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EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '93

Keele, England, July 20–29, 1993

The European Summer Meeting of the Association for Symbolic Logic was held at Keele University, Staffordshire, on 20–29 July, 1993. The program included twenty invited hour talks, two invited short courses and forty-eight contributed papers. There were also demonstrations of computerized logic teaching software, by Stephen Read, Steve Reeves and Richard Bornat. The program committee consisted of Dov Gabbay, Wilfrid Hodges (chair), Martin Hyland, Thomas Jech, David Makinson, Stephen Read, Charles Steinhorn, Anne Troelstra and John Truss. The local organizers were Jayne Beardmore, Peter Fletcher, Chris Johnson, John Stell, Alan Treherne (chair) and Roger Whitehouse from the Computer Science and Mathematics Departments at Keele.

The following invited addresses were presented at the meeting:

Avon Avron, *Calculi of hypersequents and their use*.

Andreas Baudisch, *A new uncountably categorical group*.

Andreas Blass, *Ubiquitous cardinals*.

Kosta Došen, *An introduction to substructural logics*.

Randall Dougherty, *On left-distributive algebras and elementary embeddings*.

Lou van den Dries, *O-minimality and tame topology (three-hour course)*.

Ronald Jensen, *An alternative to Bukovský-Namba forcing*.

D. H. J. de Jongh, *Propositional logics and theories*.

Ulrich Kohlenbach, *Analyzing proofs in analysis*.

Sabine Koppelberg, *Set theoretic questions on Boolean algebras*.

Daniel Lascar, *The reconstruction problem*.

Daniel Lehmann, *Cut elimination in a nonmonotonic logic*.

Alain Louveau, *Countable Borel equivalence relations*.

Daniele Mundici, *A logical analysis of Ulam's game with lies*.

Hans Jürgen Ohlbach, *Aspects of logic engineering*.

Evgenia Rabinovich, *On structures interpretable in fields*.

Harold Simmons, *Logic and computation—taking the Curry-Howard correspondence seriously (four-hour course)*.

Robert Staerk, $3 + 1 = 2 + 2$*: From logic programs to inductive definitions*.

Michiel van Lambalgen, *On a new notion of indiscernibles in set theory*.

Jaap van Oosten, *Axiomatizing realizability*.

Alex Wilkie, *On the decision problem for the real exponential field*.

Boris Zil'ber, *Model-theoretic dimensions in complex and ultrametric analysis*.

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Abstracts of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
WILFRID HODGES, PROGRAM CHAIR

Abstracts of contributed papers

MARAT M. ARSLANOV, *An interplay between d-r.e. and r.e. sets*.
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A set $A \subseteq \omega$ is called d-r.e. if there are r.e. sets A_1 and A_2 such that $A = A_1 \backslash A_2$. A Turing degree is called a d-r.e. degree if it contains a d-r.e. set; it is called properly d-r.e. if it is d-r.e. but not r.e.

Recently it was shown (S. B. Cooper and others) that the partial order of the d-r.e. degrees is not dense. Earlier the author proved the following statements.

THEOREM 1. *For each r.e. degree $\boldsymbol{a} < \mathbf{0}'$ there exists a properly d-r.e. degree $\boldsymbol{b}$ such that $\boldsymbol{a} < \boldsymbol{b} < \mathbf{0}'$.*

THEOREM 2. *For any high r.e. degree $\boldsymbol{a} \neq \mathbf{0}, \mathbf{0}'$ there exists d-r.e. degrees $\boldsymbol{b}$ and $\boldsymbol{c}$ such that $\boldsymbol{b} < \boldsymbol{a} < \boldsymbol{c}$ and $\boldsymbol{a}$ is the only r.e. degree between $\boldsymbol{b}$ and $\boldsymbol{c}$.*

Studying related questions, the author proved the following two theorems.

THEOREM 3. *There is a low properly d-r.e. degree $\boldsymbol{h}$ such that for any r.e. degree $\boldsymbol{a}$, if $\boldsymbol{a} < \boldsymbol{h}$ then $\boldsymbol{a} < \boldsymbol{b} < \boldsymbol{h}$ for some r.e. $\boldsymbol{b}$.*

THEOREM 4. *There is a properly d-r.e. degree $\boldsymbol{d}$ such that for any r.e. degree a, if $\boldsymbol{d} < \boldsymbol{a}$ then $\boldsymbol{d} < \boldsymbol{b} < \boldsymbol{a}$ for some r.e. degree $\boldsymbol{b}$.*

JOHN T. BALDWIN AND MASANORI ITAI, *Properties of K-generic models*.
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Let $(K, \leq)$ be a pair where K is a class of finite relational structures and $\leq$, *strong submodel*, is a relation on the members of K such that $A \leq B$ implies $A \subseteq B$. If $(K, \leq)$ satisfies the amalgamation property and the joint embedding property, there exists a structure M called $(K, \leq)$-generic with the following properties:

i) every finite member of K can be strongly embedded in $\boldsymbol{M}$,
ii) every finite substructure of M is in K,
iii) if A and B are finite strong submodels of M that are isomorphic, say by f, then there is an automorphism of M extending f.

Hrushovski constructed, as a $(K, \leq)$-generic model, a stable $\aleph_0$-categorical pseudoplane refuting a conjecture of Lachlan. He also defined, as yet another $(K, \leq)$-generic model, a strongly minimal set of a new type giving a counter example to a Zil'ber's conjecture. By generalizing the idea, Baldwin was able to define a family of almost strongly minimal, non-Desarguesian projective planes.

We study some model theoretic properties of various $(K, \leq)$-generic models, e.g., determining their Morley ranks, $I(T, \aleph_0)$ where T is the theory of a $(K, \leq)$-generic model.

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ALESSANDRO BERARDUCCI AND BENEDETTO INTRIGILA, *Some new results on easy terms*.
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An *easy* term, is a lambda term X such that for every closed lambda term Y, the $\lambda\beta$-calculus plus the equation $X = Y$ is a consistent lambda theory, written $\mathrm{Con}(X = Y)$. G. Jacopini proved that $\Omega \equiv (\lambda x.xx)(\lambda x.xx)$ is easy (see [2] for references). By some results of Visser and Statman we know that easy terms are suitable candidates for representing inside lambda calculus the undefined value of a partial recursive function (see [1]). However in some contrast with the intuition "easy = completely undefined", in [2] it is proved that if we identify all easy terms, then we obtain new equalities which equate an easy term with a non-easy term. We prove that if $N_1 X = N_2 X$ where N_1 and N_2 are distinct normal forms, then X is non-easy, actually there is a closed normal form N such that $\neg\mathrm{Con}(X = N)$. Motivated by this result we introduce the following notation: a term is *n.f.-easy* if for every closed normal form N, $\mathrm{Con}(X = N)$. It turns out that in the $\lambda\mathbf{I}$-calculus, n.f.-easy implies easy, however we show that this fails in the full $\lambda\beta$-calculus. We also prove that there is a closed lambda term X which can be consistently equated to every closed lambda term with the exclusion of the identity $\lambda x.x$ (or any given projector $\lambda x_1 \dots x_n.x_i$). Both results are consequences of a general result which states that given two closed terms A and B with incompatible Böhm trees, there is a closed term X depending only on A, which can be consistently equated to B but not to A. This is proved by a general method to obtain Church-Rosser extensions of lambda calculus by which we obtain various other consistency results. Finally we prove that for X to be easy it suffices that X can be consistently equated with every closed lambda term whose Böhm tree has height at most one.

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JÖRG BRENDLE, *Extraspecial p-groups and Kurepa trees*.
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Given a group G, we let $Z(G)$ denote its center, G' its commutator subgroup, and $\Phi(G)$ its Frattini subgroup (the intersection of all maximal proper subgroups of G). Given $U \leq G$, we let $N_G(U)$ stand for the normalizer of U in G. A group G is *FC* iff every element $g \in G$ has finitely many conjugates. A p-group E is called *extraspecial* iff $\Phi(E) = E' = Z(E) \simeq \mathbb{Z}_p$, the cyclic group with p elements.

When generalizing a characterization of center-by-finite groups due to B. H. Neumann, M. J. Tomkinson asked the following question. Is there an it FC-group G with $|G/Z(G)| = \kappa$ but $[G : N_G(U)] < \kappa$ for all (abelian) subgroups U of G, where κ is an uncountable cardinal. We consider this question for $\kappa = \omega_1$ and $\kappa = \omega_2$. It turns out that the answer is largely independent of ZFC, and that it differs greatly in the two cases.

More explicitly, for $\kappa = \omega_1$, it is consistent with, and independent of, ZFC that there is a group G with $|G/Z(G)| = \omega_1$ and $[G : N_G(A)] \leq \omega$ for all abelian $A \leq G$. We do not know whether the same statement is still consistent if we drop *abelian*. On the other hand, for $\kappa = \omega_2$, the non-existence of groups G with $|G/Z(G)| = \omega_2$ and $[G : N_G(A)] \leq \omega_1$ for al (abelian) $A \leq G$ is equiconsistent with the existence of an inaccessible cardinal. In particular, there is an extraspecial p-group with this property if there is a Kurepa tree.

MARK A. BROWN, *Stit in Time*.
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Belnap and Perloff [2] continue development of their version of *stit* theory: the theory of the operator "See To It That" for a logic of action, in which for agent α and formula P one can always meaningfully form the formula $[\alpha\text{stit} : P]$. Their underlying semantics, based on models with forward-branching time, is extremely rich and subtle, but the corresponding system—with the *stit* operator (relativized to agents) as the only modal operator—is difficult, perhaps impossible, to characterize completely by syntactical means.

In this paper, I consider an approximation to *stit* theory, intended to preserve as much as possible of the richness and subtlety of Belnap and Perloff's account, but cast in a multimodal language in which the *stit* operator is a defined, not a primitive, modal operator. The multimodal language contains modal operators

$\langle\mathrm{F}\rangle P$, interpreted to mean that at some moment in the future P;

$\langle\mathrm{P}\rangle P$, interpreted to mean that at some moment in the past P;

$\langle\mathrm{S}\rangle_\alpha P$, interpreted to mean that at some alternative moment in the present, (i.e., at some simultaneous moment on another of time's branches); consistent with α's abilities, P

and $\boxdot_\alpha P$, interpreted to mean that α *does* P; in a rather minimal sense of *does*.

The operators $\langle\mathrm{F}\rangle$, $\langle\mathrm{P}\rangle$, and $\langle\mathrm{S}\rangle_\alpha$ are all normal modal operators, corresponding to binary relations (*forwards, backwards* and *sideways in time*) by means of which we can describe the relevant aspects of the tree of branching histories. They combine to give a rich temporal framework in our language. The action operator $\boxdot_\alpha$, also a normal modal operator, is conceived along lines indicated in Brown [4], but relativized to agents.

The resulting multimodal system can help us understand, and explicitly represent, some of the interrelationships between agent, action, and time, in an approximation to *stit* theory, and can also help us explore the connections between *stit* theory and the simpler, but more simplistic, system of Brown [3]. The paper is concerned with the limitations, as well as the potential, of a multimodal approach.

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ANTONINO DRAGO, *Constructive mathematics and theoretical physics: From the wave equation to a general method.*

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In past two decades some authors found out by constructive mathematics (CoM) undecidability results in several theories of theoretical physics (TP) : statistical mechanics, thermodynamics, Hamiltonian mechanics, quantum mechanics, chaos theory [2]. In particular, Pour-El and Richards directed the attention on counter-examples of non-computable solutions of wave equation [4]. These counter-examples belong to the set of weak solutions, which Bishop's CoM is unable to represent, whereas by essentially adding Markoff's principle Aberth's CoM states them as undecidable—essentially by his Th. 6.1 [1]; i.e., some may be obtained by means of more hypotheses, some others cannot be obtained at all. However, all weak solutions are obtained when one adds a non-constructive hypothesis, i.e., extended Church's Thesis [5].

By generalizing to any differential equation of TP, we can say that almost all solutions which are relevant to TP—the regular ones—are represented by CoM. However, since singular solutions too may be relevant to TP, the correct mathematical conclusion to draw is that we cannot obtain all physical solutions by a general algorithm of a CoM. Hence, the correct philosophical conclusion with respect to a deductive physical theory of TP is that CoM does not allow to consider any differential equation as a universalistic principle. In other words, TP cannot be reduced to a mere mathematical technique; or, even, mathematical-physics is essentially more restricted than TP.

All that qualify the general method for introducing Aberth's CoM in a deductive physical theory, as illustrated in [3]. There, the CoM is the Aberth's version. I can justify this choice by the following argument.

In the history of TP the authors of some theories rejected the deductive organization. The original texts stated by a double negated statement the crucial problem of the theory. L. Carnot's classical mechanics (1782), classical chemistry, Lobachevsky's non-Euclidean geometry, S. Carnot's thermodynamics were centered respectively on the following problems: "A body cannot change its state of motion if not by other bodies"; "It is not possible that matter is divisible at infinity"; "It is not true that heat is not work"; "It is not absurd the hypothesis of two parallel lines". In classical logic one is allowed to cancel the two negations; one obtains as affirmative statements just the principles of the corresponding, deductive formulations of the same theories: "Any body perseveres in its state of motion"; "There are atoms"; "There exist two parallel lines"; "Heat is equivalent to work". Aberth's CoM, by including—through Markoff's principle—a typical formula for canceling double negations, results to be adequate to the above affirmative statements of the principles in the deductive formulations. On the contrary, the founders of the above theories rejected the affirmative statements for lack of experimental evidence. By accepting $\neg\neg A \neq A$, i.e., the characteristic law of non-classical logic, they organized their formulations in a not at all deductive way; thus the CoM resulting to be adequate to them is the Bishop's one, whose logic—being without Markoff's principle—is a non-classical one. In other terms, the radical variation of meaning suffered by principles w.r. to the option on kind of organization, requires that a suitable version of CoM be introduced in a physical theory.

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ROY DYCKHOFF, *Loop-free construction of counter-models for intuitionistic propositional logic*.

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The standard method of constructing counter-models to invalid formulae of intuitionistic propositional logic requires a loop checker, essentially because Contraction is a primitive rule of the standard Gentzen sequent calculus for this logic. We showed in [1] how to reformulate the calculus to avoid contraction—similar ideas were first presented in 1952 by Vorob'ev and more recently in 1992 by Hudelmaier and in 1991 by Lincoln *et al.* Here we make explicit the associated algorithm, without loop-checking, for the construction of Kripke counter-models.

Part of the work described here is joint work with Luis Pinto. A detailed exposition will appear in the Proceedings of the Second International Gauss Symposium (editor Rudolf Fritsch), held in Munich in August 1993.

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DANIEL DZIERZGOWSKI, *A saturation theorem for Kripke structures*.

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We propose an extension of the classical notion of recursively saturated structures (Barwise and Schlipf) to Kripke structures for intuitionistic logic.

Let $\mathcal{L}$ be a countable language. We define $\mathcal{D}$ to be a *countable $\mathcal{L}$-collection* iff $\mathcal{D}$ is a countable set of pairs $\langle \Phi, (x, x_1, \ldots, x_n) \rangle$, where Φ is a countable set of formulae whose free variables are among $x, x_1, \ldots, x_n$.

Let $\mathcal{M} = ((\mathcal{M}_k)_{k \in K}, (K, \leq, 0))$ be a Kripke structure (K is partially ordered by $\leq$, for all $k \in K$, $0 \leq k$ and $\mathcal{M}_k \subseteq \mathcal{M}_l$ if $k \leq l$). We suppose that $\mathcal{L} \subseteq \mathcal{L}_{\mathcal{M}_0}$, and say that $\mathcal{M}$ is *$\mathcal{D}$-saturated* iff for all $k \in K$, for all $\langle \Phi, (x, x_1, \ldots, x_n) \rangle \in \mathcal{D}$, and for all $c_1, \ldots, c_n \in \mathcal{M}_k$,

$$\text{there exists } m \in \mathcal{M}_k \text{ s.t. } \mathcal{M} \Vdash_k \Phi[m, c_1, \ldots, c_n]$$

$$\text{iff for all finite } \Psi \subseteq \Phi, \text{ there exists } m \in \mathcal{M}_k \text{ s.t. } \mathcal{M} \Vdash_k \Psi[m, c_1, \ldots, c_n].$$

Then we are able to prove that, *if $\mathcal{M}$ is a countable Kripke structure and $\mathcal{D}$ is a countable $\mathcal{L}$-collection, then $\mathcal{M}$ has a countable $\mathcal{D}$-saturated elementary extension.*

On the other hand, let $\mathcal{M} = ((\mathcal{M}_k)_{k \in K}, (K, \leq, 0))$ and $\mathcal{N} = ((\mathcal{N}_k)_{k \in K}, (K, \leq, 0))$ be two countable Kripke structures (both are indexed by the same $(K, \leq, 0)$. Let $\mathcal{M} \oplus \mathcal{N}$ denote the "disjoint union" of $\mathcal{M}$ and $\mathcal{N}$: $\mathcal{M} \oplus \mathcal{N} = ((\mathcal{M}_k \oplus \mathcal{N}_k)_{k \in K}, (K, \leq, 0))$. For classical structures, it can be proved that if $\mathcal{M} \oplus \mathcal{N}$ is recursively saturated then $\mathcal{M}$ and $\mathcal{N}$ are isomorphic as soon as $\mathcal{M} \equiv \mathcal{N}$. For Kripke structures for intuitionistic logic, this is likely to be false; in [2], we could only prove a weak form of this result, where, furthermore, $\equiv$ has been replaced with a stronger relation related to Fraïssé's characterization of elementary equivalence by means of partial isomorphism (classically this stronger relation is equivalent to $\equiv$; see also [1]).

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KENJI FUKUZAKI, *Definability results and a proof of a Nullstellensatz for regular rings*.
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We show some definability results for (commutative von Neumann) regular rings.

THEOREM. *For any $t, q, n \in N$ there exists $m_1(t, q, n)$ such that if R is any regular ring and $g, f_1, \ldots, f_t$ are polynomials in $R[x_1, \ldots, x_n]$ with $\deg f_i \leq q$ for each i and $g \in (f_1, \ldots, f_t)$, then there exist polynomials $h_1, \ldots, h_t \in R[x_1, \ldots, x_n]$ such that $\deg h_i \leq \deg g + m_1(t, q, n)$ for each i and $g = \sum_{i=1}^{t} h_i f_i$.*

THEOREM. *For any $t, q, n \in N$ there exists $m_2(t, q, n)$ such that if R is any regular ring and $g, f_1, \ldots, f_t$ are polynomials in $R[x_1, \ldots, x_n]$ with $\deg f_i \leq q$ for each i and $g \in \sqrt{(f_1, \ldots, f_t)}$, then $g^{m_2(t,q,n)} \in (f_1, \ldots, f_t)$.*

These are analogues of definability results for fields proved by G. Hermann. Using these results we give another proof of a Nullstellensatz for regular rings proved by Saracino and Weispfenning.

THEOREM. *Let R be a regular ring. R' a monically closed regular ring extending R, and $g, f_1, \ldots, f_t$ be in $R[x_1, \ldots, x_n]$. Let $I = (f_1, \ldots, f_t)$. We assume $I \cap R = \{0\}$. Then there exist $r \in N$ and $h_1, \ldots, h_t \in R[x_1, \ldots, x_n]$ such that $g^r = \sum_{i=1}^{t} h_i f_i$ (i.e., $g \in \sqrt{I}$) iff g vanishes at every common zero of $f_1, \ldots, f_t$ in R'.*

S. S. GONCHAROV, *Families with positive computable enumerations*.
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Systematic studying of computable enumerations was initiated by A. N. Kolmogorov. As the result of intensive investigating by a large number of authors, the enumeration theory questions formed a new actual direction in classical recursion theory and reflected in the monograph [1].

In this paper we study the problems of existence and quantity of minimal enumeration of families of r.e. sets raised in [4]. These questions were also studied in [2]–[5]. In [3] we proved that there exists the family of r.e. sets that has exactly two non-equivalent one-one computable enumerations. Remind that enumeration ν is called positive provided the set $\{(n, m) : \nu(n) = \nu(m)\}$ is r.e. set.

THEOREM 1. *For every $n \in N$, there exists a family of r.e. sets with exactly n non-equivalent* *positive computable enumerations.*

THEOREM 2. *There exists a family of r.e. sets with the unique but not the least positive computable enumeration.*

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VALENTIN GORANKO, *Refutation systems in modal logic*.
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A refutation system for a formal theory is a deductive system which infers the non-valid (refutable) statements of the theory. Refutation systems originate from Aristotle's attempt at systematic refutation of the non-valid syllogisms not by means of counterexamples, i.e., "refuting models", but rather applying certain rules of inference like *modus tollens*: if A implies B and B is refuted then A is refuted too. The idea of refutation systems was carried further by Łukasiewicz, Slupecki and his school, Scott, Skura and others.

In the first part of this investigation, *sentential* refutation systems are constructed and proved to be complete for a number of modal logics including **K**, **GL** (the logic of provability in Peano arithmetic), **T**, **S4**, **S4Grz** and others. Thus, complete syntactical characterizations in Łukasiewicz's sense are established for these logics and, in particular, purely syntactic decision procedures for them are obtained.

In the second part, *sequential* refutation systems in Gentzen style are introduced for the classical propositional calculus and for some normal modal logics including **K** and **GL**.

In conclusion, generalized deductive systems are discussed which involve both operators for provability and refutability, and employ combined inference rules for them.

JONANNA GRYGIEL, *Freely generated filters in Boolean algebras*.
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A set A of elements of a Boolean algebra is said to be independent provided that $a_1 \cdot \ldots \cdot a_n \leq b_1 + \ldots + b_k$ does not hold for any system of different elements $a_1, \ldots, a_n, b_1, \ldots, b_k$ from A.

We say that a filer F is freely generated if an independent set of generators for F exists.

$m(F)$ denotes the minimal cardinality of the set of generators for F. We can prove the following results:

(1) Every countably generated filter in any atomless Boolean algebra is freely generated.
(2) If $m(F)$ is a regular and uncountable cardinal and F is a filter of a free Boolean algebra, then F is freely generated.

The connection between the existence of the set of free generators for a filter F and $m(F_\tau)$, where F_τ is an image of F in τth Cantor-Bendixson derivative of a given Boolean algebra will be also considered.

Application of the notion of independence to classical logic gives us the natural strengthening of the notion of logical independence. The results obtained in algebraical way can be easily applied to Lindenbaum algebras and give us an answer to questions about the algebraic axiomatizability of sets of formulae.

SACHIO HIROKAWA, *The proofs of $\alpha \to \alpha$ in $P - W$*.
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A syntactic structure is investigated for the system of pure implicational relevant logic $P - W$ [1]. The system is defined by the axioms $B = (b \to c) \to (a \to b) \to a \to c$, $B' = (a \to b) \to (b \to c) \to a \to c$ and $I = a \to a$. Martin and Meyer [3] showed a semantic proof for the following conjectures. But no syntactic proof is yet known.

Belnap's conjecture: If $\alpha \to \beta$ and $\beta \to \alpha$ are derivable in $P - W$ then $\alpha = \beta$.

Power's conjecture: No instance of $a \to a$ is derivable from B and B'.

Trig, Bunder and Hindley [4] defined the set of 'hereditary right-maximal linear' λ-terms which exactly corresponds to the set of $BB'I$-combinators. Thus the theorems in $P - W$ coincides the set of types of those λ-terms. By the analysis of type assignment to these λ-terms, we show that $\alpha \to \alpha$ is a type of such a λ-term iff the λ-term is $\beta\eta$-reducible to $\lambda x.x$. We prove this characterization using the underivability of $\alpha \to \alpha$ from B and B'.

Interesting is that we can prove this underivability syntactically from our characterization of the proofs of $\alpha \to \alpha$. We can prove Power's conjecture using our characterization and the characterization of invertible λ-terms (Dezani [2]).

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LEON HORSTEN, *Reflecting in epistemic arithmetic*.
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One of the principal aims of Shapiro's system of Epistemic Arithmetic (EA) is to give an axiomatic investigation of the notion of provability in principle in an arithmetical context (see [1]). By an old argument of Kaplan and Montague, it follows that for no consistent extension T of EA we have

$$\vdash_T Pr_T(\ulcorner A \urcorner) \to \Box A \tag{1}$$

or even

$$\vdash_T \Box(Pr_T(\ulcorner A \urcorner)) \to \Box A \tag{2}$$

Nevertheless, for some consistent extensions of EA (for example EA itself), (1) and (2) are true reflection principles, expressing seemingly weak logical relations between the concept of absolute provability or provability in principle and provability in a formal system. In other words, the argument of Kaplan and Montague reveals a strong proof-theoretical limitation of consistent extensions of EA.

For modal-structural formalizations of arithmetic (see [1]), the situation turns out to be different. We construct two (sound) epistemic formalizations T_1 and T_2 of arithmetic on a modal-structural basis such that T_1 contains (1) and (2) and T_2 contains (2). These systems contain enough arithmetic for the two incompleteness theorems to be proved for them, whereby they cannot prove their own Gödel sentence or their own consistency. Yet an easy fixed-point calculation shows that T_1 and T_2 can prove that their Gödel sentence and consistency are provable in principle.

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TANELI HUUSKONEN, *M-languages are not closed under negation*.
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The so-called *M-languages* $M_{\kappa\lambda}$, where $\kappa, \lambda \in \mathrm{Card} \cup \{\infty\}$, have conjunctions and disjunctions of size less than κ and well-ordered quantifier strings of length less than λ. Their definition does not allow for a syntactic negation, as there does not seem to be any satisfactory semantics. This leaves open the question as to whether every M-language is closed under negation, and if not, whether every M-formula has a semantic negation in some possibly larger M-language.

We prove that the following $M_{\omega_1\omega_1}$-sentence has no semantic negation in any $M_{\kappa\lambda}$:

$$\forall x_0 \exists y_0 \forall x_1 \exists y_1 \cdots \exists u \left[\bigwedge_{n<\omega} x_n < u \wedge \forall v[v < u \to \bigvee_{n<\omega} v < y_n] \wedge A(u) \right],$$

where $<$ and A are a binary and a unary relation symbol, respectively.

The proof is based on the fact that the falsity of the above sentence in well-ordered models is equivalent to the existence of certain stationary sets, and that stationarity reflects worse than the truth of a sentence in an M-language.

TAPANI HYTTINEN, *A short survey on strong nonstructure theorems*.
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By a strong nonstructure theorem we mean a theorem which says that if a theory belongs to some class of theories then it has very equivalent nonisomorphic models. Usually the equivalence is measured by the length of the Ehrenfeucht-Fraïssé games in which $\exists$ has a winning strategy (see Definition 1 below). These theorems are called nonstructure theorems because intuitively the models must be complicated if they are very equivalent but still nonisomorphic. Also structure theorems usually imply that certain degree of equivalence give isomorphism.

1 DEFINITION. *Let t be a tree and κ a cardinal. The Ehrenfeucht-Fraïssé game of length t between models $\mathcal{A}$ and $\mathcal{B}$, $G_t^\kappa(\mathcal{A}, \mathcal{B})$, is the following. At each move α:*

(i) *player $\forall$ chooses $x_\alpha \in t$, $\kappa_\alpha < \kappa$, and either $a_\alpha^\beta \in \mathcal{A}$, $\beta < \kappa_\alpha$ or $b_\alpha^\beta \in \mathcal{B}$, $\beta < \kappa_\alpha$, we will denote this sequence by X_α.*

(ii) *if $\forall$ chose from $\mathcal{A}$ then $\exists$ chooses $b_\alpha^\beta \in \mathcal{B}$, $\beta < \kappa_\alpha$, else $\exists$ chooses $a_\alpha^\beta \in \mathcal{A}$, $\beta < \kappa$, we will denote this sequence by Y_α.*

$\forall$ must move so that $(x_\beta)_{\beta \leq \alpha}$ forms a strictly increasing sequence in t. $\exists$ must move so that $\{(a_\gamma^\beta, b_\gamma^\beta) \mid \gamma \leq \alpha, \beta < \kappa_\gamma\}$ is a partial isomorphism from $\mathcal{A}$ to $\mathcal{B}$. The player who first has to break the rule loses.

We write $\mathcal{A} \equiv_t^\kappa \mathcal{B}$ iff $\exists$ has a winning strategy for $G_t^\kappa(\mathcal{A}, \mathcal{B})$. Among the strong nonstructure theorems we mention in this talk is the following.

2 THEOREM. ([1]) *Let $\lambda = \mu^+$, $cf(\mu) = \mu$, $\kappa = cf(\kappa) < \mu$, $\lambda^{<\kappa} = \lambda$ and $\mu^\kappa = \mu$. Assume T is an unsuperstable theory, $|T| \leq \lambda$ and $\kappa(T) > \kappa$. Then there are models $\mathcal{A}, \mathcal{B} \models T$ of cardinality λ such that $\mathcal{A} \equiv_{\mu\times\kappa}^\lambda \mathcal{B}$ and $\mathcal{A} \not\cong \mathcal{B}$.*

Under the assumptions of Theorem 2 the claim of the theorem is virtually the best possible, i.e., the theorem does not remain true if in the claim we replace $\mu \times \kappa$ by $\mu \times \kappa + 2$.

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TAKAO INOUÉ, *Partial interpretations of Leśniewski's epsilon in modal and intensional logics*.
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The propositional fragment $\boldsymbol{L}_1$ of Leśniewski's ontology $\boldsymbol{L}$ was introduced in [5] (for $\boldsymbol{L}_1$, see also [4]). In this paper, we propose the following sound translation from $\boldsymbol{L}_1$ to Lemmon-Scott's normal propositional modal logic $\boldsymbol{K}$.

(i) $I(A \vee B) = I(A) \vee I(B)$,

(ii) $I(\sim A) = \sim I(A)$,

(iii) $I(\varepsilon ab) = p_a \wedge \Box(p_a \equiv p_b)$,

where p_a and p_b are propositional variables corresponding to name variables a and b, respectively. The translation is not an embedding. But we prove that it is faithful for a certain large decidable set of formulas (i.e., decent formulas) of $\boldsymbol{L}_1$. The (partial) faithfulness of the translation is generalized to an arbitrary extension of $\boldsymbol{K}$ under some conditions. With respect to similar translations, analogous theorems hold for Von Wright-type deontic logics, the system ***PrL*** of provability logic and its subsystem, for example. This paper is an endeavor to understand Leśniewski's epsilon on the basis of my recognition that Leśniewski's epsilon would be a variant of truth-functional equivalence. In the Appendix, an axiomatic rejection for $\boldsymbol{L}_1$ is proposed as a case study of [1]–[3].

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PETER KOEPKE, *A proof of the Martin-Steel theorem based on embedding normal forms*.
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In "A Proof of Projective Determinacy" by D. A. Martin and J. Steel, ***Journal of the AMS***, vol. 2, no. 1, 1989, the argument is based on the interplay between homogeneous trees and iteration trees in the presence of Woodin cardinals.

A set $A \subseteq ({}^{\omega}\omega)^n$ is the projection of a (sufficiently complete) homogeneous tree iff it possesses an *Embedding Normal Form with Witnesses* (ENFW), which means that there is a commutative system

$$\langle M_s \mid s \in ({}^{<\omega}\omega)^n \rangle, \langle \pi_{st} \mid s \subseteq t \in ({}^{<\omega}\omega)^n \rangle$$

and *witnesses* $f_s \in M_s$ such that:

(a) $M_0 = V$, M_s is transitive, $\pi_{st} : M_s \to M_t$ is elementary, and $f_s : ({}^{\omega}\omega)^n \to On$;

(b) $\vec{b} \in A$ implies that $\langle M_s \mid s \subset \vec{b} \rangle$ has a well-founded direct limit;

(c) $\vec{b} \notin A$ implies that $\pi_{st}(f_s)(\vec{b}) > f_t(\vec{b})$ for $s \subset t \subset b$, $s \neq t$.

The *critical point* of this system is the least ordinal moved by the maps π_{st}.

In analogy to results on projections of homogeneous trees we can give (rather short) proofs of:

(1) If A has an ENFW then A is determined.

(2) If $A \subseteq ({}^{\omega}\omega)^{n+1}$ has an ENFW with critical point $> \delta$, where δ is a Woodin cardinal, then $B = \{\vec{y} \in ({}^{\omega}\omega)^n \mid \neg\exists x \in ({}^{\omega}\omega)(x, \vec{y}) \in A\}$ has an ENFW with arbitrary high critical point below δ.

This immediately implies the Martin-Steel Theorem: If there are infinitely many Woodin cardinals then every projective set of reals is determined.

In my talk I am going to sketch some aspects of the proofs of (1) and (2) which are specific to the use of ENFWs.

OLIVER KULLMANN, *Methods for 3-SAT-decision in less than* $2^{0.59 \cdot n}$ *steps*.
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In [2] and [3] the upper bound $2^{0.6943 \cdot n}$ (worst case for the number of steps in abstraction from a (small) polynomial factor in the number of literal-occurrences) for 3-SAT-decision was demonstrated (n = number of variables). Starting out from Schiermeyer [4] (who achieved the upper bound $2^{0.659 \cdot n}$) we improved the bound to $2^{0.5893 \cdot n}$ (with a better understanding of what's going on).

For us the step forward in [4] depends on a refined measure of complexity for the clause sets, which causes that beyond the decrease of the number of variables in the branchings of the algorithm there comes another (structural) simplification of the clause sets into account (namely there arise new 2-clauses).

We now develop this (implicit) basis idea and introduce the following new methods (which yield our results):

1. We generalize the 'autarkness-lemma' ([3]) and get a natural and flexible way of reacting if there are not 'enough' 2-clauses.

2. We introduce the new 'blocking-lemma' which gives the opportunity of eliminating or adding new clauses. With this we can fill the gaps left from the autarkness-method (and probably we go beyond the realm of resolution.)
3. We can use certain global structures of the whole evaluation tree to 'accept' the new 2-clauses.
4. We refine the measurement of the clause-sets.

The general tool for the estimation of running time is the concept of a 'measure' for trees (which substitutes the ordinary calculation of recursion equations with the benefit of great simplification)—only with this 'global' concept (in contrast to the 'local' recursion equations) one can use global regularities of the behavior of the algorithm (see point 3 above).

This result goes on with the results from [1], where the upper bounds $2^{\frac{1}{9}\cdot l}$ and $2^{\frac{1}{3}\cdot k}$ for whole KNF (l = number of literals, k = number of clauses) with short algorithms (and proofs) were obtained.

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CLAUDE LAFLAMME, *Bounding and dominating number of families of functions on* $\mathbb{N}$.
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We survey a few general mathematical problems which have an equivalent form in terms of the structure of certain families of functions (or subsets) on the natural numbers.

The goal is to classify these families in ZFC, but forcing is useful to test what can't be done. The task appears to be difficult since not only any partial order of size at most ω_1 embeds in $\langle \omega^\omega, \leq^* \rangle$ but any partial order of size at most continuum consistently embeds. We feel that the bounding and dominating numbers is a good place to start, as it has been useful in the classification of gaps. A classification of unbounded families has essentially been completed a few years ago but the bounded case appears to be much more difficult and interesting.

LARISA MAKSIMOVA, *Explicit definition of implicitly definable in some modal logics*.
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In [1] the following analogue of the Beth Theorem was proved ($\vdash_L$ denotes derivability in L by modus ponens and necessitation rules).

Theorem 1. *Let L be any normal modal logic containing K4. Let* $\boldsymbol{p}$ *be a list of variables and x and y be outside of* $\boldsymbol{p}$. *For any formula* $A(\boldsymbol{p}, x)$, *if* $A(\boldsymbol{p}, x), A(\boldsymbol{p}, y) \vdash_L (x \leftrightarrow y)$, *then there exists a formula* $B(\boldsymbol{p})$, *such that* $A(\boldsymbol{p}, x) \vdash_L (x \leftrightarrow B(\boldsymbol{p}))$.

But the proof of this theorem given in [1] does not contain any way of constructing an explicit form of $B(\boldsymbol{p})$. In this paper we construct an explicit definition for some modal logics extending K4.

Denote by K4A a logic, which is characterized by the class FTA of all finite frames, both transitive and antisymmetric. For $n < \omega$, let $\mathrm{K4A}_n$ be characterized by the frames from FTA of height at most n. Denote $\boxdot A = (A \,\&\, \Box A)$.

THEOREM 2. *Let L be any normal modal logic containing $K4A_n$. Let $\boldsymbol{p}$ be a list of variables and x and y be outside of $\boldsymbol{p}$. For any formula $A(\boldsymbol{p}, x)$, if $A(\boldsymbol{p}, x)$, $A(\boldsymbol{p}, y) \vdash_L (x \leftrightarrow y)$, then $A(\boldsymbol{p}, x) \vdash_L (x \leftrightarrow B_n(\boldsymbol{p}))$, where*

$$B_n(\boldsymbol{p}) = \vee\{(C_{2i+1}(\boldsymbol{p}) \,\&\, \neg C_{2i}(\boldsymbol{p})) \mid 0 \leq 2i \leq n+2\},$$
$$C_0(\boldsymbol{p}) = 1,$$
$$C_1(\boldsymbol{p}) = \boxdot A(\boldsymbol{p}, T),$$
$$C_{i+1}(\boldsymbol{p}) = \boxdot A(\boldsymbol{p}, C_1(\boldsymbol{p}) \leftrightarrow \cdots \leftrightarrow C_i(\boldsymbol{p})).$$

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ALBERTO MARCONE, *Bqo theory and reverse mathematics*.
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The notion of better quasi-ordering (bqo) has been introduced by Nash-Williams in the 60's and has been studied by both combinatorists and logicians. We study bqos from the point of view of *reverse mathematics* ([3]) by trying to establish which subsystems of second order arithmetic are necessary to prove the theorems of bqo theory. We concentrate on one of the results proved by Nash-Williams ([1]) which we denote by **NWT** and which asserts that if a quasi-ordering Q is a bqo then $\tilde{Q}$ (the set of all countable sequences of elements of Q ordered by embeddability) is also a bqo.

Nash-Williams' original proof of **NWT** and all its refinements rely on the so-called *minimal bad array lemma*, a powerful technique that appears to require rather strong set-existence axioms (e.g., the system $\mathbf{\Pi}^1_2\text{-}\mathbf{CA}_0$). By using an entirely new proof we prove **NWT** in the weaker system $\mathbf{\Pi}^1_1\text{-}\mathbf{CA}_0$. Our proof uses hereditarily indecomposable sequences and a weak version of the minimal bad array lemma which suffices to prove a generalization of the classical Higman's theorem.

We also obtain some partial results toward the proof of **NWT** in the even weaker system $\mathbf{ATR}_0$ (the only step missing is a proof of the generalized Higman's theorem). If this proof exists it would settle the status of **NWT** because it follows from some recent results by Shore ([2]) that **NWT** implies $\mathbf{ATR}_0$ over the weak base system $\mathbf{RCA}_0$.

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JUAN CARLOS MARTÍNEZ, *Forcing construction of very thin-thick boolean algebras*.
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A *superatomic Boolean algebra* is a Boolean algebra in which every subalgebra is atomic. It is known that a Boolean algebra B is superatomic iff its Stone space $S(B)$ is scattered. The Cantor-Bendixson process for topological spaces can be transferred to the context of Boolean algebras, obtaining in this way a sequence of ideals. Suppose that B is a Boolean algebra. Then, for every ordinal α, the ideal I_α is defined by transfinite induction as follows: $I_0 = \{0\}$; if $\alpha = \beta + 1$, I_α is the ideal generated by $I_\beta \cup \{b \in B : b/I_\beta$ is an atom in $B/I_\beta\}$; and if α is limit, $I_\alpha = \bigcup\{I_\beta : \beta < \alpha\}$. Then, B is superatomic iff $B = I_\alpha$ for some α.

The *height* of a superatomic Boolean algebra B, $\mathrm{ht}(B)$, is the least ordinal α such that B/I_α is finite (which means $B = I_{\alpha+1}$). For every ordinal $\alpha < \mathrm{ht}(B)$ let $\mathrm{wd}_\alpha(B)$ be the cardinality of the set of atoms in B/I_α. Then, for an infinite cardinal κ, B is called *κ-very thin-thick* if $\mathrm{ht}(B) = \kappa^+ + 1$, $\mathrm{wd}_\alpha(B) = \kappa$ for $\alpha < \kappa^+$, and $\mathrm{wd}_{\kappa^+}(B) = \kappa^{++}$.

Roitman proved that the existence of an ω-very thin-thick superatomic Boolean algebra is consistent with the axioms of set theory. Using the notion of a simplified morass introduced by Velleman, we shall present a generalization of Roitman's theorem to larger cardinals.

CHRISTIAN MICHAUX AND ROGER VILLEMAIRE, *Expanding predicates in arithmetic and Cobham's theorem*.
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A subset L of natural numbers is said to be expanding if the distance between consecutive elements of L is not bounded.

We show that $L \subseteq \mathbb{N}$ is not first-order definable in Presburger Arithmetic iff there exists a subset of $\mathbb{N}$ first-order definable in $\langle \mathbb{N}, +, L\rangle$, which is expanding.

For example, if L is the Thue-Morse predicate (i.e., n belongs to L iff the number of 1's in the binary expansion of n is even), it is easy to show that the subset P_2 of the powers of two is first-order definable in $\langle \mathbb{N}, +, L\rangle$. Using a first-order version of Büchi's theorem (see [2]) we get a new proof of Cobham's theorem ([1]).

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CHRIS MILLER, *A growth dichotomy for o-minimal expansions of ordered fields*.
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Let $\mathcal{R}$ be an o-minimal expansion of an ordered field with underlying set R. There is a striking dichotomy in the orders of growth at $+\infty$ of $\mathcal{R}$-definable unary functions. In one case, $\mathcal{R}$ is *exponential*; i.e., there exists a unique nonconstant differentiable function $E : R \to R$, definable in $\mathcal{R}$ without parameters, with $E(x+y) = E(x)E(y)$ for all $x, y \in R$ and $E(0) = 1$. In the o-minimal context, such a function has most of the properties we associate with the real exponential function. If $\mathcal{R}$ is not exponential, then $\mathcal{R}$ is *power bounded*; i.e., every $\mathcal{R}$-definable function $f : R \to R$ is eventually dominated by some definable multiplicative homomorphism $g : (0, +\infty) \to (0, +\infty)$. Such a function g is called a *power function* for $\mathcal{R}$, and these functions behave essentially like real power functions $x \mapsto x^r : (0, +\infty) \to (0, +\infty)$.

In either case, the set of all power functions for $\mathcal{R}$ can be made into an ordered field, (with composition as multiplication, and pointwise multiplication in R as addition). If $\mathcal{R}$ is exponential, this field is isomorphic to R, and $\lim_{x\to+\infty} E(x)/g(x) = +\infty$ for all power functions g. If $\mathcal{R}$ is power bounded, this field is isomorphic to a subfield of the prime model of $\mathcal{R}$, every power function is definable *without* parameters, and for every $\mathcal{R}$-definable function $f : R \to R$, not ultimately identically 0, there exists a power function g and $c \in R$, $c \neq 0$, such that $\lim_{x\to+\infty} f(x)/g(x) = c$.

MARGARITA OTERO, *The amalgamation property in normal open induction*.
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Let OI denote the fragment of Peano Arithmetic with the induction restricted to open formulae. It is known that OI fails to have the joint embedding property, a result due to Wilkie.

On the other hand, we have proved that if we require our models to be normal, that is, integrally closed in their fraction fields, the corresponding theory NOI, extending OI, has the joint embedding property.

Here we prove that NOI does not have the amalgamation property.

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JOHN PAIS, *The topology of nonmonoticity*.
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One current popular model-theoretic approach to the study of the mathematical properties or "logic" (e.g., [2], [3]) of nonmonotonic inference operators focuses its effort on the task of providing a semantics for various conditional logics. In this case, the primary allure of conditional logic is its well-known ability to reflect any extant *residual classical monotonicity*. However, since its model theory is too weak to provide a *faithful representation* ([5]) of *intrinsic nonmonoticity* ([6]), one can ascribe too much significance to this fact at the expense of not focusing on the problem of a general mathematical characterization of nonmonotonicity.

This paper presents an abstract framework for representing nonmonotonic inference operators based on *exterior topology* ([4]), which is a topological approach employing nonmonotonic exterior operators in contrast to monotonic interior or closure operators. A metric concept of "closest" new information is developed in terms of exterior (inference) operators inducing *symmetric revision spaces*. These operators are then compared to inference operators constructed using the minimal elements of a preference relation and inducing *cumulative revision spaces*. It is shown that the former have properties preserving more residual monotonicity, while the later relinquish this residual monotonicity and instead possess more intrinsic nonmonotonicity. Furthermore, this intrinsic nonmonotonicity is now easily described in terms of standard topological concepts. Finally, it is shown that exterior topology yields a framework subsuming both the AGM logic of theory change ([1]) and approaches employing preferential consequence relations.

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VALERIA DE PAIVA AND LUIZ CARLO PEREIRA, *A new proof system for intuitionistic logic*.
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Sequent Calculus formulations of Intuitionistic Logic (LJ) are usually derived from those of Classical Logic (LK) by restricting the form of the sequents, so that they are of the form

$$\Gamma \vdash \Delta$$

where Δ contains at most one formula (Gentzen 1933).

A less restrictive system, variously known as LJ$'$ or IL>, relaxes this condition. In LJ$'$ the rules are as with LK (i.e., they permit multiple conclusions) except for the rules for implication and universal quantification on the right, which are of the form

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \qquad\qquad \frac{\Gamma \vdash A(a)}{\Gamma \vdash \forall x A(x)}$$

(with the usual proviso in the universal quantification rule). Note that B (as well as $A(a)$) is a single formula. It can be shown that LJ$'$ satisfies cut-elimination and that it is equivalent to LJ.

This paper proposes a further relaxation of the form of the rules above. We define a new formulation,, Full Intuitionistic Logic (FIL) which, under certain very natural conditions, allows for multiple conclusions in the above two rules. We show that FIL satisfies cut-elimination and that it is equivalent to LJ'.

We also consider a natural deduction-like formulation of FIL and, via the Curry-Howard Correspondence, a term calculus. This provides a new form of typed lambda-calculus, which suggests applications to functional programming.

To conclude we discuss briefly the application of this formulation to intuitionistic modal and linear logic.

HAYDEE WERNECK POUBEL AND LUIZ CARLOS PEREIRA, *A functorial approach to abstract introduction and elimination rules*.
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A natural extension of Natural Deduction was defined by Schröder-Heister where not only formulae but also rules could be used as hypothesis and hence discharged. It was shown that this extension induces the definition of abstract introduction and elimination rules and that the set $\{\wedge, \vee, \rightarrow, \perp\}$ of intuitionistic sentential operators form a *complete* set of operators modulo the abstract introduction and elimination rules, i.e., that any operator whose introduction and elimination rules are instances of the abstract rules can be defined in terms of $\{\wedge, \vee, \rightarrow, \perp\}$.

The aim of this work is to extend to Abstract Natural Deduction the well-known connections between Proof Theory and Category Theory. More precisely, we shall show how to systematically associate to any operator ϕ, whose introduction and elimination rules are instances of the abstract introduction and elimination rules, an adjunction between cartesian closed categories with finite coproducts whose objects are rules instead of formulae.

STEPHEN READ, *In praise of classical harmony*.
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It is sometimes claimed that classical logic is incoherent and unacceptable because it is not a conservative extension of its negative fragment; or at least, that the rules for classical negation are not in harmony, for proofs of certain theses, notably Peirce's Law, can be proved only via a maximal formula, its double negation, introduced by one negation rule, and eliminated by another. (See, e.g., M. Dummett, "The philosophical basis of intuitionistic logic", reprinted in his ***Truth and other enigmas***, Duckworth, 1978, esp. p. 221.)

The objection depends crucially on the presentation of classical logic which is considered. This is a familiar point. The standard natural deduction rules (in the system NK) for '&', '$\vee$' and '$\rightarrow$' are insufficient to yield proofs of Peirce's Law and its kin. They become provable only when negation is added. But this problem does not affect, for example, standard sequent calculus, LK. The reason is that LK allows multiple conclusions, and Peirce's Law is provable directly by two applications of $\rightarrow$ right, and one each of $\rightarrow$ left, Thinning and Contraction. Also truth-tables (or semantic trees), arguably the real basis of classical logic (i.e., the study of structures, proofs being only a means to that end), yield all the negation-free formulae of classical logic directly.

The purpose of the present paper is to propose a minor revision of NK whereby all the negation-free theses of classical logic will become provable without use of the rules for negation. The revised rules can be challenged, on grounds of impurity; but nonetheless, the rules are in harmony, in that all proofs in '&', '$\vee$' and '$\rightarrow$', complete for the theses of classical logic, are normalizable. Indeed, it is an easy step to obtain a full calculus for classical logic with negation in which all the rules are in harmony. Every proof can be put in normal form by eliminating maximal formulae, formulae introduced by introduction rules and acting as major premise of elimination rules.

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There exist two sources of motivation for Basic Logic; constructivism and constructivity. The approach through constructivism is expounded in [3]. In that paper we observe that the first-order logic of the well-known constructivisms is Intuitionistic Predicate Calculus IQC. Most explanations of the logical constants are variations upon the Brouwer-Heyting-Kolmogorov proof interpretation. Our modification of this interpretation yields the first-order logic Basic Predicate Calculus BQC, a proper subsystem of IQC.

The approach through constructivity precedes the one through constructivism, but was originally restricted to Basic Propositional Calculus BPC, see [5]. The traditional world of constructivity consisted of two related, but quite distinct, parts. One part included the forms of constructive mathematics alluded to above; the other involved Gödel's incompleteness theorem and the associated modal logics for provability, in particular PrL (identical to the system G of [2]), that satisfy Löb's rule. The syntax of Intuitionistic Propositional Calculus IPC differs in an essential way from that of PrL with its $\Box$, but IPC corresponds in a natural way to the modal logic S4. Albert Visser found a propositional logic, FPC, with a syntax identical to the one for IPC, to complete the informal equation

$$\frac{\mathrm{IPC}}{\mathrm{S4}} = \frac{\mathrm{FPC}}{\mathrm{PrL}}.$$

This enabled him to interpret implication as formal provability [5]. In [4] Craig Smoryński uses Basic Modal Logic BML, better known as K4, as the natural generalization of both S4 and PrL. Similarly, Visser went beyond the equation above by introducing the system BPC satisfying the informal equations

$$\frac{\mathrm{BPC}}{\mathrm{K4}} = \frac{\mathrm{IPC}}{\mathrm{S4}} = \frac{\mathrm{FPC}}{\mathrm{PrL}}.$$

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VLADMIR V. RYBAKOV, *Recognizing of admissibility via branching property and drop-point property*.
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Starting with the solution of Harvey Friedman problem of effective recognizing admissible for intuitionistic **H** inference rules in [1], a number of similar results for individual various logics were obtained. But, so far, no general results on decidability or admissibility for a sufficiently rich classes of logics have been found (except for extensions of **S4.3** and tabular logics). This paper presents such new results of algorithmic recognizing admissibility in classes of modal and intermediate logics. They were obtained using and developing the displacement method discovered in [1] and [2].

THEOREM 1. *If a modal logic $\lambda \supseteq$ **K4** has the m-branching property and the effective m-drop point property and there is an algorithm which recognizes finite λ-frames (in particular, if λ is finitely axiomatizable) then there is an algorithm which checks admissibility of inference rules in λ.*

THEOREM 2. *If an intuitionistic intermediate logic λ has the strong effective m-drop point property and the m-branching property and the finite λ-frames are effectively recognizable (if λ is finitely axiomatizable, for instance), then λ is decidable with respect to admissibility.*

These theorems cover all known up to now results on recognizing admissibility and give a lot of new ones. For instance,

COROLLARY 3. *All logics below are decidable with respect to admissibility.*

(a) ***K4, K4.1, K4.2, K4.3, S4.1, S4.2, Grz.2, KC*** $:=$ ***H*** $+\neg p \vee \neg\neg p$.
(b) *any finitely axiomatizable logic λ of finite depth, where **K4** $\subseteq \lambda$ or λ is an intermediate logic.*
(c) *Logics $\mathbf{K4}_{b,n}$, $\mathbf{S4}_{b,n}$, $\mathbf{Grz}_{b,n}$, $\mathbf{GL}_{b,n}$ with branching limited by given n and the Gabbay-De-Jong intermediate logics $\mathbf{D}_n$.*
(d) *modal logics $\mathbf{K4I}_n$, $\mathbf{S4I}_n$, $\mathbf{GLI}_n$, and intermediate logics $\mathbf{HI}_n$, where each of the logics is the logic of all finite rooted corresponding to the abbreviation frames with not more than n maximal clusters.*

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V. YU. SAZONOV, *On polynomial-regular codings of hereditarily-finite sets*.
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Let $\theta : \{0,1\}^* \to HF$ be any coding of the universe HF of *hereditarily finite sets* by binary strings. Define $\mathcal{P}_\theta$ to be the class of operations $F : HF \to HF$ such that $F\theta = \theta f$ for some polynomial-time computable (shortly, P-) function $f : \{0,1\}^* \to \{0,1\}^*$. Let $\theta \leq_P \eta := (\theta = \eta f$ for some $f \in P)$.

Set-theoretic Δ-*language* (cf. [1]) consists of Δ-terms $a, b, \ldots$ and Δ-*formulas* $\phi, \psi, \ldots$ and is defined by the following clauses:

$$\begin{aligned} \Delta\text{-}\mathit{formulas} &:= a \in b \mid \neg\phi \mid \phi \& \psi \mid \phi \vee \psi \mid \phi \Rightarrow \psi \mid \\ &\quad\; \forall x \in a.\phi \mid \exists x \in a.\phi \\ \Delta\text{-}\mathit{terms} &:= \langle\text{set-variables}\rangle \mid \{a, b\} \mid \cup\{b(x) : x \in a \& \phi(x)\} \\ &\quad\; \mid \left[p = p \cup \{x \in a : \phi(x, p)\}\right]. \end{aligned}$$

Here x and $p \notin FV(a)$. These constructs have the evident semantics in HF (and even in any universe V for ZF), except probably *inductive* Δ*-separation* $\left[p = p \cup \{x \in a : \phi(x, p)\}\right]$. This is considered as a term (not formula!) to denote the result of stabilization (in $\leq \mathrm{card}(a)$ steps) of the monotonic sequence $\emptyset = p_0 \subseteq p_1 \subseteq \ldots \subseteq a$, where $p_{n+1} := p_n \cup \{x \in a : \phi(x, p_n)\}$.

THEOREM 1. *There exists* $i : HF \to \{0,1\}^*$ *such that arbitrary* P*-function* $f : \{0,1\}^* \to \{0,1\}^*$ *satisfies* $fi = iF$ *for some* Δ*-definable* $F : HF \to HF$.

P-regular coding system of HF is any four-tuple of mappings $\theta, i^R : \{0,1\}^* \to HF$ and $\theta^R, i : HF \to \{0,1\}^*$ satisfying the following five conditions:

1. $\theta\theta^R = 1_{HF}$ and $ii^R = 1_{\{0,1\}^*}$,
2. theorem 1 holds for i,
3. the predicate $\theta(a) \in \theta(b)$ is P-computable,
4. two P-computable mappings $a \mapsto a_1, \ldots, a_k$ and $a_1, \ldots, a_k \mapsto a$ between codes and lists of codes exist such that both satisfy the equality $\{\theta(a_1), \ldots, \theta(a_k)\} = \theta(a)$, and
5. $\theta^R i^R, i\theta$ and $\theta^R\theta : \{0,1\}^* \to \{0,1\}^*$ are in P.

(Example: *well-founded graphs collapsing* [1].) Let Δ_θ be Δ-language extended by operations $\tilde{\theta} := \theta i$ and $\tilde{\theta}^R := i^R\theta^R : HF \to HF$.

THEOREM 2. *For all P-regular codings θ, η:*

(1) $\Delta_\theta \equiv \mathcal{P}_\theta$*, i.e.,* Δ_θ*-language represents exactly* $\mathcal{P}_\theta$*-operations (and* $\mathcal{P}_\theta$ *predicates) in* HF*;*

(2) $\eta \not\leq_P \theta$ *implies* $\mathcal{P}_\theta \not\subseteq \mathcal{P}_\eta$ *and* $\mathcal{P}_\eta \not\subseteq \mathcal{P}_\theta$ *(with counter-examples* $\tilde{\theta}^R$ *and* $\tilde{\eta}$*, respectively; in fact,* $\eta \leq_P \theta \Leftrightarrow \tilde{\eta} \in \mathcal{P}_\theta \Leftrightarrow \tilde{\theta}^R \in \mathcal{P}_\eta$*).*

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V. L. SELIVANOV, *A universal hierarchy of formulas*.
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In our contributed papers for the two previous Logic Colloquiums as well as in [1] we considered a universal hierarchy of subsets of ω which is in a sense a "finite analog" of the Wadge hierarchy. It was defined abstractly, namely with any sequence $\{L_n\}_{n<\omega}$ of distributive lattices satisfying $L_n \cup \check{L}_n \subseteq L_{n+1}$ (where $\check{L}_n = \{\overline{x} \mid x \in L_n\}$ we related a sequence $\{S_\alpha\}_{\alpha<\varepsilon_0}$ (where $\varepsilon_0 = \sup sup\{\omega, \omega^\omega, \omega^{\omega^\omega}, \ldots\}$) satisfying

$$S_\alpha \cup \check{S}_\alpha \subseteq S_\beta \text{ for } \alpha < \beta < \varepsilon_0 \text{ and } \bigcup_{\alpha<\varepsilon_0} S_\alpha = \bigcup_{n<\omega} L_n.$$

By universal hierarchy of formulas (modulo a given theory T) we mean the sequence $\{S_\alpha\}$ associated with the distributive lattices L_n of Σ^0_{n+1}-formulas (modulo equivalence in the theory T). This hierarchy refines the classical quantifier-alternation hierarchy. the aim of our talk is to discuss some interesting properties of the universal hierarchy, namely:

a) $S_\alpha \cap \check{S}_\alpha = \bigcup_{\beta<\alpha}(S_\beta \cup \check{S}_\beta)$ for all $\alpha < \varepsilon_0$, so the universal hierarchy cannot be further refined;
b) there is a clear model-theoretic characterization of S_α-formulas extending the classical characterization of Σ^0_{n+1}-formulas;

c) it is closely related to the universal hierarchy $\{\Sigma_\alpha\}_{\alpha<\varepsilon_0}$ of subsets of ω, which can be used to give some explanation of the fact that natural definable index sets are usually m-complete in one of Σ_α, Π_α.

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A. STERN AND S. SWIERCZKOWSKI, *Connected theories of order*.
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A theory T is called *connected* if T interprets in the union $T_1 \cup T_2$ of two theories T_1, T_2 with *disjoint languages* only if T interprets in T_1 or T_2. Interpretations are meant here in the very general sense (multidimensional and with parameters) introduced by J. Mycielski in 1977 (general reference: a survey in the ***American Mathematical Society Memoirs 86*** (1991) by J. Mycielski, P. Pudlak and A. Stern).

THEOREM. *Let T be a theory which proves the axioms of linear ordering and which provides for each model $\mathcal{M}$ of T and every non-empty interval $(a,b) \subseteq |\mathcal{M}|$ a definable injection $|\mathcal{M}| \to (a,b)$. Then T is connected.*

Here T can be any extension of the theory of ordered fields. In particular, it follows that RCF is connected (as it was conjectured in the ***Memoirs 86***). J. Mycielski has shown that a finitely axiomatizable theory T is connected iff:

(*) For any structures $\mathcal{A}, \mathcal{B}$, T interprets 1-dimensionally in the theory $\mathrm{Th}(\mathcal{A}\dot{\times}\mathcal{B})$ of the disjoint product $\mathcal{A}\dot{\times}\mathcal{B}$ iff T interprets in $\mathrm{Th}(\mathcal{A})$ or in $\mathrm{Th}(\mathcal{B})$.

Using (*), one can reduce the above theorem to the following:

LEMMA. *For any structures $\mathcal{A}, \mathcal{B}$ and any $(\mathcal{A}\dot{\times}\mathcal{B})$-definable linear ordering of $\mathcal{A}\dot{\times}\mathcal{B}$, there is a non-trivial open interval in $\mathcal{A}\dot{\times}\mathcal{B}$ which projects finitely on $\mathcal{A}$ or on $\mathcal{B}$.*

(Since in (*) the equality in T need not interpret as equality in $\mathcal{A}\dot{\times}\mathcal{B}$, one actually has to prove a generalization of this lemma concerning a linear ordering of $(\mathcal{A}\dot{\times}\mathcal{B})/\equiv$, where $\equiv$ is any $(\mathcal{A}\dot{\times}\mathcal{B})$-definable equivalence relation.)

FRANK O. WAGNER, *Hyperstable Theories*.
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Generalizing the work in [1] on theories without dense forking chains, we define an analogue of Krull dimension (which generalizes Shelah degree). We call a theory *hyperstable*, if every type is nonorthogonal to a formula which has this dimension. In such a theory, every type is domination equivalent to a product of regular types, which may however be infinite. Examples for hyperstable theories are those of [1], but there the weight must be finite. The regular types in a hyperstable theory are arranged in a hierarchy; every type has ordinal hereditary weight with respect to any type in the highest level it is not hereditarily orthogonal to. Every regular type has a domination equivalent representative based on a finite set. Lachlan's conjecture holds for these theories, and in between any two models there is a realization of a regular type. In the nonmultidimensional case, hereditary weight with regard to any level of the hierarchy is definable and continuous on some $\emptyset$-definable set of nonzero weight. Similarly if p is regular nontrivial, hereditary p-weight is definable and continuous on some set of nontrivial weight.

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HEINRICH WANSING, *Sequent systems for simple tense logic*.
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This talk deals with sequent calculi for the most important systems of simple tense logic within a modified version of Nuel Belnap's display logic. Various axiomatic extensions of the minimal normal tense logic K_t are captured by adding purely structural rules to a display calculus for K_t, so that the proof-theoretic meaning of the connectives, as laid down in their introduction rules, remains unchanged. The display calculi under consideration enjoy cut-elimination and the subformula property.

MICHAEL ZAKHARYASCHEV, *Decidability via canonical formulas*.
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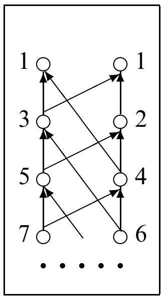
This paper presents two new results on the finite model property and decidability of logics in NExt**K4**, Ext**K4**, and Ext**Int**, the classes of normal and quasi-normal extensions of **K4** and extensions of intuitionistic logic **Int**, respectively. Both of them were obtained using the apparatus of the canonical formulas introduced in [2].



Let $\mathfrak{F}$ be a finite frame. By induction on n define a notion of *n-stable point* in $\mathfrak{F}$. Each point of depth 1 is *1-stable* in $\mathfrak{F}$, and a point is $n+1$-*stable* if it is not m-stable, for any $m \leq n$, and does not see at least one n-stable point in $\mathfrak{F}$. Say that an antichain $\mathfrak{d}$ in $\mathfrak{F}$ is *n-stable* if it contains at least one n-stable point *in the subframe of $\mathfrak{F}$ generated by $\mathfrak{d}$* and no $n+1$-stable point in the subframe. For example, each point in every generated subframe of the frame in the picture is n-stable, where n is the number located near the point.

Theorem 1. *If, for some $d > 0$, a logic $L \in NExt$**K4** or $L \in Ext$**Int** can be axiomatized by a (finite or infinite) set of modal canonical formulas $\alpha(\mathfrak{F}, \mathfrak{D}, \perp)$ or, respectively, by a set of intuitionistic canonical formulas $\beta(\mathfrak{F}, \mathfrak{D}, \perp)$ in which every closed domain $\mathfrak{d} \in \mathfrak{D}$ is n-stable in $\mathfrak{F}$ for some $n \leq d$, then L has the finite model property.*

Example. As is known, every intermediate logic with additional axioms of one variable can be axiomatized by one or two intuitionistic canonical formulas of the form $\beta(\mathfrak{F}, \mathfrak{D}^{\#}, \perp)$ where $\mathfrak{F}$ is a generated subframe of the frame depicted above and $\mathfrak{D}^{\#}$ is the set of all antichains in $\mathfrak{F}$. Therefore, by Theorem 1, every such logic has FMP, and so is decidable. This fact was first proved in 1977 by Sobolev, who used a completely different technique. Theorem 1 gives also a positive solution to the conjecture of Shehtman [1] on FMP of the logics

$$\begin{aligned}\mathbf{D4G}_n &= \mathbf{D4} \oplus \Box \bigvee_{j=0}^{n} \Box^{+}(p_j \wedge \bigwedge_{i \neq j} \neg p_i) \to \bigvee_{j=0}^{n} \Box\neg(p_j \wedge \bigwedge_{i \neq j} \neg p_j) \\ &= \mathbf{K4} \oplus \alpha(\bullet, \emptyset, \perp) \oplus \alpha(\overbrace{\circ \cdots \circ}^{n+1}\!\!{}_{\bullet}, \mathfrak{D}^{\#}, \perp).\end{aligned}$$

THEOREM 2. *Every quasi-normal logic $L \in Ext\boldsymbol{K4}$, which is axiomatizable by a finite number of normal and quasi-normal modal canonical formulas, without closed domains (i.e., formulas of the form $\alpha(\mathfrak{F}, \emptyset, \bot)$ and $\alpha^{-}(\mathfrak{F}, \emptyset, \bot)$), is decidable, though may have no Kripke frames at all, like Solovay's system $\boldsymbol{S}(\boldsymbol{K4} \oplus \alpha(\circ, \emptyset)) + \alpha(\bullet, \emptyset)$.*

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Abstracts of papers submitted by title

ALEXANDER CHAGROV AND MICHAEL ZAKHARYASCHEV, *You can't independently axiomatize all modal logics*.
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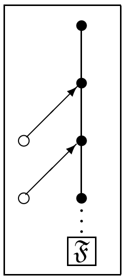
An old open problem of the modal logic folklore is to prove or disprove that every normal logic has an independent axiomatization (with modus ponens, necessitation and substitution as the only inference rules). The following simple observation opens a way for solving this problem.

LEMMA 1. *If a logic L_1 has an independent axiomatization then, for every finite axiomatizable logic $L_2 \subset L_1$, $L_2 \neq L_1$, there is an immediate predecessor of L_1 containing L_2.*

Let L_2 be the minimal logic containing **K4** and the following seven formulas:

$$\Diamond^{+}\Box\bot, \quad \gamma \to \Diamond\gamma, \quad \neg\beta_0, \quad \Box^{+}(q \to \psi) \vee \Box^{+}(\neg q \to \psi), \quad \Diamond\alpha \to \Diamond^{+}\beta \vee \Diamond^{+}\alpha',$$



where

$$\Box^{+}\phi = \phi \vee \Box\phi, \Diamond^{+}\phi = \phi \vee \Diamond\phi, \psi \in \{\alpha, \beta, \gamma\},$$
$$\alpha = p \wedge \Box\neg p, \alpha' = \alpha(\Diamond p/p), \alpha'' = \alpha'(\Diamond p/p),$$
$$\beta = \Diamond\Diamond\alpha \wedge \neg\Diamond\alpha', \beta_0 = \beta(\top/p), \beta' = \beta(\Diamond p/p),$$
$$\gamma = \Diamond\beta' \wedge \Diamond\alpha'' \wedge \neg\Diamond\beta, \gamma' = \gamma(\Diamond p/p).$$

By L_1 we denote the logic which is characterized by the class of all general frames for L_2 that are based on the Kripke frames of the form shown in the picture (points in $\mathfrak{F}$, if any, see all the depicted irreflexive points $\bullet$ and no depicted reflexive $\circ$).

LEMMA 2. *L_1 is a proper extension of L_2 having no immediate predecessor containing L_2.*

THEOREM 3. *L_1 has no independent axiomatization.*

We can also construct logics without independent axiomatization which contain **Grz, GL** and intuitionistic propositional logic; however their description is somewhat bulky. As a consequence of these results we obtain an extended answer to a question of Blok [1]:

THEOREM 4. *The lattices of varieties of interior (Grzegorczyk, diagonalizable, Heyting) algebras are not strongly atomic.*

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S. T. FEDORYAEV, *Decidable algorithmic problems on relatively complemented distributive lattices which cannot be simultaneously decidable*.
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Here we study the algorithmic problems with mentioned properties along with the spectrum property and the correlation problem of algorithmic dimensions [2, 1] for the relatively complemented distributive lattices. By the algorithmic problem on a constructive model $\mathfrak{A}$ we mean the relation on $\mathfrak{A}$ that is invariant with respect to the automorphism group of this model [2] (equivalently, that is definable in $L_{\omega_1\omega}$).

Let $\mathcal{A}$ be the class of all relatively complemented distributive lattices with zero element admitting a constructivization with the decidable set of atoms.

THEOREM. *For every lattice $D \in \mathcal{A}$ with infinitely many atoms, there exists a countable family W of algorithmic problems on D such that every algorithmic problem in W is decidable with respect to some constructivization of D, but no problems $W_1, W_2 \in W$ can be simultaneously decidable with respect to any constructivization of D.*

COROLLARY 1. *The algebraic reducibility structure L (see* [1]*) of a lattice $D \in \mathcal{A}$ (or a lattice $\langle D, F\rangle$ in the language extended by Fréchet ideal) with infinitely many atoms has a countable set of elements such that no pair of these ones has a lower bound. In particular, L has no least element.*

Let dim-D be one of the algorithmic dimensions [2] (i.e., algebraic, program, uniform, or auto-dimension) of a lattice D.

COROLLARY 2. *For any lattice $D \in \mathcal{A}$, the following statements are equivalent:*
(i) *dim-$D \neq 1$;*
(ii) *dim-$D = \omega$;*
(iii) *D contains infinitely many atoms.*

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L. GORDEEV, *Strong normalization and realizability*.
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Let T be an intuitionistic formal system in the language of set theory (with equality) enriched by the canonical abstraction-term operator $\{x \mid A\}$ for certain formulas A. Nonlogic axioms of T are of the form $(z \in \{x \mid A\} \leftrightarrow A[x/z])$ as corresponding to $\exists y \forall x(x \in y \leftrightarrow A)$ where y does not occur free in A. Let SEQT extend the canonical intuitionistic predicate sequent calculus (with cut) in the language of T by the corresponding rules of inference (τ

— arbitrary abstraction term):

$$\frac{A[x/\tau], \Gamma \Rightarrow B}{\tau \in \{x \mid A\}, \Gamma \Rightarrow B} \qquad \frac{\Gamma \Rightarrow A[x/\tau]}{\Gamma \Rightarrow \tau \in \{x \mid A\}},$$

Strong normalizability of T expresses that every primitive recursive (say) chain of successive (canonical) cut-reductions in SEQT terminates. [This expression reduces by standard techniques to an equivalent purely combinatorial sentence.]

With any such T we correlate an appropriate (classical, say) formal system $T^{\times}$ whose language has objects of two sorts: (a) natural numbers and (b) sets. Moreover, the language of $T^{\times}$ includes standard number theoretic operations $0, 1, +, \cdot$, and it does not include abstraction terms. As compared to T, atomic formulas of $T^{\times}$ include $m = n$ as well as $(n, x) \in y$ instead of $x \in y$, where m, n and x, y are numbers and sets, respectively. Nonlogic axioms of $T^{\times}$ are the ones of Peano arithmetic (PA) and certain set formation axioms $\exists y \forall n \forall x((n, x) \in y \leftrightarrow A)$ where y does not occur free in A.

THEOREM. *The consistency of $T^{\times}$ implies strong normalizability of T, provably in* PA.

In particular, let T be the intuitionistic variant of Quine's New Foundations minus Extensionality (NF^{-}). The corresponding theory $T^{\times}$ preserves stratification, and its consistency reduces to the existence of κ-iterated power-set for some cardinal κ being a solution to the familiar proposition $(\forall m < \omega)(\kappa \rightarrow (\omega)^{m}_{\omega})$ of infinite combinatorics.

On the other hand, strong normalizability of classical (sequent calculus corresponding to) theory NF^{-} easily follows from strong normalizability of the intuitionistic one.

COROLLARY. *Strong normalizability of (either intuitionistic or classical variant of) Quine's New Foundation minus Extensionality follows from the consistency of Zermelo's set theory extended by the existence of power-set hierarchy along $\aleph_\omega$, provably in* PA.

TAKAO INOUÉ AND ARATA ISHIMOTO, *A uniform formulation of some syllogistic systems with or without Leśniewski's epsilon*.

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We shall be here concerned with a uniform treatment of some syllogistic systems including classical propositional logic **CP**, in particular the Brentano-type syllogistic with Leśniewski's epsilon ε, denoted by **B**-ε, which has already been presented by the second named author in Japan in 1991. The system **B**-ε contains both the Brentano-type syllogistic and the propositional fragment $\mathbf{L}_1$ of Leśniewski's ontology **L** introduced in [5] (cf. [4]). Starting with **CP**, the systems to be presented are developed, step by step, by way of additional reduction rules and postulates. All the systems considered are rendered both in tableau method (in the style of [4]) and Hilbert-style formulation provided with Hilbert-style axiomatic rejection with Hintikka formulas as axioms as a case study of [1]. Some meta-theorems on cut elimination theorem and Ł-decidability for those systems are proved as a case study of [2] (cf. [3]).

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K. ZH. KUDAIBERGENOV, *On resplendency and homogeneity*.
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We indicate some conditions for "resplendency implies homogeneity" that are more or less similar to the standard notions of stability theory and compute the spectra of resplendency for some classes of superstable theories. The problem of finding such conditions and computing such spectra was posed by J. Baldwin.

GUN-WON LEE, *A construed number*.
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The use of number words has been already a part of our natural language as we can see in "lucky seven." It was said as that the number words are used as both *referential* and *predicative*. In order to have a stable theory the restriction of the uses of number words as predicative use only has been thought as an adequate at least temporarily in the formal languages. For this purpose, the number construed in set theory has been utilized widely already in the predicative use. And the use of the numbers unconstrued can be as usual to allow both usages of the number words in the above.

As in Quine, the construed numbers are the numbers construed in terms of Zermelo-Fraenkel-type set theory or von Neumann-type set theory as used usually in pure mathematics. Here these numbers aer different from the usual numbers in our ordinary life which have been used in various ways without the original meaning of them but as a 'name' also, which has been said to be "referential" in Cheng [4]. As in Belair [1], in order to construct a handy model it is in practice useful to use the Zermelo-Fraenkel-type set theory or von Neumann-type set theory without giving any useful meanings or interpretations at all but the artificially defined formal definitions only.

In principle, this method is useful in a constructive method to define a system. By doing so, it can be verified finitely. However, it cannot be the whole system which can handle number theory in general. What we have here is that (1) this system cannot say anything in general about the logistic thesis at all, (2) this system does not give any positive answers to the decision problems in general, but it tried not to get involved in those problems, (3) this system did not provide any way to skirt the logical or semantic paradoxes at all, but it constructs step by step avoiding the elements which can possibly generate those contradictions in the axiomatic systems, and (4) finally as an artificial language, this system cannot give any algorithms at all in the practical applications or measurements, however this system itself keeps its own analyticity, well-definedness, or well-order.

It is undeniable also that this construed number has been regarded as the duration which can be the paradigm of all the various disciplines also. However I can hardly find enough evidence to assert that it can be the absolute duration conceived up to now especially as in Newton, I think.

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S. I. MARDAEV, *Smallest-fixed-point theorem for intuitionistic propositional logic and Grzegorczyk logic*.
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Modal propositional formulas are formed in the usual way from variables, the constant $\perp$ and the connectives $\&, \vee, \neg, \Box, \Diamond$. Connectives $\supset, \equiv$ are usual abbreviations.

THEOREM 1. *Let $\phi(p, q_1, \dots, q_n)$ be a modal formula such that all occurrences of p are positive. There is a formula $w(q_1, \dots, q_n)$ such that*

$$\vdash_{Grz} \omega(q_1, \dots, q_n) \equiv \phi(\omega(q_1, \dots, q_n), q_1, \dots, q_n),$$

$$\vdash_{Grz} \Box(p \equiv \phi(p, q_1, \dots, q_n)) \supset (\omega(q_1, \dots, q_n) \supset p).$$

Propositional formulas are formed in the usual way from variables, the constant $\perp$, and connectives $\&, \vee, \neg, \supset$.

THEOREM 2. *Let $\phi(p, q_1, \dots, q_n)$ be a propositional formula such that all occurrences of p are positive. There is a formula*

$$\vdash_{Int} v(q_1, \dots, q_n) = \phi(v(q_1, \dots, q_n), q_1, \dots, q_n),$$

$$\vdash_{Int} (p \equiv \phi(p, q_1, \dots, q_n)) \supset (v(q_1, \dots, q_n) \supset p).$$

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MAARTEN MARX, *Relational semantics for (conjugated) modal logics*.
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Modal logics are usually presented using Kripke semantics. One well known exception is **S5** which one can also present without using an accessibility relation. Recent research in Lambek calculus and Arrow logic has shown us that a complete relational semantics can be given for them. Both Lambek calculus and Arrow Logic might also be viewed as modal logics. In [1] we present a generalization of these results to modal logics with n-adic operators. We end with a discussion and some applications of this generalization.

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LARRY MATHEWS, *Model theory of p-adically closed rings*.
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We develop some basic model theory for p-adically closed rings, extending some fundamental work on these structures by Belair in [1] and [2]. We consider the theory $\mathrm{Th}(p\mathrm{CR})$ of

p-adically closed rings in a suitable first-order language in which it has elimination of quantifiers. We prove that Th(pCR) has prime models over arbitrary subsets, and also identify the substructure of models of this theory.

We derive some elementary consequences of quantifier elimination, namely that Th(pCR) is *t-minimal* (see [3]), model-theoretically bounded, and satisfies the Exchange Principle. We also prove that Th(pCR) has definable Skolem functions in a suitable extended language.

We also prove some basic results concerning the structure of definable sets in models of Th(pCR). We prove a cell decomposition theorem for definable sets, similar to that for p-adically closed fields in [4]. From this we deduce a theory of dimension for definable sets.

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A. S. MOROZOV, *Properties of automorphism groups of some recursive models*.
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A model M is called *recursive* (*decidable*) provided its universe is a recursive set of natural numbers and its (elementary) diagram is decidable.

Here we study properties of automorphism groups of models with specific properties.

THEOREM. *Let M be a recursive model and let G be a subgroup in $AutM$. Denote $I_n(M) \rightleftharpoons \{\langle\overline{a},\overline{b}\rangle \in |M|^n \times |M|^n \mid \exists f \in G. f(\overline{a}) = \overline{b}\}$. Let the model $(M, I_n)_{n<\omega}$ be decidable. Then the following statements are valid:*

1. *The $\forall$-positive theory of G is a Π^0_1-set.*
2. *If all G-orbits are finite then the $\forall$-negative theory of G is a Σ^0_1-set.*

The conditions of the Theorem is often the case in recursive linear orders, fields, Boolean algebras.

D. E. PAL'CHUNOV, *Elementary types semigroup of boolean algebras with distinguished ideals*.
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Boolean algebras with distinguished ideals (we call them I-algebras) are considered in the signature $\langle \cup, \cap, -, 0, 1, I_1, \ldots, I_\lambda \rangle$, where I_j are unary predicates which distinguish ideals; number λ is fixed.

Let $S \rightleftharpoons \{\mathbb{A} \mid \mathbb{A} \text{ is an } I\text{-algebra}\}/\equiv$ be the set of elementary types of I-algebras. Let $\mathfrak{S} \rightleftharpoons (S, \times)$, where $\times$ denotes the direct product: $[\mathbb{A}]_\equiv \times [\mathbb{B}]_\equiv = [\mathbb{A} \times \mathbb{B}]_\equiv$.

Notice that $\mathbb{S}$ is a commutative semigroup with unit; the unit of S is the class consisting of the degenerated Boolean algebra with $I_j = \{0\}$.

For $a, b \in \mathbb{S}$ we write $a \leq b$ if $(\exists c)(b = a \times c)$.

Designate $N(a) \rightleftharpoons (\forall c, d)(a = c \times d \rightarrow (a = c \vee a = d))$.

Remark. The relation $\leq$ is a partial order on $\mathbb{S}$.

Consider the following predicates $L(a)$, $F(a)$, and $C(a)$ on the set

$$S : \begin{cases} L([\mathfrak{A}]_{\equiv})\ \&\ \text{iff the } I\text{-algebra } \mathfrak{A} \text{ is local (see [1])}, \\ F([\mathfrak{A}]_{\equiv})\ \&\ \text{iff } \mathfrak{A} \text{ is finitely axiomatizable}, \\ C([\mathfrak{A}]_{\equiv})\ \&\ \text{iff } \mathfrak{A} \text{ is } \omega\text{-categorical}. \end{cases}$$

THEOREM. *Locally, finite axiomatizability and ω-categoricity of I-algebras can be expressed in the language of the semigroup $\mathbb{S}$ by the following first-order formulae:*

$$L(a) \sim (\forall b \leq a)(\exists c < b)(\forall d < b)((c \leq d \,\&\, N(d)) \to c = d),$$

$$\begin{aligned} F(a) \sim\ & L(a) \,\&\, (\forall b, d, e) \\ & ((e = d \times e \,\&\, a \neq e \,\&\, d = d \times d \,\&\, b \leq d \,\&\, b \neq b \times b \\ & \,\&\, N(b) \,\&\, N(d)) \to (\exists c)(N(c) \,\&\, b < c < d)), \end{aligned}$$

and

$$C(a) \sim (\forall b \leq a)(F(b)).$$

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D. SINGH, *An extension of the program KRIPKE*.
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In this paper I propose an extension of the Program KRIPKE of [2] by interpreting a multiset of formulae, viz., $[A, B^m, C^n, \dots]$ as an ordinal tree, viz., $[A \cdot B^m \cdot C^n \cdots]$, where A, B and C can be treated as events and A^k denotes A occurring k times. This extension can enable KRIPKE to capture temporal environments by way of interpreting an ordinal tree as a temporal data structure.

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ALDO URSINI, *An easy way to phase semantics for linear logic*.
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The aim is to give a simple interpretation of the basic connectives of (classical or intuitionistic, commutative or non-commutative) linear propositional logics. If R is a binary relation on a set M, $a \in M$ and $X \subseteq M$, let $R(a) = \{x \mid aRx\}$, $R(M) = \{x \mid \text{ for some } a \in M, aRx\}$. A *phasoid* is a structure $\langle M, R, \cdot, 1\rangle$ where $\cdot$ is a binary operation in M with neutral element 1, R is a binary relation on M and: (i) for all $a \in M, X \subseteq M$, $R(a) \subseteq R(M)$ iff $\check{R}(a)] \subseteq \check{R}(M)$; (ii) for all $a, b, c \in M$, $(ab)Rc$ iff $aR(bc)$ (Here $\check{R}$ is the converse of R; $a\check{R}b$ iff bRa.) The R-facts are those $X \subseteq M$ such that if $R(a) \subseteq R(X)$, then $a \in X$. The basic linear operations on R-facts are defined very simply. For instance:

$$X^{\perp} = \{x \mid R(x) \cap X = \emptyset\};$$

$$X \text{ PAR } Y = \{x \mid \text{ for any } b, c \in M, \text{ if } (bc)Rx \text{ then } b \in \check{R}(Y) \text{ or } c \in \check{R}(X)\}.$$

Various completeness results are obtained.

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YU. G. VENTSOV, *Effectively infinite algorithmic dimension of recursive models*.
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We study recursive models of infinite algorithmic dimension [4]. A class of recursive models of Δ^0_2-effectively infinite algorithmic dimension is characterized in terms of effective density of some subset of the set of finite isomorphisms which can not be extended up to an automorphism. This class is sufficiently wide for it to contain such well-known classes, defined by Goncharov [1, 2], as of branching and unbounded recursive models. According to [3], this is the best result on effectively infinite dimension we can expect.

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XIAOKANG YU, *Measurability of analytic sets, formalized in second-order arithmetic*.
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Measurability of Borel sets of compact complete separable metric spaces, formalized in the language of second-order arithmetic, is established in the subsystem ATR_0 with the aid of Borel codes. Measurability of analytical sets, coded in the language of second-order arithmetic, can be established in the subsystems Π^1_1-CA_0.

SHU-GUO ZHANG, *A characterization of strongly normal ideals on $P_\kappa\lambda$*.
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PROPOSITION 1. *If $I_{\kappa\lambda} \subseteq I$ and I is a normal ideal over $P_\kappa\lambda$ (in fact a little weaker assumption is enough) then I is κ-complete.*

LEMMA 2 [JOHNSON]. *If $\mu < \kappa$, $\mu < \nu \leq \lambda$ and I is normal, then I is (μ, ν)-distributive iff whenever $A \in I^+$ and $f : A \to P(\nu)$ is such that for each $x \in A$, $f(x) \in [x]^{\leq\mu}$, there is a $B \in P(A) \cap I^+$ such that $f \upharpoonright B$ is constant.*

We give a characterization of strongly normal ideals over $P_\kappa\lambda$ as follows, by using Lemma 2.

THEOREM 3. *If κ is weakly inaccessible, a normal ideal I over $P_\kappa\lambda$ is strongly normal iff it is (μ, λ)-distributive for each $\mu < \kappa$.*

COROLLARY 4. *If $\lambda^{<\kappa} = \lambda$ and κ is λ-ineffable or almost λ-ineffable or has the λ-Shelah property, then the associated ideals $NIn_{\kappa\lambda}$, $NAIn_{\kappa\lambda}$ and $NSh_{\kappa\lambda}$ are (μ, λ)-distributive for every $\mu < \kappa$.*