5. A countable set is said to have cardinality $\aleph_0$ and the set of real numbers has cardinality $\aleph$. Furthermore, since a set of cardinality $\aleph_0$ is "smaller" than a set of cardinality $\aleph$ we say that $\aleph_0 < \aleph$. Now for the question: Is there a cardinal number that is greater than $\aleph$? Why?

Proof: Let $I = \{x : 0 < x < 1\}$ and let $F = \{f : f : I \to I\}$. Suppose that $F$ has the same cardinality as $I$. Then there is a mapping

$$\alpha : I \to F
\quad x \to f_x$$

that is is one-to-one and onto. Each $f_x$ is a map of $I$ to $I$ and any map from $I$ to $I$ is an $f_x$ for some $x$.

We construct a map $g$ from $I$ to $I$ that is not a $f_x$ for any $x$. To construct $g$ we must define $g$ at $x$, $g(x)$ for all $x$ in $I$. Do this by defining

$$g(x) = xf_x(x) \text{ for all } x \text{ in } I.$$  

Then, since $0 < x < 1$, $g(x) \neq f_x(x)$, so $g \neq f_x$ for all $x$. A contradiction.