1. (a) Use differentiation formulas to calculate $f'(1)$ for 

\[ f(x) = \frac{x}{\sqrt{x^2 + 3}} \]

(b) Use differentiation formulas to calculate the second derivative of 

\[ f(x) = \sin(\sin x) \]

(a) \[ f'(x) = \frac{x}{(x^2 + 3)^{3/2}} \]
\[ f'(x) = \frac{(x^2 + 3)^{1/2} - x \frac{1}{2} (x^2 + 3)^{-1/2} (2x)}{x^2 + 3} \]
\[ f'(1) = \frac{2 - \frac{1}{4}}{1} = \frac{3}{4} = \frac{3}{8} \]

(b) \[ f''(x) = (\cos(\sin x))(-\sin x)(\cos x)(\cos x) \]
\[ + (\cos(\sin x))(-\sin x) \]

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2. The slope of the tangent line to the curve 
\[ \sqrt{y} + axy^2 = 2 \]
for \( x = 0 \) is 8. Find the value of \( a \).

If \( x = 0 \) then \( \sqrt{y} = 2 \) \( \Rightarrow y = 4 \)

\[ \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx} + ay^2 + ax^2y \frac{dy}{dx} = 0 \]

at \( x = 0 \)

\[ \frac{1}{2}(4^{-\frac{1}{2}})(8) + a(4)^2 = 0 \]

\[ 2 + 16a = 0 \quad a = -\frac{1}{8} \]
3. Use the definition of the derivative and properties of limits to prove that if

\[ f(x) = \sqrt{2x^2 - 1} \]

then \( f'(1) = 2 \).

\[
f'(1) = \lim_{x \to 1} \frac{\sqrt{2x^2 - 1} - 1}{x - 1} \cdot \frac{\sqrt{2x^2 - 1} + 1}{\sqrt{2x^2 - 1} + 1}
\]

\[ = \lim_{x \to 1} \frac{(2x^2 - 1) - 1}{(x - 1)(\sqrt{2x^2 - 1} + 1)}
\]

\[ = \lim_{x \to 1} \frac{2x^2 - 2}{(x - 1)(\sqrt{2x^2 - 1} + 1)}
\]

\[ = \lim_{x \to 1} \frac{2(x-1)(x+1)}{(x-1)(\sqrt{2x^2 - 1} + 1)}
\]

\[ = \frac{2(1+1)}{\sqrt{2} + 1} = \frac{4}{\sqrt{2}} = 2
\]
4. There is a number $c > 0$ such that the triangle in the first quadrant bounded by the axes and the tangent line to the curve $y = x^{-3}$ at $(c, c^{-3})$ has area equal to 6. Find the value of $c$.

\[
y'' = -3y^{-4} \\
y - c^{-3} = 3c^{-4}(x - c) \\
-c^{-3} = -3c^{-4}(x - c) \\
c/3 = x - c \\
\chi = 4c/3 \\
\chi = 0 \\
y - c^{-3} = 3c^{-1} \\
y = 4c^{-3} \]

\[
\text{Area} = \left(\frac{1}{2}\right)\left(\frac{4c}{3}\right)\left(\frac{4}{c^3}\right) = 6
\]
\[
\frac{8}{3c^a} = 6 \\
c^a = \frac{8}{18} = \frac{4}{9} \\
c = \sqrt[3]{\frac{4}{9}}
\]
5. Use the definition of the derivative and properties of limits to prove that if

\[ f(x) = \frac{1}{g(x)} \]

when \( g(x) \neq 0 \), then

\[ f'(x) = -\frac{g'(x)}{(g(x))^2} \]

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
  &= \lim_{h \to 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h} \\
  &= \lim_{h \to 0} \frac{g(x) - g(x+h)}{h} \cdot \frac{1}{g(x+h)g(x)} \\
  &= -\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \cdot \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \\
  &= -g'(x) \left(\frac{1}{g(x)}\right)^2
\end{align*}
\]