Your name and signature

Student ID #

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Notes (please read carefully before starting the exam):

- Keep your desktop clean. Put your textbooks and notebooks in your bag and keep them closed.

- One handwritten 5x7 index card or one half of A4 paper (one side) are allowed. Calculators are NOT allowed.

- In order to receive credit, you must show all of your work; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

- Read the questions carefully. Make sure you answer the questions asked in the problems.

- You may use the back side of the exam papers, as well as the last blank page.

- You have 50 minutes to complete the midterm.
Problem 1. (10 points) Find an explicit formula for the analytic function $f(z)$ that has the Maclaurin expansion $\sum_{k=0}^{\infty} k z^k$. 
Problem 2. (10 points) Find the Taylor series and radius of convergence for $f(z) = \sin(\pi z)$ around $z = 1$. 
Problem 3. (a) (10 points) Find the Taylor series and radius of convergence for \( g(z) = (z^2 - 2z) \sin(\pi z) \) around \( z = 1 \).

(b) (5 points) Find \( g^{(4)}(1) \) with \( g \) as in (a).

(c) (5 points) Compute

\[
\int_{|z-2|=3} \frac{g(z)}{(z-1)^2} \, dz
\]
Problem 4. (20 points) Find the Laurent series centered around $z = 4$ of

$$\frac{z + 2}{z(z - 4)^3}$$

in the following regions:

(a) $0 < |z - 4| < 4$.
(b) $|z - 4| > 4$. 
Problem 5. (a) (10 points) Compute
\[ \oint_{|z-4|=3} \frac{z+2}{z(z-4)^3} \, dz. \]

(b) (5 points) Compute
\[ \oint_{|z|=3} \frac{z+2}{z(z-4)^3} \, dz. \]