Suppose that $a, b, c, d$ are positive numbers such that $a/b < c/d$. Prove that
\[ \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}. \]

Proof: Start with the first inequality
\[ \frac{a}{b} < \frac{a+c}{b+d} \]
and work backwards. Since $a, b, c, d$ are all positive we can multiply the inequalities below without changing their sense:
\[ \begin{align*}
\frac{a}{b} &< \frac{a+c}{b+d} \\
\iff
a(b+d) &< b(a+c) \\
\iff
ab + ad &< ba + bc \\
\iff
ad &< bc \\
\iff
\frac{a}{b} &< \frac{c}{d}
\end{align*} \]

Since the last inequality is true, we can reverse directions of the argument to establish the first.

The proof of the second inequality in the statement of the problem is similar.