Let \( p(x) \) be a polynomial with a root at \( x = r \). When will \( p(x) \) be differentiable at \( x = r \). 

Solution: \( p(x) \) will be differentiable at \( x = r \) iff and only if \( r \) is a root of multiplicity \( m \) where \( m \geq 2 \).

To see this suppose that \( r \) is a root of multiplicity \( m \) where \( m \geq 1 \). Then \( p(x) = (x-r)^m g(x) \), where \( g(x) \neq 0 \).

Next, set \( F(x) = |p(x)| = |(x-r)^m g(x)| \). Then

\[
\frac{F(x) - F(r)}{x-r} = \frac{|p(x)| - |p(r)|}{x-r} = \frac{|p(x)|}{x-r} - \frac{|x-r|^m |g(x)|}{x-r} \]

\[
= \begin{cases} 
|g(x)| & \text{for } x > r \\
-g(x) & \text{for } x < r 
\end{cases}
\]

Suppose that \( m = 1 \): Then

\[
\frac{F(x) - F(r)}{x-r} = \begin{cases} 
g(r) & \text{as } x \to r^+ \\
-g(r) & \text{as } x \to r^- 
\end{cases}
\]

That is,

\[
\frac{F(x) - F(r)}{x-r} \to \begin{cases} 
g(r) & \text{as } x \to r^+ \\
-g(r) & \text{as } x \to r^- 
\end{cases}
\]

Since \( g(r) \neq 0 \) this says that \( F(x) \) is not differentiable at \( x = r \).

If \( m \geq 2 \) then \( |x-r|^{m-1} \to 0 \) as \( x \to r \), so

\[
\frac{F(x) - F(r)}{x-r} = \begin{cases} 
|x-r|^{m-1} |g(x)| & \text{for } x > r \\
-x| |^{m-1} |g(x)| & \text{for } x < r 
\end{cases} \to 0 \text{ as } x \to r
\]

That is, \( F \) is differentiable at \( x = r \), and \( F'(r) = 0 \).