There are eight questions on this examination.

Calculators, notes and books may not be used in this examination.

You may not receive full credit for a correct answer if insufficient work is shown.

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<th>QUESTION</th>
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1. (10 points)

Evaluate the following.

\[ a) \int_0^2 (2x^3 - \pi x + \cos(x)) \, dx \]

\[ b) \frac{d}{dx} \int_0^{x^2} \sqrt{u} \, du \]

\[ c) \int \frac{x^3}{\sqrt{x^2 + 3}} \, dx \]
2. (10 points)

Consider a function $f(x)$ defined in the following manner.

$$f(x) = \begin{cases} 
2x^3 - 4x + 5 & \text{if } x \leq 1, \\
a\sqrt{x} + b & \text{if } x > 1.
\end{cases}$$

Find values for $a$ and $b$ such that $f(x)$ is differentiable at $x = 1$. 

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3. (10 points)

Let \( f(x) = x^4 - 4x^3 \).

a) Find the critical points of \( f(x) \).

b) Find the \( x \) values where \( f(x) \) has a local minimum or maximum.

c) Find the intervals where \( f(x) \) is concave up and where it is concave down.

d) Find points of inflection for \( f(x) \).
4. (10 points)

A car is traveling 84 ft/sec begins to decelerate at a constant rate of 14 ft/sec$^2$. After how many seconds does the car come to a stop and how far will the car have traveled before stopping?
5. (10 points)

A 6 ft tall man walks away from a 15 ft lamp post at a speed of 3 ft/sec. At what rate is the length of his shadow increasing?
6. (10 points)

Let \( f(x) = x^2 - 5x - 6 \) and \( F(X) = \int_{0}^{X} f(t) \, dt \).

a) find the critical points of \( F(x) \) and determine whether they are local minima or maxima.

b) Find the points of inflection of \( F(x) \) and determine whether the concavity changes from up to down or vice versa.
Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$. 

7. (10 points)
8. (10 points)

Consider curves $f(x) = \sqrt{x}$ and $g(x) = x^2$.

a) Find the area in between the curves.

b) Find the volume of the resulting solid if the enclosed area is rotated about the $y$-axis.
9. **(10 points BONUS)**

This is a BONUS problem. Only work on this problem after you have finished rest of the exam.

Cowboy Clint wants to build a dirt road from his ranch to the highway so that he can drive to the city in the shortest amount of time. The perpendicular distance from the ranch (point A) to the highway (point B) is 4 miles, and the city (C) is located 9 miles due east from down the highway from point B. Where should Clint join the dirt road to the highway if the speed limit is 20mph on the dirt road and 55mph on the highway? That is: Find the distance from point B to point X where the dirt road should join the highway.