The basic problem is to show that the set of functions from the integers to the set \( \{1, 2\} \) is uncountable.

Solution: To define \( f \) from the integers to \( \{1, 2\} \)

\[
f: \{1, 2, \ldots, n, \ldots\} \rightarrow \{1, 2\}
\]

you must first decide what set of integers will be mapped onto 1. Then the complement will be mapped onto 2.

So, defining the function is equivalent to selecting a (possibly infinite) subset of \( \{1, 2, \ldots, n, \ldots\} \). Suppose we select a subset \( A \), letting \( a_n = 1 \) if \( n \) is in \( A \) and \( a_n = 0 \) if \( n \) is not in \( A \). Thus, selecting a subset of \( \{1, 2, \ldots, n, \ldots\} \) is equivalent to selecting a sequence of 0s and 1s.

But the set of all sequences \( a = \{a_1, a_2, \ldots, a_n, \ldots\} \) where each \( a_n \) is a 0 or 1 is uncountable:

Suppose the set were countable. Then we could write

\[
A = \{b_n, n = 1, 2, \ldots\}
\]

where each

\[
b_n = \{b_{n1}, b_{n2}, \ldots, b_{nn}, \ldots\}
\]

is a sequence of 0s and 1's. In addition every sequence of 0s and 1s is a \( b_n \) for some \( n \).

Construct a new sequence \( c = \{c_1, c_2, \ldots, c_n, \ldots\} \) 0s and 1s by setting

\[
c_n = 1 \text{ if } b_{nn} = 0
\]

\[
c_n = 0 \text{ if } b_{nn} = 1
\]

Then \( c \) is not equal to \( b_n \) for any \( n \), a contradiction.

Defining \( f \) from the integers to any finite set is the same problem because you must first define set of integers that map onto 1 and, by the above argument, this can be done in an uncountable number of ways.