Question 1

(a) [3 points] Give the definition of the inverse of a matrix $A$ (if it exists).

(b) [4 points] Find values for $a, b, c$ such that

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} a & b & c \\ 2/5 & 1/5 & 1/5 \\ 6/5 & 13/5 & 8/5 \end{pmatrix}.$$ 

(c) [3 points] Give an example of a $2 \times 2$ matrix $B$ such that $B \neq I_2$ and $B = B^{-1}$. 

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Question 2

Let

\[ A = \begin{pmatrix}
1 & 1 & 2 \\
-2 & 0 & 6 \\
0 & 2 & 10
\end{pmatrix}. \]

(a) [3 points] Suppose that \( \vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \) is in the kernel of \( A \). Write an equation that relates the elements of \( \vec{v} \) to the columns of \( A \).

(b) [4 points] Suppose that \( \vec{v} \neq 0 \)? Find an example of such a \( \vec{v} \).

(c) [3 points] Find rank(\( A \)), and give a basis for the image of \( A \)
Question 3

(a) [2 points] Define a subspace of a vector space.

(b) [4 points] Suppose that $V$ is a subspace of $\mathbb{R}^3$, and $\vec{v}, \vec{w} \in \mathbb{R}^3$, such that $\vec{v} \in V$ and $\vec{w} \notin V$. Is $\vec{v} + \vec{w} \in V$?

(c) [4 points] Let $V$ be defined by the equation $v_1 = v_2$. Find a basis for $V$. 

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