Quiz for Thursday, October 7

1. Use substitution to evaluate
\[ \int_0^{13} \frac{dx}{\sqrt{(1 + 2x)^2}}. \]

*Solution:* Let \( u = 1 + 2x, du = 2dx. \) Then
\[ \int_0^{13} \frac{dx}{\sqrt{(1 + 2x)^2}} = \frac{1}{2} \int_1^{27} u^{-\frac{3}{2}}du = \frac{3}{2} \left[ u^{-\frac{1}{2}} \right]_1^{27} = 3. \]

2. Sketch the region between the curves \( y = 12 - x^2 \) and \( y = x^2 - 6, \) and find the area of this region.

*Solution:* The two parabolas meet at \((\pm 3, 3)\). Therefore,
\[ \text{Area} = \int_{-3}^{3} (18 - 2x^2)dx = 18x - \frac{2}{3}x^3 \bigg|_{-3}^{3} = 72. \]

Quiz for Thursday, October 12

1. Sketch and find the area of the region bounded by the parabola \( y = x^2, \) the tangent line to this parabola at \((1, 1), \) and the \( x \)-axis.

*Solution:* The equation of the tangent line is \( y = 2x - 1. \) Therefore,
\[ \text{Area} = \int_0^{\frac{1}{2}} x^2dx + \int_{\frac{1}{2}}^{1} (x - 1)^2dx = \frac{1}{12}. \]

Alternatively, one can integrate with respect to \( y \) and end up with a single integral:
\[ \text{Area} = \int_0^{1} \left( \frac{y + 1}{2} - \sqrt{y} \right)dy = \frac{1}{12}. \]

2. Use slices to find the volume of the solid obtained by rotating the region bounded by \( y = x \) and \( y = \sqrt{x} \) about the line \( y = 1. \) Be sure to sketch the region.

*Solution:*
\[ \text{Volume} = \pi \int_0^{4} (1 - x)^2 - (1 - \sqrt{x})^2 dx = \frac{\pi}{6}. \]

Quiz for Thursday, October 14

1. Use slices to find the volume of the solid obtained by rotating the region bounded by \( y = x, y = 0, x = 2, \) and \( x = 4 \) about the line \( x = 1. \) Be sure to sketch the region.

*Solution:*
\[ \text{Volume} = \pi \int_0^{2} (9 - 1)dy + \pi \int_2^{4} (9 - (y - 1)^2)dy = 25\frac{1}{3}\pi. \]

2. Use shells to find the volume of the solid obtained by rotating the region bounded by \( x = 4y^2 - y^3 \) and \( x = 0 \) about the \( x \)-axis. Be sure to sketch the region.

*Solution:*
\[ \text{Volume} = 2\pi \int_0^{4} y(4y^2 - y^3)dy = \frac{512}{5}\pi. \]