(15) 1. Define the following:
   (a) The series $\sum_n a_n$ converges.
   
   (b) The series $\sum_n a_n$ converges absolutely.
   
   (c) The series $\sum_n a_n$ converges conditionally.

(10) 2. State the integral test for convergence of series.
(10) 3. Compute
(a) \[ \lim_{x \to 2} \frac{\sqrt{2x} - 2}{\log(x - 1)} \]

(b) \[ \lim_{x \to 1} \frac{x^4 - 4x^3 + 8x - 5}{x^3 - 3x + 2} \]

(20) 4. For each of the series below, determine whether it converges absolutely, converges conditionally, or diverges. Explain briefly.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Reason</th>
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<tbody>
<tr>
<td>(a) [ \sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\log n} ]</td>
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<tr>
<td>(b) [ \sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n} ]</td>
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<tr>
<td>(c) [ \sum_{n=1}^{\infty} \frac{\sqrt{n \log n}}{n^2 + 2} ]</td>
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<tr>
<td>(d) [ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 50}} ]</td>
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<tr>
<td>(e) [ \sum_{n=1}^{\infty} \frac{(-1)^n}{(\log n)^2} ]</td>
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</table>
5. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by

\[ y = e^x, y = e^{-x}, x = 1 \]

about the \( y \)-axis.

6. Find the value of \( c \) such that the area of the region enclosed by the parabolas \( y = x^2 - c^2 \) and \( y = c^2 - x^2 \) is 576.
(15) 7. Find the volume common to two spheres, each with radius $r$, if the center of each sphere lies on the surface of the other sphere.

(10) 8. Evaluate the following integrals:

(a) $\int \frac{\cos x}{2 + \sin x} dx$

(b) $\int_{0}^{2} x^3 \sqrt{x^2 + 4} dx$
(15) 9. Find the area of the surface obtained by rotating the curve 

\[ 9x = y^2 + 18, \quad 2 \leq x \leq 6, \]

about the \( x \)-axis.

(10) 10. (a) Use properties of the logarithm to expand \( \ln \sqrt{a(b^2 + c^2)} \).

(b) Solve the equation \( 2 \ln x = \ln 2 + \ln(3x - 4) \) for \( x \).
(15) 11. (a) Find the radius of convergence and the interval of convergence for each of the two series

\[
\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n}}
\]

\[
\sum_{n=1}^{\infty} \frac{n(x - 4)^n}{n^3 + 1}
\]

(b) A function \( f \) is defined by

\[
f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + x^6 + \cdots.
\]

Find the interval of convergence of the series, and find an explicit formula for \( f(x) \).

(10) 12. By differentiating the geometric series \( \sum_{n=0}^{\infty} x^n \), find the following sums explicitly:

(a) \( \sum_{n=0}^{\infty} n x^n \)

(b) \( \sum_{n=0}^{\infty} n^2 x^n \)
(15) 13. (a) Find the Taylor series for $f(x) = \frac{1}{\sqrt{x}}$ centered at $a = 9$.

(b) Express the following indefinite integral as an infinite series:

$$\int \frac{e^x - 1}{x} dx$$

(15) 14. Suppose $f$ and $g$ are one-to-one and twice differentiable functions that are inverses of each other.

(a) Express $g''(x)$ in terms of $f'(g(x))$ and $f''(g(x))$.

(b) Suppose $f$ is decreasing and concave upward. What can you conclude about the concavity of $g$?
(15) 15. (a) State the root test for convergence of series.

(b) Prove the root test for convergence of series.

Some Formulas

\[
\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}, \quad \cos^2 \theta = \frac{1 + \cos(2\theta)}{2}, \quad \sin \theta \cos \theta = \frac{\sin(2\theta)}{2},
\]
\[
\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}, \quad \frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}, \quad \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^2},
\]
\[
\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \sec^2 \theta = \tan^2 \theta + 1.
\]