Review, Problem 11

Let \( a_n \) be defined by \( a_0 = 678912 \) and

\[ a_{n+1} = 3a_n + 1 \text{ for } n = 1, 2, 3, \ldots \]

Find the limit of the sequence \( \{a_n\} \)

Solution: A finite limit does not exist. To see this, iterate:

\[ a_1 = 3a_0 + 1 \]
\[ a_2 = 3a_1 + 1 = 3^2a_0 + 3 + 1 \]
\[ a_3 = 3a_2 + 1 = 3^3a_0 + 3^2 + 3 + 1 \]

These first few values show that \( a_n \rightarrow \infty \)