Prove that if \( n \) is a positive integer the \( n^2 + 5n \) is divisible by 6.

The proof is by induction on \( n \).

Since \( 1^2 + 5\times 1 = 6 \), the assertion is true for \( n = 1 \).

The inductive step: Suppose that \( n^2 + 5n \) is divisible by 6.

Next,
\[
\begin{align*}
(n+1)^2 + 5(n+1) &= \\
&= n^3 + 3n^2 + 3n + 1 + 5n + 5 \\
&= n^2 + 5n + 3n^2 + 3n + 6 \\
&= (n^2 + 5n) + 3n(n+1) + 6
\end{align*}
\]

Then:

- by the inductive hypothesis \( (n^2 + 5n) \) is divisible by 6;
- since \( n \) and \( n+1 \) are consecutive integers, one of them is even, so \( n(n+1) \) is divisible by 2 and \( 3n(n+1) \) is divisible by 6.

Consequently \( (n + 1)^2 + 5(n+1) \) is divisible by 6.

This completes the inductive step.