1. Use differentiation formulas to find \( \frac{dy}{dx} \):

(a) \( y = \frac{(1-x)^{1/3}}{\tan x} \)

\( dy \) \( dx \) = \( \frac{1}{3} (1-x)^{-2/3} (-1) \tan x - (1-x)^{1/3} \sec^2 x \)

\( \frac{dy}{dx} \) = \( \frac{\frac{1}{3} (1-x)^{-2/3} (-1) \tan x - (1-x)^{1/3} \sec^2 x}{\tan^2 x} \)

(b) \( xy + \sqrt{y} = \sin^3(x^2) \)

\( \frac{dy}{dx} \) \( x \frac{dy}{dx} + \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 3 \sin^2(x^2) \cos^2(x^2)(2x) \)

\( x + \frac{1}{2} y^{-1/2} \) \( \frac{dy}{dx} \) = \( 6x \sin^2(x^2) \cos^2(x^2) - y \)

\( \frac{dy}{dx} = \frac{6x \sin^2(x^2) \cos^2(x^2) - y}{x + \frac{1}{2} y^{-1/2}} \)
2. Find the value of $a$ for which the limit

$$\lim_{x \to 2} \frac{2x^2 + ax - 1}{x - 2}$$

exists and calculate that limit.

$2(2)^2 + a(2) - 1 = 0$

$2a + 7 = 0 \quad a = -\frac{7}{2}$

$$\lim_{x \to 2} \frac{2x^2 - \frac{7}{2}x - 1}{x - 2} = \lim_{x \to 2} \frac{(x-2)(2x + \frac{1}{2})}{x - 2}$$

$$= \lim_{x \to 2} 2x + \frac{1}{2} = \frac{9}{2}$$
3. Prove that the equation
\[(\sin x)^k = k \cos x\]
has a solution for each positive integer k.

Let \( f(x) = (\sin x)^k - k \cos x \)

\[f'(x) = k(\sin x)^{k-1} \cos x - k \sin x\]

so \( f(x) \) is differentiable, so continuous

\[f(0) = -k < 0, \quad f(\frac{\pi}{2}) = (1)^k - 0 > 0\]

so \( f(x) = 0 \) for some \( x \) by the Intermediate Value Theorem.
4. Use the definition of the derivative and properties of limits to show that if 

\[ f(x) = \frac{1}{(x+1)^2} \]

then \( f'(1) = -\frac{1}{4} \).

\[
\begin{align*}
    f'(1) &= \lim_{x \to 1} \frac{\frac{1}{(x+1)^2} - \frac{1}{4}}{x - 1} \\
    &= \lim_{x \to 1} \frac{4 - (x+1)^2}{4(x+1)^2(x-1)} \\
    &= \lim_{x \to 1} \frac{-x^2 + 2x + 3}{4(x+1)^2(x-1)} \\
    &= \lim_{x \to 1} \frac{(x-1)(x+3)}{4(x+1)^2(x-1)} \\
    &= \lim_{x \to 1} \frac{-x-3}{4(x+1)^2} = \frac{-4}{(4)(4)} = -\frac{1}{4}
\end{align*}
\]
5. (a) Write the equation of the tangent line to the graph of
\[ f(x) = 3x^2 - x + 1 \]
at the point \((c, f(c))\).

(b) Use the equation of part (a) to find all values of \(c\) such that
the tangent line to the graph of \(f(x)\) at \((c, f(c))\) passes through the
point \((1, -3)\).

\[
\begin{align*}
(a) & \quad f'(x) = 6x - 1 \\
& \quad f(c) = 3c^2 - c + 1 \\
& \quad y - (3c^2 - c + 1) = (6c - 1)(x - c) \\
(b) & \quad (-3) - (3c^2 - c + 1) = (6c - 1)(1 - c) \\
& \quad -3c^2 + c - 4 = -6c^2 + 7c - 1 \\
& \quad 3c^2 - 6c - 3 = 0 \\
& \quad c^2 - 2c - 1 = 0 \\
& \quad c = \frac{2 \pm \sqrt{4 + 4}}{2} = 1 \pm \sqrt{2}
\end{align*}
\]