Suppose that $x \geq -1$ and $n$ is a positive integer. Prove that

$$(1 + x)^n \geq 1 + nx.$$ 

The proof is by induction.

By inspection, it is true for $n = 1$.

As for the inductive step:

$$(1 + nx)^n \geq 1 + nx \Rightarrow$$

$$(1 + x)^{n+1} = (1 + x)(1 + x)^n \geq (1 + x)(1 + nx) = 1 + (n+1)x + nx^2$$

$$\geq 1 + (n+1)x + 0 = 1 + (n+1)x$$

The fact that $1 + x \geq 0$ is used in multiplying the inequality above. The fact that $x^2 \geq 0$ for all $x$ is also utilized.

In short

$$(1 + x)^{n+1} \geq 1 + (n+1)x,$$

so the inductive step is true.