and $r_{2,2}$ form a basis in $S4$ for all admissible in $S4$ quasi-characteristic inference rules.*

COROLLARY 2. *All admissible in $S4$ inference rules are valid on a finite modal algebra $\mathcal{A}$ if and only if the rules $\Box\alpha \vee \Box w/\Box\beta \vee \Box w$, where w is a new variable having no occurrences in α and β, and α/β is a rule from the list Tr_M, $r_{1,1}$, $r_{2,2}$, are valid on $\mathcal{A}$.*

These results are in a contrast with the following

THEOREM 5. *Any quasi-characteristic rule $r(\mathcal{A})$ for arbitrary finite subdirectly irreducible modal algebra $\mathcal{A}$, where $\|\mathcal{A}\| > 1$, is admissibile in the modal system $K4$.*

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► IRINA A. SEMAKINA, *Being of a sense in a point*.
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The problem of sense in modern poststructural philosophy is connected with correlation of form and content. J. Deleuze wrote in "Logic of sense" that sense is neutral in relation to state of things and sentences, that describing this reality. In this case the place of sense is one that is found out of language and things, out of form and content and its being is being of bound between them. Sense as a bound called surface, event as such, effect. It is void word and void thing. It is a nonsense that exists as surface. Topological surface is a void position of sense. This problem is insoluble because we can think nothing father void space. The "void" is a limit concept of that way of thinking which suggests that structure of sense's being is concept of that way of thinking which suggests that structure of sense's being is based on non-structural foundation. The search of foundation of sense is directed to search one that is out of structure and exists as an uncertainty that is unknown.

We offer to consider the being of sense as one that permits to exist to things and words. It is found in a point of identity of form and content of sense. Thus, the point is position of sense, place and state. Being of sense acquires form and that is why it has definite position or place and is found somewhere. But so far as it is found somewhere it is contented somewhere or is found in something. Sense is formed and is kept by form, thus, sense "is", "presents", and "has place to be".

When the sense is formed it become definite. It indicate to one that is formed is indefinite at the one time. The form indicate to one that is kept and held in it. Content of sense, thus, within the limits of form and is definite. At the same time it is out of limits because it is held in form and cannot be formed at all and to be full in form.

Thus, we can tell about being of sense as a bound which has a certain way of existence. It is a definition of itself when sense defines itself and exists as a limit. It defines itself as a bound that is cannot be determine and exist as uncertainty. Its indefinite being forces to define itself constantly. It exists as definite uncertainty.

► D. SINGH, *Some reflections on WORD problem using multiset theory*.
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This paper attempts at explicating the WORD problem by defining a WORD as an ordered multiset.

[1] WAYNE D. BLIZARD, *Multiset theory*, ***Notre Dame Journal of Formal Logic***, vol. 3 (1989), no. 1, pp. 36–66.

[2] JOHN L. BRITTON, *The WORD problem*, ***Annals of Mathematics***, vol. 77 (1969), no. 1, pp. 16–32.

► DIMITER VAKARELOV, *Proximity modal logic*.
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We give a possible world semantics and complete axiomatization of a modal logic with a binary modality $\diamond(P, Q)$, which can be read "P is near Q from certain point of view". Since in possible world semantics sentences are identified with sets of possible worlds, a natural way to assign an appropriate meaning of $\diamond(P, Q)$ is to use the topological notion of proximity relation between sets and proximity spaces (see [1]). A typical example of a proximity relation can be defined in a pseudo-metric space (X, d), putting for subsets $A, B \subseteq X$ $A\delta B$ (A near B) if and only if the distance between A and B is zero. The obtained logic is denoted by PML—Proximity Modal Logic. Formally, semantical structures for PML are systems of the form (X, D), where X is a non-emptyset of possible worlds and D is called a distribution—a function assigning to each world x a pseudo-metric d_x. Let the corresponding proximity relation between subsets of X be denoted by δ_x. In this way each possible world has its own proximity relation and for the subsets A and B $A\delta_x B$ means that "A is near B from the point of view of x". Now the semantics of $\diamond(A, B)$ is the following

$$x \Vdash \diamond(P, Q) \text{ if and only if } \{\, y : y \Vdash P \,\}\delta_x\{\, y : y \Vdash Q \,\}.$$

Informally, P is near Q at the world x if the set of the worlds in which P is true is near the set of the worlds in which Q is true from the point of view of x.

In the axiomatization of PML we will use also the universal modality $\blacksquare P$.

The following set of axioms and rules form a complete axiomatization of PML with respect to the given semantics ($\Box(P, Q) =_{\text{def}} \neg \diamond (\neg P, \neg Q)$).

Axioms and rules for PML

All Boolean tautologies

$\Box(P \implies Q, R) \implies (\Box(P, R) \implies \Box(Q, R))$,

$\Box(P, Q \implies R) \implies (\Box(P, R) \implies \Box(P, R))$,

$\blacksquare(P \implies Q) \implies (\blacksquare P \implies \blacksquare Q)$,

$\blacksquare P \implies P$,

$\blacksquare P \implies \blacksquare\blacksquare P$,

$\blacklozenge\blacksquare P \implies P$,

$\diamond(P, Q) \implies \blacklozenge P \wedge \blacklozenge Q$,

$\diamond(P, Q) \implies \diamond(Q, P)$,

$\blacklozenge(P \wedge Q) \implies \diamond(P, Q)$.

(MP) $\frac{A, A \implies B}{B}$, (Nor) $\frac{A}{\blacksquare A}$.

Using filtration we prove that PML is decidable. An axiomatization of PML with respect to a class of structures (X, D) in which the distribution D is a constant function, is also given.

[1] S. A. Naimpally and B. D. Warrack, ***Proximity spaces***, Cambridge University Press, 1970.

► ANDRÉS VILLAVECES, *Categoricity spectrum for abstract elementary classes: Reducing the set-theoretical hypotheses*.

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In the eighties, Makkai and Shelah studied the categoricity spectrum for abstract elementary classes in the presence of compact cardinals. Later, Kolman and Shelah were able to obtain results in that direction assuming only the existence of a measurable cardinal; their result provided a bounded portion of the categoricity spectrum. I will present recent results of Shelah and myself on uniqueness of disjoint amalgamation that extend the known categoricity spectrum from measurable cardinals, and I will also present categoricity results under weaker set theoretical hypotheses.