12. What would be a "reasonable" definition for \(\sup(A)\) if \(A\) were the empty set?

One solution: Recall, the number \(M\) is an upper bound for the set \(A\) if

\[ a \leq M \quad \text{for all } a \text{ in } A. \]

So, say, \(-10^{10}\) is an upper bound for a set with no elements,

\[ a \leq -10^{10} \quad \text{for all } a \text{ in } A, \]

since there are no elements in \(A\). The number \(-10^{10}\) can be replaced by any negative number.

At this point we could be cautious and say a empty set does not have a least upper bound. We could also throw caution to the wind and say the least upper bound is \(-\infty\). But then we will have the problem of defining what it means for \(-\infty\) to be a number.

Caution is probably the most reasonable choice here.