1. Find the (absolute) maximum and minimum of 
\[ f(x) = \frac{\sin x + \cos x}{2} \]
on the interval \([0, \pi]\), showing your work.

2. Use the Mean Value Theorem to prove that if \(f(1) = 3\) and \(f'(x) < -1\) for all \(x \geq 0\), then \(f(4)\) is negative.

3. A function is defined by 
\[ f(x) = \frac{x}{\sqrt{2x^2 + 1}}. \]
(a) Find the horizontal asymptotes of \(f(x)\), if it has any.
(b) Given that 
\[ f'(x) = \frac{1}{\sqrt{(2x^2 + 1)^3}}, \]
determine the intervals on which \(f(x)\) is concave up and the intervals on which \(f(x)\) is concave down.

4. Given that 
\[ f''(x) = \sin x + 6x, \]
\(f'(\pi) = 1\) and \(f(\pi) = 0\), find \(f(x)\).

5. A kite 100 feet above the ground moves horizontally at a rate of 8 feet per second. At what rate is the angle \(\theta\) between the kite string and the (horizontal) ground changing when the length of the string is 200 feet? (Note: Think of the kite string as a straight line from a point on the ground to the kite.)