Meeting 2: Meeting Mr. No and drawing conclusions

January 14, 2010

Mister No is a funny character who replies with the *opposite* to every statement that he hears.

It does not matter to Mr. No whether the statement is a truth or a lie.

Here are some examples of his answers:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Mr. No’s response (Opposite statement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All children like ice cream</td>
<td>Some children don’t like ice cream</td>
</tr>
<tr>
<td>All balloons are red</td>
<td>Some balloons are not red</td>
</tr>
<tr>
<td>All fairy tales have happy end</td>
<td>Some fairy tales do not have happy end</td>
</tr>
</tbody>
</table>

1. Can you predict what Mr. No will respond to each of the following statements?

<table>
<thead>
<tr>
<th>Statement</th>
<th>Mr. No’s response (Opposite statement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students like math</td>
<td></td>
</tr>
<tr>
<td>All stars are very far away</td>
<td></td>
</tr>
<tr>
<td>All lemons are sour</td>
<td></td>
</tr>
</tbody>
</table>

2. Can you describe how Mr. No constructs the opposite of each of the statements above?
3. What would Mr. No reply to each of the following statements?

<table>
<thead>
<tr>
<th>Statement</th>
<th>Mr. No's response (Opposite statement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some cats are purple</td>
<td></td>
</tr>
<tr>
<td>Some planets have rings</td>
<td></td>
</tr>
<tr>
<td>Some cars are fast</td>
<td></td>
</tr>
</tbody>
</table>

As you have discovered, the statements

All ... are ...
Some ... are not ...

are opposites of each other. In addition, the statements

Some ... are ...
All ... are not ...

are opposites of each other.

4. What is the opposite of “There are...”? For example, how would Mr. No respond to “There are boys who likes pizza”? Start your sentence with “There are...”.

5. What is the opposite of “There is a...”? For example, how would Mr. No respond to “There is a boy who likes pizza”? Start your sentence with “There are...”. What do you notice about this problem and the previous problem?
6. Mr. No tells you a story. As usual, instead of telling how it should be, he tells the exact opposite. Can you rewrite (or retell) the original story?

Once upon a time there were no magic animals living in the enchanted forest. Some of them were not friends with each other. All animals were not very powerful. They did not help others. When there was no danger, some animals did not get together and protect the baby animals. This is not how some of the baby animals did not grow up in the enchanted forest and could not learn their magic powers.

7. Mr. Yes is good friends with Mr. No. A typical conversation between Mr. Yes and Mr. No goes something like this:

- Mr. Yes says something
- Mr. No responds in his usual way
- Mr. Yes cannot believe what Mr. No just said, so he repeats what Mr. No said as a question
- Mr. No responds
- Mr. Yes is pleased and says goodbye.

Write the conversation that Mr. Yes and Mr. No would have if Mr. Yes starts with the statement: “All days in Los Angeles are sunny days.” Why is Mr. Yes pleased in the end?
8. A statement can be either true or false. If a statement is true, this means its opposite is false. If a statement is false, this means its opposite is true.

For example, the statement: *All days in Los Angeles are rainy days.* is a false statement. A true statement is the opposite statement: *Some days in Los Angeles are not rainy days.*

If a statement is false, we can find an example that demonstrates this. This is called a counter-example. For example:

*Yesterday, it did not rain in Los Angeles.*

is a counter-example to *All days in Los Angeles are rainy days.*

For each of the statements below, state whether it is true or false. If it is false, provide the opposite statement and a counter-example.

(a) “All numbers are even numbers”.

(b) “Some numbers are not whole numbers.”

(c) “Every even number is followed by an odd number.”

(d) “Every child has a sister.”

(e) “All cats are black.”
9. Junior circle students always bring pencils to the math circle. Today, some students brought 3 pencils and some brought 4 pencils. Altogether, there are 7 students and 25 pencils. Nikki wants to know how many students brought 3 pencils and how many brought 4. Can you help her?

10. Nine points at the nodes form a square (see the picture below)

(a) Can you add several more points at the nodes in the above picture so that they form a bigger square that contains the given nine points? How many points did you add?

(b) Can you add just 4 points to the above picture so that you have a square that contains the given one?
11. There are 5 cities in a country. The cities are points lying on a circle. Each two cities are connected by a road.

(a) How many roads are there?

(b) One more city is built so that the number of cities is 6. The new city is connected by roads to all other cities. How many roads are there now?

(c) What if the number of cities increases to 7?

(d) How many roads are you adding each time?

12. Shannon wrote the following on the board:

\[ A \times B = AB. \]

She says that

- \( A \) and \( B \) are one-digit numbers;
- \( AB \) is the two-digit number consisting of digits \( A \) and \( B \);
- the sign \( \times \) denotes multiplication

Can you list all possibilities for \( A \) and \( B \) so that the equation above is correct?