
Problems 24,25,26,31(use conditioning, not problem 27, which I will discuss in class 2/15),32 on pages 124-133, and problem 17 on page 249.

H1. Suppose \( n \) balls are distributed at random into \( r \) boxes. Let

\[
X_i = \begin{cases} 
1 & \text{if box } i \text{ is empty;} \\
0 & \text{otherwise.}
\end{cases}
\]

(a) Compute \( EX_i \).
(b) For \( i \neq j \), compute \( EX_iX_j \) and \( \text{Cov}(X_i, X_j) \).
(c) Let \( S = \) the number of empty boxes. Relate \( S \) to the \( X_i \)'s, and use part (a) to compute \( ES \) and \( \text{Var}(S) \).

H2. Toss a biased coin (with probability \( p \) of heads and \( 1 - p \) of tails) until the first head appears. Let \( N \) be the number of tosses required to get the first head. Now toss a fair die \( N \) times, and let \( S \) be the sum of results of the \( N \) tosses of the die. Use conditioning to compute \( ES \).

H3. A miner is trapped in a mine containing three doors. The first door leads to a tunnel that leads him to safety after three hours. The second door leads him back to the mine after five hours. The third door leads him back to the mine after seven hours. Each time, the miner is equally likely to choose each of the three doors. Let \( X \) be the number of hours it takes him to get to safety, and \( Y \) the number of the door he initially chooses.

(a) Compute \( E[X|Y = i] \) for \( i = 1, 2, 3 \) in terms of \( EX \).
(b) Compute \( EX \).

H4. Consider an experiment that has three possible outcomes that occur with probabilities \( p_1, p_2, p_3 \). Suppose that \( n \) independent repetitions of the experiment are made, and let \( X_i \) denote the number of times that outcome \( i \) occurs \( (i = 1, 2, 3) \).

(a) What is the distribution of \( X_1 + X_2 \)?
(b) Compute \( P(X_2 = k \mid X_1 + X_2 = m) \).
(c) What is \( E(X_2 \mid X_1 + X_2 = m) \)?