Warm-up exercises (do not hand in)

Problem 1

Let
\[
A = \begin{pmatrix}
1 & 0 & -1 & 3 \\
3 & 1 & -2 & 4 \\
-1 & -1 & 2 & 2
\end{pmatrix}, \quad
B = \begin{pmatrix}
5 & 6 & -4 & 0 \\
4 & 1 & -3 & 2 \\
0 & 3 & 1 & -1
\end{pmatrix}, \quad
C = \begin{pmatrix}
5 & 6 & 0 \\
4 & 1 & 2 \\
0 & 1 & -1 \\
-2 & 3 & 3
\end{pmatrix}
\]

(a) Find at least three entries of the matrices \(A - B\) and \(BC\), by hand.

(b) Find the complete matrices \(A - B\) and \(BC\), using a computer. There are many kinds of calculators, software, webpages that can do that, and being able to check answers and quickly do computations on matrices and vectors will be very helpful for your learning this quarter. I want you to be modern scientists. Examples of software to use are Matlab, Mathematica, or free resources such as: Octave and Wolfram Alpha online (see https://www.wolframalpha.com/examples/Matrices.html). Let me know what software you are using!

Problem 2

Use Gauss-Jordan elimination to find all solutions for the following systems of linear equations. Imagine the vector (sub)space and the vectors in it to understand why there are no/∞/1 solution(s).

(a) \[
3x + 6y = 3 \\
2x + 4y = 0
\]

(b) \[
x - y = 1 \\
2x - 2y = 4
\]

(c) \[
x - y = 3 \\
x + z = 6 \\
x - y - z = 0
\]

Problem 3

For which values of \(a\) and \(b\) is \[
\begin{pmatrix} 2 \\ -4 \\ a \\ b \end{pmatrix}
\] a linear combination of the vectors \[
\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} ?
\]
Hand-in Exercises

Problem 1

If $A$ is a matrix in reduced row echelon form, and a column is removed, is it still in reduced row echelon form? What if a row is removed? Justify your answer carefully.

Problem 2

Consider three linearly independent vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$. Are the vectors $\vec{v}_1, \vec{v}_1 + \vec{v}_2, \vec{v}_1 + \vec{v}_2 + \vec{v}_3$ also linearly independent? (Explain.)

Problem 3

Show that there is a nontrivial relation on the set of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ if and only if at least one of the vectors $\vec{v}_j$ is a linear combination of the other vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{j-1}, \vec{v}_{j+1}, \ldots, \vec{v}_m$.

Problem 4

For which values of $a$, $b$ and $c$ are the vectors

$$\begin{bmatrix} a \\ 0 \\ b \end{bmatrix}, \begin{bmatrix} 0 \\ c \end{bmatrix}$$

linearly independent?
Optional Exercises (do not hand in)

Problem 1  Application

Balance the chemical reaction $CH_4 + O_2 \rightarrow CO_2 + H_2O$.

Problem 2  Application

(See also example 1.1.23 on page 6 of the book.) Consider a two-commodity market. Unit prices are $P_1$ and $P_2$, demands for the commodities are $D_1$ and $D_2$, and supplies are $S_1$ and $S_2$. Relationships are given by

\[
D_1 = 30 - P_1 - P_2 \\
D_2 = 100 - 2(P_1 + P_2) \\
S_1 = 4P_1 - 40 \\
S_2 = 3P_2 - 20
\]

Explain why these are complementary products and show that the system is at equilibrium when $P_1 = 10$ and $P_2 = 20$.

Problem 3  Mathematics

Prove that the intersection of two subspaces of $V$ is also a subspace of $V$.

Problem 4  Mathematics

Prove that the union of two subspaces of $V$ is also a subspace of $V$ if and only if one of the subspaces is contained in the other.