This is the seventh homework for Math 114s in the winter quarter of 2016. It is due Friday March 11th in class.

1. Prove that for a cardinal $\kappa$, $\kappa^+$ is regular.
2. Prove that $\text{cf}(\alpha)$ is a regular cardinal, that is $\text{cf}(\text{cf}(\alpha)) = \text{cf}(\alpha)$.
3. Prove that if $\kappa$ is singular, then $\text{KIL}(\kappa)$ is false.
4. Let $\kappa$ be an infinite cardinal. Prove that there is a tree of height $\kappa$ with no cofinal branch.
5. We say that a tree $T$ is normal if for every $\alpha < \beta$ below the height of $T$ and every $t \in \text{Lev}_\alpha T$, there is an $s \in \text{Lev}_\beta T$ such that $s > t$. Let $\kappa > \omega$ be a regular cardinal. Prove that there is a normal tree of height $\kappa$ with no cofinal branch.
6. Let $T$ be the tree of height $\omega_1$ with countable levels and no cofinal branch that we constructed in class. Define a function $f : T \to \mathbb{Q}$ by $f(t) = \max t$. Show that if $s < t$ in $T$, then $f(s) < f(t)$.
7. Let $T$ be a tree of height $\omega_1$. Suppose that there is a function $f : T \to \mathbb{Q}$ such that if $s < t$, then $f(s) < f(t)$. Prove that $T$ has no cofinal branch.
8. Assume the continuum hypothesis. Prove that there is a linear order $(L, <)$ of size $\omega_1$ such that for any countable $A, B$ subsets of $L$ such that every element of $A$ is less than every element of $B$, there is an $l \in L$ which is above every element in $A$ and below every element in $B$.
9. Assume the continuum hypothesis. Prove that $\text{KIL}(\omega_2)$ is false. Hint: Try to repeat the argument from proving that $\text{KIL}(\omega_1)$ is false, but using the linear order from the previous exercise in place of $\mathbb{Q}$. Hint 2: Break up the limit step into cases based on the cofinality of the limit ordinal.
10. Let $f : \kappa \to \kappa$. Show that the set $\{ \gamma < \kappa \mid f^{\gamma} \subseteq \gamma \}$ is a club.
11. Suppose that there is a train which stops at stations indexed by each ordinal $\alpha < \omega_1$ in order. At each stop 1 person gets off and $\omega$ many people get on. Show that there is an ordinal $\gamma < \omega_1$ where the train is empty when it stops (before the passenger exchange) at station $\gamma$. 

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