Let \( A \) be the set of positive integers and \( S \) be the set of all finite subsets of \( A \). Prove that \( S \) is countable. Hint: a set of \( k \) positive integers means a set of \( k \) distinct integers. Furthermore, order is not considered: \( \{1,2,3\} = \{3,2,1\} = \{2,3,1\} \) etc.

Solution: Let \( k \) be fixed and \( S_k \) be the set of all subsets having exactly \( k \) elements.

Let \( \{n_1, n_2, \ldots, n_k\} \) be a typical set of \( k \) positive integers. Since there are \( k! \) ways of writing out this set we choose the one that has \( n_1 < n_2 < \ldots < n_k \).

Next we set up the map from \( S \) to \( N_k \), where \( N_k \) is the cartesian product of \( k \) copies of the positive integers, by

\[
\{n_1, n_2, \ldots, n_k\} \to (n_1, n_2, \ldots, n_k)
\]

This is a one-to-one map of \( S_k \) into \( N_k \). Since \( N_k \) is countable and the image of \( S_k \) is an infinite subset of \( N_k \), \( S_k \) is countable.

Since \( S \), the set of all finite subsets of \( A \), is the union of a countable number of countable sets it follows that \( S \) is countable.