Consider the sequence $a_1 = 1/2$, $a_2 = 1/4$, $a_3 = 1/2$, $a_4 = 3/4$, the next seven terms are $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, $7/8$... and so forth. What are the limit points of the sequence.

Solution: All the numbers in the closed interval $[0, 1]$ are limit points of this sequence.

Note first: the sequence contains all numbers of the form

$$\frac{1}{2^2}, \frac{1}{2^2}, \frac{2}{2^2}, ..., \frac{k}{2^2}, \frac{k+1}{2^2}, ..., \frac{2^n-1}{2^n}$$

Second, the difference between consecutive members of the sequence displayed above is $1/2^n$. Thus if $x$ is any number in $[0, 1]$ and $n$ is chosen so that $1/2^n < \varepsilon$ then the interval

$$(x - \varepsilon, x + \varepsilon)$$

contains at least one number of the form $k/2^n$. Thus any $x$ in $[0, 1]$ is a limit point of the sequence.