Squares and Cubes

1. Below is a sequence of squares of increasing sizes:

   a. Write down the number of little squares under each big square. What is the pattern in the sizes of the big squares?

   b. What pattern do you notice in the sequence of square sizes, for example, are the sizes even numbers, odd numbers, or do they alternate?

   c. Fill in the table below for the differences in the sizes of the squares. Do you notice a pattern? How do these differences relate to the sides of the shaded squares?

| Size of 2^{nd} square – Size of 1^{st} square | Size of 3^{rd} square – Size of 2^{nd} square | Size of 4^{th} square – Size of 3^{rd} square | Size of 5^{th} square – Size of 4^{th} square |
2. Bob has a box full of 1 centimeter by 1 centimeter by 1 centimeter cubes. (This means that each side of the cube has length 1 centimeter).
   a. He wants to make a 2 centimeter by 2 centimeter by 2 centimeter cube out of these little cubes. How many cubes does he need?

   b. Now he wants to make a 3 centimeter by 3 centimeter by 3 centimeter cube out of these little cubes. How many cubes does he need?

3. Imagine a cube. Complete the following sentences:
   a. The number of vertices in a cube is:

   b. The number of edges in a cube is:

   c. The number of faces in a cube is:
4. Using a cube, make a “wrapper” for a cube: take your cube, roll it on the paper below and trace out a wrapper.

5. Which of the following shapes can you fold into a cube?
6. On the picture below (on the left) mark two opposite vertices of a cube, as in the picture on the right. Is there more than one way to do this?

7. A mouse sits on a vertex of a cube made of tasty cheese. It wants to go to the opposite vertex. What is the fastest way? (The mouse can eat through the cheese).
8. A cube is made out of sticks glued together at the vertices. An ant sits at a vertex and wants to crawl to the opposite vertex to get some sugar. What is the shortest path? Is there more than one way?

9. A spider and a fly are sitting on the opposite vertices of a solid wooden cube. Find (and draw) the shortest path from the spider to the fly. Is there more than one path?
10. Starting from the lower left vertex, can you trace along the edges and connect all the vertices together, without drawing over the same edge twice and without lifting your pencil?

11. Starting from the lower left vertex, can you trace every edge of a cube without going over the same edge twice and without lifting your pencil? Why or why not?
12. Lisa has a wooden cube of size 2 centimeter by 2 centimeter by 2 centimeter. Lisa paints the cube red (on the outside only). After that, she saws the cube into smaller cubes, each with side length 1 centimeter.
   a. How many faces of each cube are painted red?

   b. How many faces, in total, are painted red? How many faces, in total, are not painted?
13. Kate has a wooden cube of size 3 centimeter by 3 centimeter by 3 centimeter. Kate paints the cube red (on the outside only). After that, she saws the cube into smaller cubes, each with side length 1 centimeter.
   a. How many cubes are there with exactly 1 red face?
   b. How many cubes are there with exactly 2 red faces?
   c. How many cubes are there with exactly 3 red faces?
   d. How many cubes are there with no red faces?
   e. Are there any cubes that are completely red? Why or why not?
14. How many 2 centimeter by 2 centimeter by 2 centimeter cubes can Kate make with the 1 centimeter by 1 centimeter by 1 centimeter cubes from the previous problem? Are there any cubes remaining?

15. Below is wrapper for a piece of candy shaped as a cube. Which of the three wrapped candies does our wrapper correspond to?
16. Below are paths an ant walked along a cube. In each picture, what does the ant’s path look like from above the cube, looking down? Trace out the path on the dotted squares below.