Problem 1

Show, for $n \times n$ matrices $A$, $B$, $C$, that similarity is transitive: i.e. if $A \sim B$ and $B \sim C$, then $A \sim C$.

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Answer. If $A \sim B$ and $B \sim C$ then there exists some invertible $S$ such that $A = SBS^{-1}$ and some invertible $T$ such that $B = TCT^{-1}$. Then $A = SBS^{-1} = STCT^{-1}S^{-1} = (ST)C(ST)^{-1} = PCP^{-1}$, where $P = ST$ is an invertible matrix. Therefore, $A \sim C$.

Problem 2

When we change from a standard basis to a new basis $\mathcal{B} = \left( \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$, this is geometrically like reflecting the plane in the vertical axis. Show that this change of basis indeed changes a horizontal shear ($A = \begin{pmatrix} 1 \\ k \\ 0 \\ 1 \end{pmatrix}$) with parameter $k$ to one with parameter $-k$.

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Answer. Let the change of basis matrix be $S = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = S^{-1}$. Then the $\mathcal{B}$-matrix is

$$B = S^{-1}AS = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -k \\ 0 & 1 \end{pmatrix},$$

as expected.