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# 1994 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

## Gainesville, Florida, March 5–8, 1994

The Annual Meeting of the Association was held at the University of Florida in Gainesville, Florida, March 5–8, 1994. The program included eight invited hour talks; Special Sessions in Set Theory, and Proof Theory and Computer Science each consisting of five invited half-hour talks; forty-eight contributed talks; and five talks contributed by title. In addition, Donald A. Martin delivered the Fifth Annual Gödel Lecture entitled, $L(\mathbb{R})$ : *A Survey*, and Charles Parsons delivered his Retiring Presidential Address entitled, *Platonism and Mathematical Intuition in Kurt Gödel's Thought*. The Ulam Lecture, sponsored by the Southeastern Logic Symposium and by the Department of Mathematics of the University of Florida, was entitled *Transparent Proofs and Limits to Approximation*, and was given by László Babai. The meeting took place shortly after the death of one of the most important mathematical logicians of the twentieth century, Stephen Cole Kleene, on January 26, 1994. Kleene's major contributions to both recursive function theory and intuitionistic mathematics were described in remarks made by Yiannis Moschovakis at the meeting. The Program Committee consisted of Steven Buechler, Manual Lerman (Chair), Dale Miller, William Mitchell, and Gila Sher. The Local Organizing Committee consisted of Douglas Cenzer, Jean A. Larson (Chair), William J. Mitchell, Greg Ray, and Rick Smith.

The following invited addresses were presented at the meeting:

Lou van den Dries (University of Illinois), *New O-Minimal Expansions of the Field of Reals*.

Jean-Yves Girard (CNRS, Marseilles), *Geometry of Interaction: A Monist Approach to Logic*.

Stewart D. Shapiro (Ohio State University), *Logical Consequence: Models and Modality*.

Robert I. Soare (University of Chicago), *Models of Arithmetic and Upper Bounds for Arithmetical Sets*.

Simon Thomas (Rutgers University), *The Cofinality of the Infinite Symmetry Group*.

James Baumgartner (Dartmouth College), *New Ultrafilter Problems on $\omega$*.

Julia Knight (University of Notre Dame), *Nest Priority Constructions*.

Harold T. Hodes (Cornell University), *Introduction and Elimination, in General and in Particular*.

The following addresses were presented at the Special Session on Proof Theory and Computer Science:

Natarajan Shanker (SRI International), *Proof Search in Intuitionistic and Linear Logic*.

Gopalan Nadathur (Duke University), *Some Applications of Proof Theory in Logic Programming*.

Douglas Howe (Bell Laboratories), *Functional Programming Logics Based on Set Theory*.

Maria Luisa Bonet (University of Pennsylvania), *On Finding Hard Tautologies for Frege Systems*.


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Radha Jagadeesan (MIT), *Games and Linear Logic*.

The following addresses were presented at the Special Session on Set Theory:

Menachem Kojman (Carnegie-Mellon University), *Negations of GCH and Universality Spectrums*.

Stevo Todorčević (University of Toronto), *Classifying Finite Partitions of $X^n$*.

Matthew Foreman (University of California-Irvine), *How Different are $\aleph_2$ and $\aleph_1$?*.

Andreas Blass (University of Michigan), *Some Cardinality Characteristics of the Continuum*.

Ernest Schimmerling (MIT), *Inner Models with Woodin Cardinals*.

Abstracts of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
MANUEL LERMAN

## Abstracts of contributed papers

STÅL AANDERAA, *A Trachtenbrot inseparability theorem for groups*.
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Given a finitely presented group $G$. Then we may define two sets of words in the generators of $G$. Let $A$ be the set of words $w$ in the generators of $G$ such that $w$ becomes the identity in $G$. Let $B$ be the set of words $w$ in the generators of $G$ such that there exists a finite homomorphic image of $G$ in which $w$ becomes different from identity. Then $A$ and $B$ are both r.e. sets and they are disjoint. I can prove that there exists a finitely presented group $G$ such that the sets $A$ and $B$ are recursively inseparable, i.e., there exists no recursive set $C$ of words such that $A$ is a subset of $C$, and such that $C$ and $B$ are disjoint.

TUNA ALTINEL, *Simple groups of finite Morley rank which contain a strongly embedded subgroup*.
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We prove results related to the Cherlin-Zilber conjecture which states that an infinite simple group of finite Morley rank should be an algebraic group over an algebraically closed field.

A proper definable subgroup $M$ of a group $G$ of finite Morley rank is called *strongly embedded* if it contains involutions (elements of order 2) and for any $g \in G\backslash M$, $M \cap M^g$ does not contain any involutions.

A connected nonsolvable group of finite Morley rank whose proper definable connected subgroups are nilpotent is called a bad group. A structure $\langle K, +, \cdot, A\rangle$ of finite Morley rank where $K$ is an algebraically closed field and $A$ is a proper infinite subgroup of the multiplicative group of $K$ is called a bad field.

We prove that a simple group of finite Morley rank whose proper definable simple sections are algebraic, which contains a strongly embedded subgroup, does not interpret a bad field and in which the Sylow 2-subgroups contain infinitely many involutions, is isomorphic to $\mathrm{PSL}_2(\mathrm{K})$ where $K$ is an algebraically closed field of characteristic 2.

The connected component of a Sylow 2-group of a group of finite Morley rank is the central product of a definable, connected nilpotent group of bounded exponent and a divisible abelian group. A group $G$ of finite Morley rank is said to be of mixed type if both of these subgroups of a Sylow 2-subgroup of $G$ are nontrivial. We discuss potential applications of the theorem to the groups of mixed type.

PAUL BANKSTON, *The model-theoretic topology of compact Hausdorff spaces*.
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We describe a mechanism, based on the Keisler-Shelah ultrapower theorem, which translates a substantial amount of classical first-order model theory into the setting of compact Hausdorff spaces. To achieve such a translation formally, all one needs is a category equipped with products and enough directed limits to allow an autonomous notion of ultraproduct, as well as a cardinal-valued function defined on the class of objects and reasonably like the usual cardinality in the presence of certain morphisms (e.g., monos and epis). In our situation the category is the opposite of the category of compact Hausdorff spaces and continuous maps, and the cardinal-valued function is topological weight.

Given a model-theoretic notion (or assertion) $N$, let $[N]^*$ be its translation (when possible) into the compact Hausdorff setting. For example, if $f : X \to Y$ is an [elementary embedding]$^*$, then $f$ is a continuous surjection that preserves and reflects such properties as connectedness, total disconnectedness, covering dimension, and finite cardinality. (On the other hand, $f$ need not preserve or reflect strong inductive dimension, and need not reflect either metrizability or local connectedness.)

If $N$ is a theorem of general model theory, then the assertion $[N]^*$ is not guaranteed to be a topological theorem; however, we know of no examples where this fails. Examples of success include the compactness theorem, as well as all known versions of the Löwenheim-Skolem theorem (with topological refinements). There are also some more specialized results. For example, if $f : X \to Y$ is a function between compact Hausdorff spaces and $X$ is an arc (i.e., a homeomorphic copy of the real closed unit interval), then $f$ is an [elementary embedding]$^*$ if and only if $Y$ is an arc and $f$ is a continuous monotone surjection.

STEPHEN BLOCH, *On parallel hierarchies and $R^i_k$*.
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In this paper I define hierarchies of function and relation classes, constructed from parallel complexity classes in a manner analogous to the polynomial hierarchy. There is a surprising correspondence between them and the levels of the polynomial hierarchy, and a Buss-style witnessing theorem relating the levels of these hierarchies to definability in the bounded arithmetic theories $R^i_k$; these results provide a more elegant proof of a logical characterization due to Buss, Krajíček, and Takeuti (1993). Finally, I show results analogous to those of Krajíček, Pudlák, and Takeuti (1991): if $R^{i+1}_k$ is conservative over $S^i_k$ with respect to Boolean combinations of $\Sigma^b_{i+1,k}$ formulae, then every $\Pi^b_{i,k}$ optimization problem can be approximated to within a constant factor by a finite student-teacher game at level $i+1$ of the parallel hierarchy; this in turn would imply that the parallel hierarchy, and hence the (quasi)polynomial hierarchy, collapses to level $i+2$.

MOGAN BROWN, *A characterization of $Q_3$*.
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$Q_3$ is an analog of the set of hyperarithmetical reals, $\Delta^1_1 \cap \mathbb{R}$, at level three of the analytical hierarchy. A. Kechris proved that $\Delta^1_1 \cap \mathbb{R}$ is the largest $\Pi^1_1$ set of reals (not equal to $\mathbb{R}$) which is closed downward under hyperarithmetical degrees. Using the recent result of J. Steel, that under certain large cardinal assumptions, the core model is $\Sigma^1_3$ correct, and the result of H. Woodin that assuming $\mathbf{\Delta}^1_2$ determinacy, all uncountable $\Delta^1_3$ sets of reals contain a member of every $\Delta^1_3$ degree above that of $y^0_3$, I prove the following:

THEOREM. (Assume $\mathbf{\Delta}^1_2$ determinacy) *$Q_3$ is the largest $\Pi^1_3$ set of reals (not equal to $\mathbb{R}$) closed downward under $\Delta^1_3$ degrees.*

In particular, assuming determinacy there is no uncountable $\Pi^1_3$ set of reals (not equal to $\mathbb{R}$) which is closed downwards under $\Delta^1_3$ degrees.

This solves a conjecture posed in A. Kechris, D. Martin, and R. Solovay, *Introduction to Q-theory*, ***Cabal Seminar 79–81***.

PAUL J. CORAZZA, *Justifying large cardinals with the wisdom of the ancients*.
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Large cardinals have become pervasive in set theory, but how do we justify them? Motivating principles like generalization, reflection and resemblance are sometimes helpful but still do not account for all large cardinals, especially the really big ones. Motivating principles must draw upon a fundamental intuition about the structure of the mathematical universe. One of the most explicit examples of such intuition is Cantor's notion of the Absolute Infinite. We present interesting details of the ancient Vedic conception of the Absolute Infinite that seem to suggest not only that there ought to be an elementary embedding of the universe to itself but also how to get around Kunen's proof that such embeddings don't exist! This Vedic model suggests that the "right" set theory would admit such a (modified) elementary embedding; this embedding is still strong enough to prove the existence of all large cardinals through super $n$-huge for every $n$. This model also suggests a number of unexpected research problems concerning large cardinals and the structure of the universe.

DENNIS F. CUDIA, *The thermodynamic limit*.
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The thermodynamic limit essential to the theory of information is given in the following corollary.

THEOREM 1. *If $\{Y(mx)/m : m = 1, 2, \dots\}$ is convergent to $Y_\infty(x)$ then, $Y_\infty$ is an extensive variable, i.e., $Y_\infty(nx) = nY_\infty(x)$ for all integers $n > 0$.*

COROLLARY 2. *If $\{Z(mx)^{1/m} : m = 1, 2, \dots\}$ is convergent to $Z_\infty(x)$, then $\log Z_\infty$ is an extensive variable.*

The thermodynamic limit allows three things to be done:

1) exercise the dichotomy between information $I$ and entropy $S$, that $S = -I$,
2) determine properties without using Stirling's approximation,
3) produce an extensive entropy $S_\infty$ from a non-extensive thermodynamic probability $W_M$. For notation and general background see [1].

All probability distributions discussed here are multinomial distributions. A macrostate is any $n$-tuple of positive integers $(N_1, \dots, N_n)$. The distribution corresponding to the macrostate is $p_i = N_i/N$, $i = 1, \dots, n$ where $N = N_1 + \cdots + N_n$. It is intuitively obvious that some information is lost in going from the macrostate to the distribution. That is, $I(N_1, \dots, N_n) > I(p_1, \dots, p_n)$. Since $S = -I$ it follows that $S(N_1, \dots, N_n) < S(p_1, \dots, p_n)$. In order to formulate a precise inequality recall that $W_M(N_1, \dots, N_n) = N!/(N_1! \cdot \dots \cdot N_n!)$ is the thermodynamic probability of mixing. Denote $W_m(N_1, \dots, N_n) = W_M(mN_1, \dots, mN_n)^{1/m}$ and denote

(*) $$W_\infty(N_1, \dots, N_n) = \lim W_m(N_1, \dots, N_n) = (1/p_1)^{N_1} \cdot \dots \cdot (1/p_n)^{N_n}$$

Then we have the following fundamental inequality.

THEOREM 3. $W_M(N_1, \ldots, N_n) < W_\infty(N_1, \ldots, N_n)$.

SKETCH OF PROOF. It must be shown that

$$(**) \qquad N!/N^N < (N_1! \cdot \ldots \cdot N_n!)/(N_1^{N_1} \cdot \ldots N_n^{N_n})$$

Proceeding by mathematical induction on $N$, if (**) is true for $(N_1, \ldots, N_n)$ then it is true for $(N_1 + 1, N_2, \ldots, N_n)$ provided $(N_1/N_1 + 1)^{N_1} > (N/N + 1)^N$. But this last inequality is true because

$$1 < (1 + x^{-1})^x < (1 + y^{-1})^y < e, 0 < x < y.$$

Proceeding to 2) without Stirling's approximation and omitting the macrostate arguments we have the following.

THEOREM 4. $\lim(W_{m+1}/W_m) = 1$. *Therefore the sequence* $\{W_m : m = 1, 2, \ldots\}$ *has exactly one point of accumulation.*

Using Stirling's approximation and the fundamental inequality we have the following.

THEOREM 5. *The sequence* $\{W_m : m = 1, 2, \ldots\}$ *converges to* $W_\infty$ *from below.*

Turning now to 3) the connection with the laws of thermodynamics is obtained.

DEFINITION 6. For any macrostate $(N_1, \ldots, N_n)$ the *limit thermodynamic probability of mixing* is denoted by $W_\infty(N_1, \ldots, N_n)$ and defined by (*).

Employing the Boltzmann Principle the following definition is obtained.

DEFINITION 7. For any multinomial distribution $p_1, \ldots, p_n$ and any integer $N$ such that $Np_i, i = 1, \ldots, n$ are integers, the *limit entropy of mixing* is denoted by $S_\infty(p_1, \ldots, p_n, N)$ and defined by $S_\infty(p_1, \ldots, p_n, N) = k \log W_\infty(Np_1, \ldots, Np_n)$.

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RUTH EBERLE, *Syntax and semantics of diagrams in natural deduction*.
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*Hyperproof* is a logical system, implemented in a computer program by the same name [2], [1], that extends first-order logic to include diagrammatically represented information. With the formal addition of diagrammatically represented information, proofs and other analytical reasoning problems can be solved far more naturally and efficiently than is possible using purely sentential first-order logic.

Both the diagrams and sentences of *Hyperproof* concern a domain of discourse consisting of blocks. The blocks may have certain properties, and certain relations may exist between them. The diagrams, which can be used in proofs just like sentences, are visual representations of situations consisting of such blocks. In this paper I formalize the syntax and semantics of the diagrams in *Hyperproof* as well as the rules of inference for reasoning between sentences and diagrams.

Hyperproof-like systems can also be used to reason about other domains of discourse, using diagrammatic representation that differ from those of *Hyperproof*. Certain of these differences are due to the fact that unlike sententially represented information, diagrammatically represented information is strongly context-dependent. Thus, I also discuss which aspects of the syntax and semantics are generalizable to include different domains of discourse and which are not.

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TODD EISWORTH, *Selective ultrafilters and $\omega \to (\omega)^\omega$*.
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It is a theorem of Mathias that given a selective ultrafilter $\mathcal{U}$ and a partition of $[\omega]^\omega$ into $\mathcal{X}$ and $[\omega]^\omega \setminus \mathcal{X}$, there is an infinite set $X \in \mathcal{U}$ all of whose infinite subsets lie in one piece of the partition. Assuming the existence of a Mahlo cardinal, he also proved the consistency of the stronger partition property in which $\mathcal{X}$ is only assumed to be a member of $L(\mathbb{R})$. In this talk, I shall outline a proof that if CH holds (so in particular selective ultrafilters exist), this strengthened partition theorem implies that $\aleph_1$ is inaccessible in $L$.

SY D. FRIEDMAN AND DORSHKA WYLIE, *Embedding hierarchies and the fine structure of K*.
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An approach to the theory of mice and the core model K is described, using the concept of embedding hierarchy. The aim is to obtain a hierarchy for the core model which permits a more satisfying fine structure theory than the existing hierarchies. This should facilitate simpler proofs of combinatorial principles as well as allow for a good theory of coding over core models.

WILLIAM GASARCH AND JEFFREY HIRST, *Reverse mathematics and recursive graph theory*.
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We examine several results of recursive graph theory within the proof theoretic framework of reverse mathematics. The interplay of the two approaches yields some new combinatorial arguments and recursion theoretic insights.

A typical result is the following theorem concerning graph colorings, inspired by the work of Bean [1]. We say that a graph $G$ is *$k$-chromatic* if there is a function mapping the vertices of $G$ into $k$ which always assigns different colors to neighboring vertices. If every finite subgraph of $G$ is $k$-chromatic, we say that $G$ is *locally $k$-chromatic*. The following theorem strengthens a result in [3].

THEOREM ($\mathbf{RCA}_0$). *For every $k \geq 2$, the following are equivalent:*
(1) $\mathbf{WKL}_0$.
(2) *If $G$ is locally $k$-chromatic, then $G$ is $k$-chromatic.*
(3) *If $G$ is locally $k$-chromatic, then $G$ is $2k-1$-chromatic.*

We also consider results on bounded graphs, Euler paths, and Hamiltonian paths. For an overview of recursive combinatorics, see [2]. A nice introduction to reverse mathematics is given in [4].

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A. HEATON, *Axiomatic subrecursion theory*.
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We attempt an axiomatization of the kind of subrecursive classes naturally arising in a proof-theoretic context:

$$\mathrm{REC}(<|T|) = \text{ PROVABLY RECURSIVE IN } T.$$

The axioms are a modification of the Moschovakis-Fenstad theory for "full" recursion theory, obtained by regulating the enumeration scheme according to a given well-ordering $\prec$ on codes, thus

$$f_i(e,x) = \begin{matrix} f_e(x) & \text{if } e \prec i \\ 0 & \text{otherwise.} \end{matrix}$$

We study various natural models and embeddings between them, our methods making use of subrecursive hierarchies.

JAMES E. HELMREICH, *Ultrahomogeneous expansions of ultrahomogeneous graphs*.
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We say a countable structure **M** is *ultrahomogeneous* provided every isomorphism of finite substructures of **M** extends to an automorphism of **M**. Lachlan and Woodrow have completely classified the countable ultrahomogeneous graphs. We expand the language of graphs to include a new unary predicate. In this expanded language, ultrahomogeneous 2-colorings of ultrahomogeneous graphs are found to be uniquely determined by the isomorphism type of each color.

GREG HJORTH, *A new Borel hierarchy theorem*.
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It is possible to strengthen the usual Borel hierarchy theorem by showing that ZFC proves that there is no Borel definable injection from the Borel sets at one level into the Borel sets of some lower level of the Borel hierarchy.

CHRYSAFIS HARTONAS, *Algebraic and Kripke-style semantics for substructural logics*.
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The central topic of this paper is to present a systematic approach to the algebraic and

Kripke-style semantics for logics with restricted structural rules, notably for logics on an underlying non-distributive lattice. We provide a new topological representation theorem for general lattices, using the filter space $X$. Our representation involves a Galois connection on subsets of $X$, hence a closure operator $\Gamma$, and the image of the representation map is characterized as the collection of $\Gamma$-stable, compact-open subsets of the filter space (which is characterized as a certain kind of lattice-ordered Stone space). The original lattice $L$ is thus imbedded in the complete lattice $(\Gamma X, \subseteq, \quad, \quad)$ of stable sets. The representation is shown to be functorial and is extended to a Stone-type duality.

We then consider lattices with additional, normal, operators. An operator $f$ on a lattice $L$ is normal if $f : L^{(0)} \times \cdots \times L^{(n-1)} \rightarrow L^{(n)}$ is a monotone map, where each $L^{(i)}, i \in n+1$, is either $L$ or $L^{op}$ and $f$ sends finite joins (or meets) of $L^{(i)}$ to joins (respectively, meets) in $L^{(n)}$. Every familiar operator, such as $\circ$ (times), $+$ (plus), $\leftarrow$, $\rightarrow$, $\neg$, $\Box$, $\Diamond$ is normal. We extend the lattice representation and duality theorems to the case of lattice-ordered algebras with normal operators.

We also raise and resolve the question of representation of operators by relations and conclude with a duality of algebraic and Kripke-style semantics. We give an extensive number of examples of representations of specific operators and provide explicit constructions of canonical Kripke frames, based on our representation arguments, for a number of logical systems, notably for Intuitionistic and Classical Linear Logics, or Noncommutative Linear Logic (without exponentials) and of fragments thereof.

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JOSÉ IOVINO, *The geometry of stable Banach spaces*.
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In [1] we introduced the concept of stability for theories of Banach spaces and Banach space structures. Significant classes of Banach spaces (e.g., Hilbert spaces, $L_p(\mu)$ spaces for $1 \leq p < \infty$) are stable.

We proved in [1] that a complete Banach space theory $T$ is stable if and only if the following condition holds in every model of $T$: If $\mathcal{R}(\overline{x}, \overline{y})$ is a definable real-valued relation and $(\overline{a}_m), (\overline{b}_n)$ are bounded sequences,

$$\lim_{n\rightarrow\infty} \lim_{m\rightarrow\infty} \mathcal{R}(\overline{a}_m, \overline{b}_n) = \lim_{m\rightarrow\infty} \lim_{n\rightarrow\infty} \mathcal{R}(\overline{a}_m, \overline{b}_n),$$

where the equality means that if the limits exist, they are equal.

On any Banach space, the function $x \mapsto \|x\|$ is a definable real-valued relation. Thus, if $T$ is a stable Banach space theory, every separable model of $T$ is "stable" in the sense of Krivine and Maurey [2]. In fact, when $T$ is model complete, the converse is also true. The examples of stable theories mentioned above admit quantifier elimination.

I shall discuss some of the conditions stability imposes on the geometry of a Banach space.

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HERMAN RUGE JERVELL, *A Ramsey theorem for trees*.
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We generalize the results of Paris-Harrington and Ketonen-Solovay to get a Ramsey theorem for certain wellfounded homogeneous trees. The techniques used come from Girard's $\Pi^1_2$-logic.

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CARL G. JOCKUSCH AND ALEXANDRA SHLAPENTOKH, *Non-standard presentations of $\mathbb{Q}$ and $\mathbb{Z}$*.
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Given two r.e. degrees $a \leq b$, there exists a one-to-one recursive function $f : \mathbb{Q} \to \mathbb{N}$ such that $f(\mathbb{Z}) \in a$, $f(\mathbb{Q}) \in b$, and there exist recursive functions $P+, P-, P\cdot, P/ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that $P + (f(x), f(y)) = f(x + y)$, $P - (f(x), f(y)) = f(x - y)$, $P \cdot (f(x), f(y)) = f(xy)$, $P/(f(x), f(y)) = f(x/y)$, assuming $y \neq 0$.

STEVEN M. KAUTZ AND PETER BRO MILTERSEN, *Relative to a random Oracle, NP is not small: An application of the independence properties of algorithmically random sequences*.

Let $A_0$ and $A_1$ be infinite binary sequences which are 1-*random*, that is, algorithmically random in the sense of Martin-Löf, Levin-Schnorr, or Chaitin. We say that $A_0$ and $A_1$ are *independent* if $A_0$ is 1-random relative to $A_1$ (the condition is symmetric, by a result of van Lambalgen). Independence properties for various kinds of subsequences of 1-random sequences have been obtained by van Lambalgen and Kautz; for example, if $A$ is 1-random and $A_0, A_1$ respectively denote the even and odd bits of $A$, then $A_0$ and $A_1$ are independent. In this paper we prove a particularly strong independence property: if $A$ is 1-random and $A_1$ is a subsequence chosen from $A$ by means of a *bounded Kolmogorov-Loveland place selection*, and if infinitely many bits are left unselected, then the sequence $A_0$ of unselected bits is 1-random relative to $A_1$, i.e., $A_0$ and $A_1$ are independent. A Kolmogorov-Loveland place selection is a very general type of recursive selection rule which may be interpreted, for example, as the sequence of oracle queries in a relativized computation.

The application we have in mind concerns *resource-bounded measure theory* as formulated by Lutz, which provides a means of probabilistically describing the relative sizes of complexity classes and of defining the "random" sequences (languages) within a complexity class. Lutz has proposed the hypothesis that NP does not have measure zero within the class of exponential time languages, meaning loosely that a randomly chosen language in exponential time has non-negligible probability of being in NP. Lutz' hypothesis implies a strong separation of P from NP and is supported by the existence of a growing body of plausible consequences which are not known to follow from the weaker assertion P $\neq$ NP. What we prove is that relative to a random oracle (in fact, a 1-random oracle) NP does not have measure zero within exponential time, improving the earlier result of Bennett and Gill that P $\neq$ NP relative to a random oracle.

FARID KHAFIZOV, *AD and ultrapowers by some measures in $L(\mathbb{R})$*.
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Assume AD. We use techniques developed by Steve Jackson in his computation of $\delta^1_5 = (\sup_\eta j_\eta(\delta^1_3))^+ = \aleph_{\omega^{\omega^\omega}+1}$ ($\eta$ ranges over the measures occurring in the homogeneous tree construction for a complete $\Pi^1_3$ set) to investigate the ultrapowers by measures, $\nu$, occurring in the homogeneous tree construction for a complete $\Pi^1_1$ set. In particular, we compute $j_\nu(\omega_n)$ for all such measures $\nu$ which seem indispensable for precise computation of $j_\eta(\delta^1_3)$.

BORIS A. KUSHNER, *On Specker series*.
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Let $\alpha$ be a non-negative recursive sequence of constructive real numbers (c.r.n.; see, [2]). The series $\Sigma\alpha(i)$ is said to be a Specker series if its partial sums are bounded above by a number $B$ and the series does not converge recursively. It is evident that the general term of a Specker Series converges to 0 classically; at the same time, as is well known, it may or may not converge to 0 recursively. The following theorem shows that general terms of all Specker series are equivalent from the point of view of recursive convergence.

THEOREM. *Let $\Sigma\alpha(i)$ be a Specker series, and $\beta$ a recursive sequence of c.r.n.s that recursively converges to 0. Then the series $\Sigma\beta(i)\alpha(i)$ converges recursively.*

COROLLARY 1. *Let $\Sigma\alpha(i)$ be a Specker series, and $s$ a c.r.n., $s > 1$. The series $\Sigma(\alpha(i))^s$ is a Specker series iff $\alpha$ converges to 0 recursively.*

COROLLARY 2. *There is a Specker series $\Sigma\alpha(i)$ such that all the series $\Sigma(\alpha(i))^s$ where $s$ is a c.r.n., $s > 1$ are Specker series.*

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CLAUDE LAFLAMME, *Strong meager properties for filters*.
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We discuss several "strong meager" properties for filters on the natural numbers between the classical Baire property and a filter being $F_\sigma$. Two such properties have been studied by Talagrand and a few more combinatorial ones are investigated. In particular, we define the notion of a $\mathrm{P}^+$-filter, a generalization of the traditional concept of P-filter, and prove the existence of a non-meager $\mathrm{P}^+$-filter. Our motivation lies in understanding the structure of filters generated by complements of members of a maximal almost disjoint family.

THOMAS E. LEATHRUM, *Completions of almost disjoint families*.

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Many interesting questions arise from considering what must be added to a nonmaximal almost disjoint family in order to make it maximal. In order to provide a general context for many such questions, define

$$adc_N(\mathcal{A}) \;=\; \min\{|\mathcal{A}'| : \mathcal{A}\cup\mathcal{A}' \text{ is an infinite maximal almost disjoint family of infinite subsets of the countable set } N\}.$$

So, for example, $\mathcal{A}$ is a maximal almost disjoint family if and only if $adc_\omega(\mathcal{A}) = 0$. The standard small cardinal invariant $a$ can be defined easily using $adc$ : $\mathfrak{a} = adc_\omega(\emptyset)$. It is natural to consider $adc_N(\mathcal{A})$ for various particular families of $\mathcal{A}$. For example, if $\mathcal{A}$ is at most countable, then $adc_\omega(\mathcal{A}) = \mathfrak{a}$. However, not all of the questions are so trivial, as can be seen with any of several examples. among the results presented here will be a condition called *nowhere maximality* such that if $\mathcal{A}$ is nowhere maximal then $adc(\mathcal{A}) \geq \mathfrak{b}$. This result has certain extensions and strengthenings, but the nowhere maximality of $\mathcal{A}$ is *not* equivalent to $adc(\mathcal{A}) \geq \mathfrak{b}$. Is there a similar condition which *is* equivalent? This suggests several other questions: Under what conditions (on $\mathcal{A}$) is $adc_N(\mathcal{A}) > \omega$? $= 2^\omega$? or $\geq \mathfrak{a}$?

STEVEN D. LEONHARDI, *A non-low$_2$ R.E. degree which does not bound any Slaman triple*.

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The high-low hierarchy is based on the Turing jump operator and provides a non-trivial partitioning of **R**, the recursively enumerable (r.e.) Turing degrees. Results of Cooper and Shore have shown that any class of the form high$_n$ or low$_n$ within this hierarchy is definable in the broader class of (not necessarily r.e.) Turing degrees. However, it is not yet known whether any such class (with $n > 0$) is definable within **R**.

Some partial progress has been made. Shore and Slaman have used both the property of bounding a nonsplitting pair, and the property of bounding what we call a Slaman triple, to separate the high from the low$_2$ r.e. degrees.

We prove the following:

THEOREM. *There exists a non-low$_2$ r.e. degree which does not bound any Slaman triple, that is, a triple **u,v,w** of r.e. degrees such that*

(i) ***u*** *is not recursive,*
(ii) ***w*** *is not below or equal to **v**, and*
(iii) *any nonrecursive $\boldsymbol{b} \leq \boldsymbol{u}$ has **w** below or equal to the join of **b** and **u**.*

This result shows that the property of not bounding a Slaman triple does not define the low$_2$ degrees within **R**, and may be viewed as (albeit weak) evidence that the low$_2$ degrees may not be definable in **R**.

ADAM H. LEWENBERG, *Elementary pairs of o-minimal structures without poles*.
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An $o$-minimal structure without poles is an $o$-minimal structure in which every total 0-definable unary function is locally bounded. Similarly, an $o$-minimal theory $T$ has no poles if every model of $T$ is $o$-minimal without poles. Fix such a theory $T$. Let $T_{\text{tame}}$ be the theory of structures of the form $(\mathcal{R}, \mathcal{N}, \text{st})$ where $\mathcal{N}$ is a proper elementary substructure of a $T$-model $\mathcal{R}$ such that $\mathcal{N}$ is Dedekind complete in $\mathcal{R}$, and st is the induced standard map on $\mathcal{R}$ induced by $\mathcal{N}$. If $T$ is universally axiomatizable and admits quantifier elimination then so does $T_{\text{tame}}$. Furthermore, $T_{\text{tame}}$ is complete; if $T$ is model complete then so is $T_{\text{tame}}$.

MARK MCKINZIE AND CURTIS TUCKEY, *Elementary properties of infinite polynomials*.
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We have explored applications of hyperfinite polynomial theory to the elementary transcendental funcitons. Our techniques yield results traditionally thought to require concepts from the calculus; we formulate our concepts in extensions of nonstandard weak arithmetic. This work has potential applications in logic, history of mathematics, and calculus reform.

CHRIS MILLER, *A conjectured dichotomy for o-minimal expansions of ordered groups*.
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Conjecture. *Let $\mathfrak{R}$ be an o-minimal expansion of an ordered group $(R, <, +)$. Then either $\mathfrak{R}$ defines a multiplication $\times : R^2 \to R$ such that $(R, <, +, \times)$ is a real closed field; or, for every $\mathfrak{R}$-definable function $f : R \to R$, there exists a definable additive function $T : R \to R$, a constant $c \in R$, and a definable function $\varepsilon : R \to R$ with $\lim_{x\to+\infty} \varepsilon(x) = 0$ such that $f(x) = c + T(x) + \varepsilon(x)$.*

It follows from a recent result of R. Poston that the conjecture is true if the elementary theory of $\mathfrak{R}$ has an archimedean model. Other evidence for the conjecture is given, and partial results are established. Some general results are proved for the case that no multiplication compatible with $(R, <, +)$ is definable in $\mathfrak{R}$; in particular, the ordered division ring of all $\mathfrak{R}$-definable additive functions is invariant under elementary equivalence.

MISAO NAGAYAMA, *Recent results in substructural implicational logics*.
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We would like to talk about recent results in substructural implication logics, such as BCK-logic and implicational relevant logics. The main result is a syntactic solution to the $P - W$ problem: The problem asks whether $a$ is identical to $b$ provided that $a \to b$ and $b \to a$ are both provable in $P - W$. E. P. Martin and R. K. Meyer originally gave an affirmative solution to this problem in [2] (for its concise version, refer to [1], pp. 375–384), however their method was semantical and not easy to grasp. Here we obtain a syntactical solution to this problem, based on a Gentzen-type system for $P - W$: The system $P - W$, without exchange rule, consists of the identity axiom, the left and right introduction rules of the implication and cuts. The left introduction rule of the implication and one of the cuts are defined by so-called *guarded merges*. The cut-elimination theorem for this system has been known among relevant logicians. Along with its proof, we obtain a normalization theorem for proofs in $P - W$, where we show that if $a \Rightarrow b$ is provable in $P - W$, we can in fact find its cut-free proof of a special form, giving us some syntactic information on $a$ and $b$. Finally we briefly discuss the normalization theorems for proofs in other logical systems, and their applications.

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ANIL NERODE AND BAKHADYR KHOUSSAINOV, *Monadic second order theories of trees and semi-regular algebras*.

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Let $r_0, \dots, r_n$ be symbols for partial unary operations. Every tree $\mathcal{A}$ over alphabet $R = (r_0, \dots, r_n)$ is a unary algebra, where the unary partial operation $r_i$ is $r_i(a) = ar_i$ if $ar_i$ belongs to $A$, undefined otherwise. We investigate the monadic second order theory it MSO($\mathcal{A}$) of such an $\mathcal{A}$ and the corresponding Rabin Automata (RA) [1, 2]. (Rabin's *SnS* deals only with the free algebra.) $\mathcal{A}$ is *regular* if empty or $\mathcal{A}$ has only finitely many non-isomorphic 1-generated subalgebras and each element has a successor. Let $L$ be a nonempty subsignature of $R$. Path $d_0d_1d_2\dots$ of $\mathcal{A}$ is an *L-path* if $(\exists n)(\forall i > n)(d_i \in L)$. A (Rabin) run is *L-successful* if its restriction to the $L$-paths is successful in Rabin's sense. This is a definition in $MSO(\mathcal{A})$ having a corresponding $L$-fragment in $MSO(\mathcal{A})$.

Theorem 1. *The following are equivalent:*

(1) *L-emptiness over $\mathcal{A}$ is decidable. (Does there exist an L-successful run of RA on $\mathcal{A}$?)*
(2) *GH game determinacy is decidable. (Who wins the Gurevich-Harrington game with corresponding winning L-acceptance condition?)*
(3) *The L-fragment of $MSO(\mathcal{A})$ is decidable.*

For ordinal $\eta$, we define $\mathbf{LQR}_\eta$. 1) $\mathbf{LQR}_0$ is all regular $L$-algebras. 2) Suppose given $\eta = \theta + 1$ and $\mathcal{A} \in \mathbf{LQR}_\theta$. If $a \in \mathcal{A}$ put $\mathcal{A}_\eta = \mathcal{A}_\theta \quad {}_{a\in\mathcal{A}}\{arx \mid a \in \mathcal{A} \,\&\, x \in \mathcal{A}_{I_a,r} \,\&\, r \in I_a\}$ into the next class. For limit ordinal $\eta$, put $\mathcal{A} \in \mathbf{LQR}_\eta$ if there exists a seqeunce $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \dots \subset \mathcal{A}_\theta \subset \dots$ such that $\forall\theta < \eta(\mathcal{A}_\theta \in \mathbf{LQR}_\theta)$, and $\mathcal{A}_\eta = \quad {}_{\theta<\eta}\mathcal{A}_\theta$. Elements $\mathcal{A}$ of ${}_\eta\mathbf{LQR}_\eta$ are the *L-quasi-regular algebras*. The *regularity degree* of $\mathcal{A}$ is the least $\eta$ with $\mathcal{A} \in \mathbf{LQR}_\eta$, and is proved finite or $\omega$. Call $\mathcal{A}$ *L-semi-regular* if the $L$-subalgebra generated by any $a \in \mathcal{A}$ is regular. We prove that $\mathcal{A}$ is $L$-semi-regular iff $\mathcal{A}$ is $L$-quasi-regular. Algebra $\mathcal{A}$ is *recursively algebraic* if there exists a recursive sequence $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \dots$ such that

(1) for each $i$, $\mathcal{A}_i \in \mathbf{LQR}_i$;
(2) for each $i$, if $x$ and $y$ generate isomorphic $L$-subalgebras in $\mathcal{A}_i$, then elements $r(x)$ and $r'(x)$ for which $r(x), r'(x) \in A_{i+1} \backslash A_i$ also generate isomorphic $L$-subalgebras;
(3) There exists a procedure which for all $i, x \in A_i$, and $r \notin L$ such that $r(x) \in A_{i+1} \backslash A_i$ finds the number of isomorphism types of the 1-generated $L$-subalgebras of he $L$-algebra generated by $r(x)$.

Theorem 2. (1) *For $\mathcal{A}$ a recursive algebraic L-semi-regular algebra with finite regularity* *degree, the L-fragment of the* $MSO(\mathcal{A})$ *is decidable.*

(2) *For* $\mathcal{A}$ *a recursively algebraic L-semi-regular algebra with infinite regularity degree, the L-degree of it* $MSO(\mathcal{A})$ *is co r.e.*

(The proof uses the GH determinacy theorem [1] at each stage of the construction.)

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ANDRÉ NIES, *Undecidable fragments of elementary theories*.
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If an elementary theory $T$ is known to be undecidable, an interesting further question is for which $k$ (if for any) the framgent of the $\Pi_k$-sentences in $T$ or the set of $\Sigma_k$-sentences in $T$ becomes undecidable: for small $k$, these sentences can be viewed as the "feasible" sentences and, if $T$ is the theory of a particular structure (e.g., a degree structure in recursion theory), they normally coincide with the sentences about the structure actually considered in mathematical research. A first result in this direction has been given in [3], where the $\Pi_3$-theory of the structure of all Turing degrees is shown to be undecidable.

Undecidability of an elementary theory $\mathrm{Th}(\mathbf{C})$, $\mathbf{C}$ a class of structures of a given type, is usually proved indirectly, by coding in a uniform way the members of a complicated class of $\mathbf{D}$ of structures. It is required that each subtheory of $\mathrm{Th}(\mathbf{D})$ containing the valid sentences is undecidable (such a theory is called hereditarily undecidable). Using some ideas from [3], we introduce a general framework where the notions involved are adapted and the coding method is refined to obtain undecidability results for fragments. The method is applied first to classes of finite graphs and finite lattices, and, in a second step, to degree structures. For instance, we show that the $\Pi_3$-theory of the class of finite distributive lattices is hereditarily undecidable, by coding the class of finite bipartite graphs by $\Sigma_1$-formulas with parameters. Next, using the fact that each finite distributive lattice is isomorphic to an initial segment of the r.e. many-one degrees ([1]), we conclue that this degree structure has undecidable $\Pi_3$-theory. The method also applies to other r.e. degree structures. Here results can be obtained by coding the class of finite bipartite graphs directly ([2]).

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GREG RAY, *Truth in an intended model*.
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Tarski gave a precise characterization of truth for formal, interpreted non-modal languages. However, in our formal characterization of the logical properties—logical truth, logical consequence—we do not use this notion of truth. Rather, we use the model-theoretic notion of truth-in-a-model. Since, presumably, logical truth is a subspecies of truth, and logical consequence involves at least truth preservation, there had better be some connection between our analysis of truth and the technical notion of truth-in-a-model. In the case of non-modal language, we bridge the gap between the two, by claiming that a sentence is true just in case it is true in an intended model. Where an intended model is a model which has as domain the actual objects of discourse, and which relates these objects in an appropriate manner. However, the same strategy applied to the case of modal languages seems to require an intended modal model whose domain includes mere possibilia — things which might have existed but do not. If such things don't exist, then there can be no intended modal models. If we insist that there are such models, then we seem committed to the actual existence of the possibilia. Building on recent work by C. Menzel, I will sketch a strategy for giving a purely extensional, model-theoretic semantics for modal lnaguages requiring nothing but pure sets and the actual objects of discourse. To do this, we must define a notion of *intended model* which does not require possibilia.

MITCH RUDOMINER, *A connection between inner model theory and descriptive set theory*.
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We discuss an extension of some work done by Donald A. Martin, John Steel, and Hugh Woodin. This work establishes connections between Inner Model Theory and Descriptive Set Theory by proving theorems of the following form: We start with a particular countable set of real numbers, $A$. This set is defined using concepts that are purely Descriptive Set Theoretic. $A$ is the se tof reals which are $\Delta$-definable from a countable ordinal parameter, for some particular pointclass $\Delta$. The set $A$ has much independent interest to Descriptive Set Theorists, who study it for their own reasons. The theorems of Martin, Steel, and Woodin then give an entirely different characterization of the set $A$ using concepts from Inner Model Theory. It is shown that $A = \mathbb{R} \cap M$, where $M$ is the canonical inner model for some large cardinal axiom. For example, let $A$ be the famous set $Q_3$. Martin, Steel, and Woodin show that $A = \mathbb{R} \cap M$, where $M$ is the canonical inner model with a Woodin cardinal.

We discuss an extension of this work to other pointclasses $\Delta$, and other inner models $M$.

We then go on to discuss one particular application. A conjecture of Steel's concerning Inner Model Operators is proved. Also a generalization of Steel's conjecture is stated and proved.

ANDRE SCEDROV, *Stochastic interaction and linear logic*.
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In this joint work with P. D. Lincoln and J. C. Mitchell we present stochastic interactive semantics for propositional linear logic without modalities. The framework is based on prover/verifier interactions considered in computational complexity theory, in which the prover has unlimited power while the verifier can only toss fair coins or perform simple tasks when presented with the given formula or with subsequent messages from the prover. The additive conjunction & is described as random choice, which reflects the intuitive idea that the verifier can perform only "random spot checks." This stochastic interactive semantic framework is shown to be sound and complete. Furthermore, the prover's winning strategies are basically proofs of the given formula. In this framework the multiplicative and additive connectives of linear logic are described by means of probabilitistic operators, giving a new basis for intuitive reasoning about linear logic and a potential new tool in automated deduction. Extended abstract is available by anonymous ftp from ftp.cis.upenn.edu as pub/papers/scedrov/ipcup.{dvi,ps}.Z or by WWW on URL http://www.cis.upenn.edu/~andre.

G.Y. SHER, *A new solution to the problem of truth (Or: how general and how substantive is Tarski's theory of truth?).*
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The idea of a theory of truth that is both general and substantive is problematic (even contradictory, according to Kant). Roughly, we can identify the problem as follows: Different things make different sentences true; a *substantive* definition of truth has to capture these differences, but a *general* theory of truth must abstract from them. In the literature, the problem is usually articulated in vague terms, but one way to give it precise content is to view it as involving the kind of consideration that Frege brought to the question "What is number?" According to Frege, an informative definition of "number" will not just distinguish between numbers and non-numbers, but will also give us the identity conditions of numbers, i.e., distinguish between one number and another. More generally, an informative definition of X has to show not only how to distinguish X's from non-X's, but also how to distinguish one X from another. For a definition of truth to be informative it has to distinguish not only between truths and falsities (or truths and non-truths) but also between one truth and another. Clearly, to say that all true sentences denote The True is not informative. But Aristotle's characterization of truth is also not informative; even Tarski's definition, many claim, is uninformative. But how can we define truth informatively? How can we define truth in a way that captures both what is common to all truths and what distinguishes between them?

What I call "the problem of truth" consists of two questions (the second possibly empty): (A) Is a general and substantive theory of truth possible? (B) What is accomplished by such a theory? In this paper I propose an outline of a positive answer to the problem of truth. I analyze Tarski's theory in light of this answer, and I explore the possibility of extending and generalizing Tarski's theory.

MASARU SHIRAHATA, *A linear conservative extension of Zermelo Fraenkel set theory*.
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In this paper, we present the system of set theory LZF with the unrestricted comprehension, formulated in linear logic. The resource awareness provided by linear logic makes it possible to have more sets than in classical set theory by the unrestricted comprehension, while avoiding the set-theoretical paradoxes.

LZF is built on top of the Zermelo-Fraenkel set theory without the axiom of regularity, or $ZF^-$. We formulate LZF as a sequent calculus with abstraction terms. The terms which correspond to the sets in $ZF^-$ are called Z-terms and the formulas which contain only Z-terms are Z-formulas. We establish the partial cut-elimination result for LZF, which implies that if a Z-formula is provable in LZF, it is provable by using only Z-terms. It follows from this that LZF is a conservative extension of $ZF^-$.

The partial cut-elimination theorem is proved in two steps. First, we eliminate all cuts over complex formulas by the standard method. Then we proceed by induction on the size of a proof to eliminate the rest of the cuts except those over Z-formulas. LZF is designed so that no contraction is involved in the second step, and this guarantees that the size of a proof gradually descreases in cut-elimination process. This nice property of linear logic has been observed by many authors and used for the cut-elimination of various systems.

WILFRIED SIEG AND JOHN BYRNES, *A mathematical explication of Turing's argument*.
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Turing addressed in [5] the fundamental question: *What are the possible processes a human computer can carry out in calculating values of a number-theoretic function?* He claimed that all such processes can be carried out by his "automatic machines" or, in modern terms, by deterministic Turing machines. Turing's consideration for this claim can be reconstructed as involving: first, a formulation of finiteness conditions (for symbolic configurations and mechanical operations); second, a proof that computational processes (satisfying the finiteness conditions) can be carried out by a Turing machine; third, the thesis that all (mechanical) processes carried out by human computors must satisfy the finiteness conditions. In [3] this is called *Turing's central thesis*. Turing made the latter highly plausible by appealing to the limitations of the human sensory apparatus; its ultimate justification, for Turing, is given by "the fact that the human memory is necessarily limited." In Turing's presentation (section 9 of [5]) the three aspects of his considerations are intermixed, and important steps in the proof are only hinted at; e.g., the reduction of computations on two-diemsnional configurations to computation on sequences is suggested by " ... I think it will be agreed that the two-dimensional character of apper is no essential of computation." Though a systematic and detailed analysis is given in [4], the proof mentioned above is still lacking mathematical rigor.

To formulate Turing's Theorem properly we have to specify the finite symbolic configurations and the mechanical operations that can be carried out on them. For this purpose we introduce *Kolmogorov machines*. These machines are inspired by the analysis of algorithms in [2], cf. also [6]; but they directly generalize Turing machines, when the latter are prsented via production systems as in [1, pp. 6–7]. These machines do not just operate on strings of symbols with one scanned element, but rather on finite labeled graphs with one *central vertex* that is distinguished by a special label; the graphs have the further property that distinct paths (containing the central vertex) have uniquely associated sequences of labels. The opeations of such machines replace distinguished subgraphs (with the central vertex) by finite graphs (also containing the central vertex); they are gien by a finite list of generalized production rules. We can prove:

THEOREM. *Computations of Kolmogorov machines can be simulated by Turing machines.*

Given this mathematical analysis, Turing's central thesis is turned into the thesis that Kolmogorov machines (that clearly satisfy the required finiteness conditions) do directly the work of human computors. The Theorem above constitutes then the general reduction of mechanical calculations to Turing machine computations.

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SŁAWOMIR SOLECKI, *Equivalence relations and the theory of continua*.
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The following is a particular case of the theorem proved in the paper.

Let $X$ be a Polish space. Let $E \subset X \times X$ be a $K_\sigma$ equivalence relation with at least two distinct equivalence classes. Assume each equivalence class of $E$ is dense. Then there is no Borel subset of $X$ which has exactly one point in common with each equivalence class of $E$.

A corollary to this theorem answers an old question in the theory of indecomposable continua. Namely, there is no Borel subset of an indecomposable continuum which has exactly one point in common with each composant.

ZORAN SPASOJEVIĆ, $(\omega_1, \omega_1)$-*gaps under* $MA(\omega_1)$.

I prove that $\mathrm{MA}(\omega_1)$ implies that every $\subset^*$-increasing $\omega_1$ sequence of subsets of $\omega$ is a lower half of some $(\omega_1, \omega_1)$-gap. This result can be seen as a refinement of the famous theorem by Hausdorff on $(\omega_1, \omega_1)$-gaps, when $\mathrm{MA}(\omega_1)$ is assumed.

KATHERINE ST. JOHN, *Substructural logics and the formal language of recursion*.
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In [1], Moschovakis interprets the meaning, or intension, of a formula as an algorithm that computes its truth value (following Frege). We outline a semantics for substructural logics based on the intensions of the Formal Language of Recursion presented in [2]. That is, we will assign to every formula a recursor that represents its meaning (ignoring timing). A natural question to ask is if $A$ and $B$ are provably equivalent, must their meanings—or some aspects of their meanings—be equal?

More precisely, say two formulas, $A$ and $B$, are *provably equivalent* in a substructural logic $S$, if $A \Rightarrow B$ and $B \Rightarrow A$ are provable in a Gentzen-style proof system for $S$. Also say $A \leftrightarrow B$ is a *linear tautology* in a substructural logic $S$, if $A$ and $B$ are propositional formulas using logical symbols only from $S$, every variable occurs exactly once in $A$ and exactly once in $B$, and $A \leftrightarrow B$ is a truth-table tautology.

We have shown for the multiplicative fragment of linear logic (following the notation of [3]):

THEOREM 1. *For arbitrary formulas of* $\{+, *\}$*, A and B are provably equivalent if and only if* $int(A) \equiv int(B)$*, where* $int(A)$ *is the intension of the formula of A, ignoring timing.*

This result extends to the system $\{+, *, \sim, \rightarrow\}$ for linear tautologies.

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KATRIN TENT, *Modules and 1-based groups*.
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We show that certain 1-based groups (e.g., totally categorical ones) can be embedded into a -definable larger group on which all structure is given by its ring of definable endomorphisms modulo the algebraic closure of $\emptyset$. In this way the larger group becomes a module which is almost quantifier eliminable and is generated by the smaller group. The ring is finite and as a consequence there is some bound $k < \omega$ such that every type is implied by its restriction to formulas with at most $k$ parameters.

JOHN THURBER, *Turing Degrees of Boolean Algebras*.
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We discuss conditions under which a Boolean algebra will or will not have a recursive copy.

One of the results is related to work by Feiner, and involves consideration of what types of sets can be coded into recursive Boolean algebras. Here sets are classified according to a measure of complexity defined by Feiner.

The results on $\text{low}_1$ and $\text{low}_2$ Boolean algebras stated in the titles of [2] and [5] depend on an isomorphism theorem of Remmel which is used to provide conditions under which two linear orders will generate isomorphic interval algebras. The techniques used suggest that a deeper understanding of the relationship between order types and the Boolean algebras they generate may be helpful in determining whether a $\text{low}_n$ Boolean algebra has a recursive copy for $n > 2$.

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DOLPH ULRICH, *A four-valued matrix validating the nine-character implication-pure theses*.
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A lemma shows that the matrix

| $C$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $*1$ | 1 | 2 | 3 | 4 |
| 2 | 1 | 1 | 3 | 1 |
| 3 | 1 | 2 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1, |

whose designated value is marked with an asterisk, includes among its tautologies the theses of the implicational fragment **IF** of the classical sentential calculus in which at most two distinct sentence letters occur. Since the matrix rejects $CCpqCCqrCpr$, there follows as an intermediate corollary the Wajsberg/Diamond-McKinsey theorem for classical implication: with substitution and detachment as rules, every complete set of axioms for **IF** must include at least one axiom containing occurrences of three or more distinct sentence letters.

Additional argument utilizes the lemma to show that the matrix also validates all pure implicational theses of length nine or less, so that every complete axiom set for **IF** must include at least one axiom eleven or more characters long. This strengthens a result of Jaśkowski [1] that every such set must contain either one axiom of length at least eleven *or else two axioms each of length at least nine*, and is best possible since $\{CCpqCCqrCpr, CCCpqpp, CpCqp\}$ is known to axiomatize **IF**.

Though only *classical* implication is explicit throughout, the proofs given and the results obtained hold as well for virtually all logics of pure implication known in the existing literature, including the various logics of pure entailment and relevant implication, the strict-implicational fragments of the standard modal logics, and the relatives of implicational intuitionism and of $C$-pure logics in the Łukasiewicz family.

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GLEN WHITNEY, *The limits of extensionality in non-deterministic models of recursion*.
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Extending models of computation from deterministic to non-deterministic operations has long posed serious theoretical difficulties, especially when investigating various types of concurrent *merges* of processes. Sets of deterministic behaviors and ordinary functions on these sets most naturally model non-deterministic processes and process transformations, respectively. Yet Brock and Ackermann [1] exhibit computation networks (critically involving *angelic* merge nodes) which cannot be adequately modeled in this fashion, seemingly entailing a loss of *extensionality*. Common responses to such difficulties include disallowing some sets as possible processes [3]; adding new processes not corresponding to any set [4]; or adding additional information to each process [1]. A recent approach by Moschovakis [2] pushes the non-extensionality into the operations; i.e., process transformations are not determined solely by their action on processes, which are simply sets of behaviors. He asks (via a specific open question) whether the non-extensionality can now be eliminated altogether.

The present work yields a negative answer to Moschovakis' question. Beyond that, it provides a broad obstruction to extensional, non-deterministic models of (the standard logic of) recursion, as in the following:

Theorem 1. *If $D$ is any complete partial order containing a copy of $\omega+1$, then no extensional recursion structure on the powerset of $D$ can assign the set $\omega$ as the meaning of some program and interpret some function symbol "or" by binary union so that the following two conditions* *are always satisfied:*

$$\begin{aligned} x \textit{ where } [x = f(x) \textit{ or } g(x)] &\supseteq x \textit{ where } [x = f(x)], \\ x \textit{ where } [x = x \textit{ or } c] &= c \cup \{\perp\}. \end{aligned}$$

This improves the Brock-Ackerman counterexample in several important ways:

1. It removes the dependency on the specific construct of angelic merge (itself intrinsically problematic to model), relying only on simple non-deterministic choice ("or").
2. It covers arbitrary structures supporting recursion, not just the "history relations semantics for computing networks" considered in [1].
3. Together with a construction I give of a structure satisfying all but the last clause of the theorem for any $D$, it helps to delineate the boundary of what can be achieved in modeling recursion and non-determinism while maintaining extensionality.

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DAN E. WILLARD, *Self-verifying axiom systems and the incompleteness theorem*.
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We introduce some weak cut-free First Order axiom systems which can *simultaneously* verify their own consistency and prove more $\Pi_1$ theorems than Peano Arithmetic. More formally, let $A$ denote any consistent recursive axiom system whose axioms are indicated by a $\Delta_0$ formula. Define IS($A$) to be a semantic tableaux deduction system containing the following three groups of axioms.

1) an initial group defining a preliminary set of functions and relations, such that Addition is the only increasing function (non-zero divison will replace multiplication as a function);
2) the statement that all $\Pi_1$ theorems proven by "the inner axiom system" $A$ are valid; and
3) the statement that the full system IS($A$), including this third sentence, is consistent.

The third axiom group can be formalized in any of several different manners. These include stating either

i) that there is no semantic tableaux proof from IS($A$) of $0 = 1$, or
ii) that the uniform version of IS($A$)'s reflection principle is valid for all $\Sigma_1$ formulae.

In either case, our main theorem will state that *if A is consistent then it will automatically follow that IS(A) is consistent*. Although our theorems require cut-free deduction, they are also valid in the presence of cuts, if the Addition function in Group-1 is replaced by Subtraction.

Our paper uses an analog of Kleene's Fixed Point Theorem to show that $\mathrm{Prf}_{\mathrm{IS}(A)}(x, y)$ can be formally defined by a $\Delta_0$ formula. It includes negative as well as positive results. The negative results are that IS($A$) is inconsistent when $A$ corresponds to Peano Arithmetic and either

a) the Group-3 axioms of $\mathrm{IS}(A)$ include the local $\Pi_1$ reflection principle, or

b) $\mathrm{IS}(A)$'s Group-1 axioms are expanded to recognize multiplication as a total function and Group-3 recognizes uniform $\Delta_0$ reflection.

Since our paper shows $\mathrm{IS}(A)$ is consistent when Group-3 employs merely uniform $\Delta_0$ reflection, the source of Result (b)'s inconsistency must be the multiplication function. A surprising aspect of Result (a) is that *although incompatible* with $\Pi_1$ reflection, $\mathrm{IS}(A)$ (itself!) *can prove that* $\{\exists y \mathrm{Prf}_{\mathrm{IS}(A)}(\ulcorner \forall v \Phi(v) \urcorner, y)\} \subset \Phi(\overline{k})$, for any *fixed constant* $\overline{k}$ and for any $\Sigma_1$ formula $\Phi(v)$. This fact implies that while $\mathrm{IS}(A)$ does not have access to $\Pi_1$ reflection, it does not have access to *a finitized* $\Pi_1$ *reflection principle*, indicating that the proof of $\forall v \Phi(v)$ is a vehicle for recognizing the validity of any finite conjunction $\Phi(0) \wedge \Phi(1) \wedge \Phi(2) \wedge \cdots \wedge \Phi(N)$.

ANDRZEJ WIŚNIEWSKI, *Some results on the reducibility of questions*.
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Let **L** be a semantically interpreted first-order language enriched with questions. Assume that for each question $Q$ of **L** there exists an at least two-element set $dQ$ of *direct answers* to $Q$; $dQ$ consists of closed declarative formulas (d-wffs) of **L**. The sets of direct answers to simple and conditional yes-no questions are of the form $\{A, \neg A\}$ and $\{A \,\&\, B, A \,\&\, \neg B\}$, respectively. Let $X$, $Y$ stand for sets of d-wffs of **L**. We say that $X$ *multiple-conclusion entails* (mc-entails) $Y$ iff each normal interpretation of **L** being a model of $X$ makes true at least one d-wff in $Y$; $X$ *entails* a d-wff $A$ iff $X$ mc-entails $\{A\}$.

DEFINITION. A question $Q$ is reducible to a non-empty set of questions $\Phi$ iff

(i) for each $A \in dQ$, for each $Q^* \in \Phi$: $A$ mc-entails $dQ^*$, and

(ii) each set made up of direct answers to the questions of $\Phi$ which contains exactly one direct answer to each question of $\Phi$ entails some direct answer to $Q$, and

(iii) no question in $\Phi$ has more direct answers than $Q$.

A question $Q$ is *safe* iff $\emptyset$ mc-entails $dQ$; otherwise $Q$ is *risky*.

THEOREM 1. (1) *Each safe question is reducible to some set of questions made up of simple yes-no questions.* (2) *If entailment is compact, then each safe question is reducible to a finite set of questions made up of simple yes-no questions.*

$A \in \mathrm{Pres}Q$ iff for each $B \in dQ$, $B$ entails $A$; the elements of $\mathrm{Pres}Q$ are called *presuppositions* of $Q$. A question $Q$ is *normal* iff $\mathrm{Pres}Q$ mc-entails $dQ$; $Q$ is *regular* iff for some $A \in \mathrm{Pres}Q$, $A$ mc-entails $dQ$.

THEOREM 2. (1) *Each regular risky question is reducible to some set of questions made up of conditional yes-no questions.* (2) *If entailment is compact, then each normal risky question is reducible to a finite set of questions made up of conditional yes-no questions.*

Moreover, if $dQ$ is a finite set, then $Q$ is reducible to a finite set of questions (made up of simple yes-no questions if $Q$ is safe, and of conditional yes-no questions otherwise).

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JIRI WITZANY, *Any behavior of the Mitchell ordering of normal measure is possible*.
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Let $U_0, U_1$ be two normal measures on $\kappa$. We say that $U_0$ is in the Mitchell ordering less then $U_1$, $U_0 \lhd U_1$, if $U_0 \in U\mathrm{lt}(V, U_1)$. The ordering is well-known to be transitive and well-founded. It has been an open problem to find a model where $\lhd$ embeds the four-element poset | |. I show that in the Kunen-Paris extension all well-founded posets are embeddable into $\lhd$ on normal measures over $\kappa$ (assuming $\kappa$ is highly measurable in the ground model). Hence there is no structural restriction on the Mitchell ordering. Moreover I show that it is possible to have two $\lhd$-incomparable measures that extend in a generic extension into two $\lhd$-comparable measures.

VLADMIR YAKHNIS, *A simplified way to complement Rabin's tree automata within Gurevich-Harrington framework*.
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Gurevich and Harrington (GH) gave a remarkably transparent procedure for complementation of Rabin's tree automata (R-automata). GH introduced slightly different tree automata (GH-automata) which are equivalent to R-automata. To every GH-automata $\mathcal{G}$ and every $\Sigma$-valued tree $v$ GH assigned a two player game (GH-game) where the players are called "automaton" and "pathfinder." $\mathcal{G}$ accepts $v$ iff "automata" wins the game. It follows from the GH Restricted Memory Determinacy Theorem (RMDT) that if the tree is not acceptable, then a winning strategy for "pathfinder" can be represented as a $\Delta$-valued tree, where $\Delta$ is a special finite alphabet.

Given an $R$-automaton $\mathcal{A}$, the first three steps of the GH construction may be summarized as follows. (1) Construct a GH automaton $\mathcal{G}$ that accepts the same set of $\Sigma$-valued trees as $\mathcal{A}$ does; (2) Convert the sentence "($\mu$ is an infinite $\{0,1\} \times \Sigma \times \Delta$-sequence) and (if we regard the $\{0,1\} \times \Sigma$-projection of $\mu$ as a path $\eta$ in a $\Sigma$-valued tree, there is a run of $\mathcal{G}$ on $\eta$ such that ("automaton" wins the respective play of the GH-game and "pathfinder" in this play follows the strategy represented by the $\Delta$-projection of $\eta$))" into an $S1S$ formula; (3) construct a Buchi $\omega$-automaton equivalent to the formula.

Monk simplified steps 2 and 3 by eliminating references to $S1S$ and by constructing a Buchi automaton from the sentence in step 2. Since Monk's construction required building a Buchi automaton for union and intersection of sets of sequences, it is still somewhat complicated.

We introduce tree automata that are slightly different from either R- or GH-automata. Our automata enjoy the same relationship with GH games as GH-automata do, and they are equivalent to R-automata as well. By replacing the GH-automata by our automata in the above construction, we remove all the complexity from steps 2 and 3, since our automata allows one to directly construct a nondeterministic Muller automaton accepting the sequences described in step 2.

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ANDRZEJ M. ZARACH, *What is a proper class?*.
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In GB, or ZF with additional predicates and the Collection Scheme for the extended language,

$$X \text{ is a proper class iff } \forall\alpha\exists x \in X(\text{rank}(x) > \alpha).$$

Thus, the concept of power classes is ZF (GB)-absolute.

If the Power Set Axiom is replaced by "$\aleph(x)$ is bound for every $x$" where

$$\aleph(x) = \{\alpha \mid \exists f(f \text{ is one-to-one function from } \alpha \text{ into } x)\},$$

then the theory is denoted by ZFH (H stands for Hartogs' Number).

In ZFH, $L =$ the class of all constructible sets is a model for ZF. In general, if $x$ is a set, then $L[x] \models$ the Power Set Axiom.

METATHEOREM: *The concept of proper class is not ZFH-absolute.*

*Construction of a counterexample:* Let $On(C) =$ the weak product (only finite functions) of $On$ copies of $C$, where $C =$ Cohen's Notion of Forcing; $\omega(C) =$ the weak product of $\omega$ copies of $C$. Let $G$ be $On(C)$-generic over $M$, where $M$ is a standard, transitive model for ZFC.

Fact 1. $\langle M[G], M, G\rangle \models$ ZFH (since the cofinality is preserved).

In this structure we define $\mathbf{A} =$ the class of all axes of $G$, and Coll$\mathbf{A} =$ the class of all finite functions from $\omega$ into $\mathbf{A}$. Let $B$ be Coll$\mathbf{A}$-generic over $\langle M[G], M, G\rangle$, so $B$ is a one-to-one function from $\omega$ onto $\mathbf{A}$.

Let $b = \{f \in \omega(C) \mid \exists F \in B(\text{dom}(f) \subseteq \text{dom}(F) \,\&\, \forall i \in \text{dom}(f)(f(i) \in F(i)))\}$. Then the $n$th axis of $b$ is $B(n)$, so $\mathbf{A} =$ the set of all axes of $b$.

THEOREM: *Let* $V_1 = \langle M[G], M, G\rangle$, $V_2 = \langle M[G][B], M, G, M[G]\rangle$, *and* $V_3 = M[b]$. *Then,* $\boldsymbol{A}$ *is definable in each of these structures, and*

(i) $V_1 \models$ *ZFH + for every cardinal* $\kappa$, *there is some one-to-one function from* $\kappa$ *into* $\boldsymbol{A}$ + $\forall a \in \boldsymbol{A}(rank(a) = \omega)$,

(ii) $V_2 \models ZF^- +$ *every infinite set is countable* + $\boldsymbol{A}$ *is not a set,*

(iii) $V_3 \models$ *ZFC+* $\boldsymbol{A}$ *is a countable set.*

Remarks. $V_1$ and $V_3$ have the same cardinals, $V_1 \subseteq V_2$ and there is no inclusion for $V_1, V_3$ and $V_2, V_3$.

$B$ and $G$ are not simultaneous classes in any universe of set theory. Otherwise,

$$\{\langle n, \alpha\rangle \mid \text{ the } n\text{th term of } B = \text{the } \alpha\text{th axis of } G\}$$

would define a one-to-one function from $\omega$ onto $On$.

## Abstracts of papers submitted by title

GUNWON LEE, *HOP-Model (study plan)*.
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In principle, the evaluation statements can be derived from the given descriptive statements as the is-ought inference in John R. Searle, with the human hiatus which makes the connections flexible.

However, we try to view it in historical perspective. If the agent $a$ had to do $x$ without any exceptions in a given situation, such as $a$ had to go out through the door $d$ if there is only one door $d$ for the room, HOP-model, $aPh$; the choice of the agent $a$, the prescription $P$ in that particular historical situation $h$, can be formulated as the historical obligatory prescription model such as the "historical definition" of Park Jong-Hong. However in practice, a lot of open questions remain.

GREGORY MCCOLM, *Dimension and connectivity*.
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In Fixed Point logic, one sets up an induction by getting a system of formulas

$$\phi_i(\boldsymbol{x}_i, S_0, S_1, \dots, S_\nu), \qquad i = 0, 1, \dots, \nu$$

(where the length of $\boldsymbol{x}_i$ equals the arity of $S_i$ for each $i$), and running an induction

$$\phi_i^{n+1}(\boldsymbol{u}) \equiv \phi_i(\boldsymbol{u}, \phi_0^n, \dots, \phi_\nu^n)$$

in the hope of reaching a fixed point $\phi_0^N, \dots, \phi_\nu^N$, where $\phi_i^{N+1} = \phi_i^N$ for all $i$. Fixed point logic consists of the formulas $\phi_0, \dots, \phi_\nu$ producing these fixed points.

We look at two popular complexity measures on fixed point logic. "Number of variables" is the maximum arity of any subformula of any of the $\phi_i$: this measure is associated with the logic $\boldsymbol{L}_{\infty\omega}(\omega)$ and is used a lot in finite model theory. "Dimension" is the maximum (first order) arity of any of the $\phi_i$, and is more difficult to deal with. We will discuss the problem of determining the number of variables and dimension of given (least) fixed point queries using pebble games.

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DAN TALAYCO, *Set theory, cohomology and the combinatorial snipe*.
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After a brief introduction to the cohomology of Hausdorff gaps, I will consider some more complex combinatorial objects which might exist. These would arise from higher cohomology groups associated with Hausdorff gaps.

GEORGE WEAVER AND DAVID LIPPEL, *The existence of finitely axiomatizable $\aleph_0$-categorical proper class II theories*.
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Let $K$ be a finite similarity type and $\mathbf{A}$ be a countably infinite interpretation of type $K$ whose complete theory is $\aleph_0$-categorical. $l_A : \{n : n \geq 1\} \to \omega$ such that $l_A(n) = m$ provided $m$ is the least member of $\omega$ such that for all $a_1, \dots, a_n$ in $A$ (the domain of $\mathbf{A}$) and all $f : \{a_1, \dots, a_n\} \to A$ if $(\mathbf{A}a_1 \dots a_n)$ and $(\mathbf{A}f(a_1) \dots f(a_n))$ are $m$-equivalent in the sense that they agree on all sentences in $m$ or fewer bound variables, then $f$ can be extended to an automorphism on $\mathbf{A}$. $l_A$ is an increasing function.

The complete theory of $\mathbf{A}$ is a *class I theory* iff for all $n$, $l_A(n) = 0$; a *class II theory* iff there is $m > 0$ and $t \geq 1$ such that for all $n \geq t$, $l_A(n) = m$; and a *class III theory* provided for all $m \geq 0$, there is $n \geq 1$ such that $l_A(n) > m$. Some class II theories can be extended to class I theories by the addition of finitely many definitions of new predicate constants. Such class II theories are called *pseudo class I theories*. Class II theories which are not pseudo class I are classed *proper class II theories*.

For each $n \geq 0$, $T_n$ is the theory of infinite Boolean algebras with exactly $n$-atoms formulated in a first order language whose non-logical vocabulary contains only individual constants and functional constants. $T_n^*$ is the theory of infinite Boolean algebras with exactly $n$-atoms formulated in a first order language whose non-logical vocabulary contains only individual constants and relational constants.

THEOREM: (1) *$T_0$ is a finitely axiomatizable class I theory;* (2) *for all $n \geq 1$, $T_n$ is a finitely axiomatizable proper class II theory;* and (3) *for all $n \geq 0$, $T_n^*$ is a finitely axiomatizable proper class II theory.*

XUNWEI ZHOU, *Set theory in geometrical logic*.
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Geometrical logic is constructed by the author, it is a geometrical approach to logic. In geometrical logic there is no propositional calculus, and predicate calculus is treated as the same discipline as set theory. In geometrical logic there are 5 operators: $\sim, \sqcap, \sqcup, \oplus, \otimes$ and 7 connectives: $\overset{-1}{\sqcap}$, $\overset{-1}{\sqcup}$, $\overset{-1}{\subseteq}$, $\overset{-1}{=}$, $\overset{-1}{\subset}$, $\overset{-1}{X}$, $\overset{-1}{\oplus}$. $\sim$ corresponds to $\sim$ in naive set theory and $\neg$ in classical logic, $\overset{-1}{\sqcap}$ to $\cap$ in naive set theory and $\wedge$ in classical logic, $\overset{-1}{\sqcup}$ to $\cup$ in naive set theory and $\vee$ in classical logic, $\overset{-1}{\subseteq}$ to $\subseteq$ in naive set theory and $\rightarrow$ in classical logic, $\overset{-1}{=}$ to $=$ in naive set theory and $\leftrightarrow$ in classical logic, $\overset{-1}{\subset}$ to $\subset$ in naive set theory and $\rightarrow$ in classical logic.

In geometrical logic, knowledge is divided into 4 leels, term level, fact level, empirical connection level, and logical connection level. Geometrical logic employs hierarchical axes shown below. The term level knoweldge is represented by the first order space of the hierarchical axes, fact by the second order space, empirical connection by the third order space, logical connection by the forth order space.

$x, y$ in the first order space of the hierarchical axes are term variables varying through elementary zeroth order terms such as $1, 2, 3$; $P, Q$ in the second order space are fact variables varying through elementary first order facts such as $= (x, y), \leq (x, y)$; $\Psi, \Omega$ in the third order space are empirical connection such as $P \overset{-1}{\subseteq} Q, \sim Q \overset{-1}{\subseteq} \sim P$. $\Lambda, \Delta$ in the fourth order space are logical connection variables varying through elementary third order logical connections such as $\Psi \overset{-1}{=} \Omega$, $\Psi \overset{-1}{\subset} \Omega$.

A point in the first order space such as $(2, 2)$ is a first order element, a point in the $P$ axis of the second order space such as $= (x, y)$ is a first order set, and we say that $(2, 2)$ belongs to $= (x, y)$, or that $= (x[2], y[2])$ is true. A point in the second order space such as $(= (x, y), \leq (x, y))$ is a second order element, a point on the $\Psi$ axis of the third order space such as $P \overset{-1}{\subseteq} Q$ is a second order set, and we say that $(= (x, y), \leq (x, y))$ belongs to $P \overset{-1}{\subseteq} Q$, or that $P[= (x, y)] \overset{-1}{\subseteq} Q[\leq (x, y)]$ is true. A point in the third order space such as $(P \overset{-1}{\subseteq} Q, \sim Q \overset{-1}{\subseteq} \sim P)$ is a third order element, a point in the $\Lambda$ axis of the fourth order space such as $\Psi \overset{-1}{=} \Omega$ is a third order set, and we say that $(P \overset{-1}{\subseteq} Q, \sim Q \overset{-1}{\subseteq} \sim P)$ belongs to $\Psi \overset{-1}{=} \Omega$, or that $\Psi[P \overset{-1}{\subseteq} Q] \overset{-1}{=} \Omega[\sim Q \overset{-1}{\subseteq} \sim P]$ is true.