Q. Let $p_t(x, y)$ be the transition probabilities for an irreducible continuous time Markov chain on a finite set $S = \{1, \ldots, N\}$. A collection of indistinguishable particles moving independently on $S$ can be thought of as a Markov chain $W_t$ on the set of configurations of particles on $S$. So, for example, if $N = 4$, then $(5, 7, 1, 0)$ corresponds to the configuration in which there are 5 particles at site 1, 7 particles at site 2, 1 particle at site 3, and none at site 4.

(a) Suppose $W_0$ has the distribution in which $\{W_0(i), 1 \leq i \leq N\}$ are independent, with $W_0(i)$ having the Poisson distribution with parameter $\lambda(i)$. Use the properties of thinning and superposition of Poisson random variables to determine the distribution of $W_t$.

(b) Using (a), find some stationary distributions for $W_t$.

(c) The chain $W_t$ is not irreducible, since the total number of particles does not change. Find the closed irreducible classes for $W_t$.

(d) For each irreducible class, find the stationary distribution for the chain restricted to that class.