Recall from last time that probability is

\[
\frac{\text{number of successful outcomes}}{\text{total number of outcomes}}
\]

1. Imagine you have a single die and you roll it once. For each of the probabilities below, describe an event that has
   a. Probability = 0?
   b. Probability = 1?
   c. Probability = \( \frac{1}{2} \)?
   d. Probability = \( \frac{1}{3} \)?
e. Probability = \( \frac{2}{3} \) ?

f. Probability = \( \frac{1}{6} \) ?

g. Can probability be less than 0? Why or why not?

h. Can probability be greater than 1? Why or why not?
2. Tina has a bag of different colored candies. In her bag, there are:
   - 3 red candies
   - 4 green candies
   - 3 blue candies

Tina decides to randomly pick a candy out of the bag.

a. What is the probability that she picks a red candy?

b. What is the probability that she picks a green candy?

c. What is the probability that she picks a red or a green candy?

d. What is the probability that she picks a blue candy?

e. How are the answers in parts (c.) and (d.) related?
f. Tina picks a green candy out of the bag and decides she doesn’t want to eat it, so she puts it back in the bag. She decides to randomly pick a candy out of the bag again. What is the probability that she picks a green candy again?

g. Tina picks a red candy out of her bag and eats it. She decides to randomly pick a candy out of the bag again. What is the probability that she picks a red candy again?

3. Nina and Gina decide to play a game. In their game, Nina has a coin and Gina has a coin. Nina flips her coin first, and then Gina flips her coin second. In the game, we write heads as the number “1” and tails as the number “2”.
   a. If Nina writes her result (1 or 2) as the tens digit and Gina write her result (1 or 2) as the ones digit, what are the possible numbers that result?
b. Now instead of writing the result as a two-digit number, they decide to write the number as a fraction:

\[
\frac{\text{number (1 or 2)}}{\text{number (1 or 2)}}
\]

Nina decides to always put her number in the numerator and Gina will always put her number in the denominator.

i. What is the probability of getting the fraction \(\frac{1}{2}\)?

ii. What is the probability of getting the number 2?

iii. What is the probability of getting the number 1?

Now Nina gets to choose where to write her number (in the numerator or denominator).

iv. Suppose Nina get a 1. Where should she put it, in the numerator or denominator, if she wants to maximize the fraction?
v. Suppose Nina gets a 2. Where should she put it, in the numerator or denominator, if she wants to maximize the fraction?

4. Nina and Gina decide to change their game. Instead of flipping a coin, they now each have a standard 6-sided die and take turns rolling their die. To simplify the game, Nina decides to always put her number in the numerator and Gina will always put her number in the denominator.

a. How many different outcomes (pairs of numbers from 1 through 6) can they get? (This is without simplifying the fraction. In the next part, we will simplify the fraction).

b. Fill out the table below with the possible fractions that result. Be sure to simplify your fractions!
c. How many different outcomes are there after you have simplified your fractions?

d. Using your table, what is the probability of getting the fraction $\frac{2}{3}$? What is the probability of getting the fraction $\frac{1}{3}$? Are they equal?

e. Using your table, what is the probability of getting the fraction $\frac{1}{2}$? What is the probability of getting the number 2? Are they equal?

f. Relate the probability of getting the fractions $\frac{m}{n}$ and $\frac{n}{m}$ for any numbers $n$ and $m$. 
5. We now play a game with the word “MATH”. In this game, we take the letters M, A, T, and H and try to make smaller words out of these letters, using each letter only once.

   a. Fill in the table below for all two-letter words. (Some of these are not real English words!)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Can you come up with a formula for the number of all two-letter words? (They do not have to be real English words). You can start by thinking of how many choices you have for the first letter. Then think of how many choices you have for the second letter (once the first is chosen).

   c. In your table above, circle the two-letter words that are real words in English.

   d. If two letters from the word “MATH” are picked at random, what is the probability that the word is a real English word?
e. Can you come up with a formula for the number of all three-letter words? (They do not have to be real English words).

f. Without making a table, what are all the three-letter words that are real English words? Why are these the only ones?

g. If three letters from the word “MATH” are picked at random, what is the probability that the word is a real English word?

h. How many ways are there of re-arranging the letters in the word, “MATH”? How does this formula relate to the formulas you found for two-letter and three-letter words?
6. Imagine we have a group of four people: Aaron, Bobby, Chris, and Darren.
   a. How many ways are there of picking a committee of two people?

   b. Now imagine that two people will be picked from these four people, one to be president, the other to be vice president. How many ways are there to pick two people for these roles?

   c. What is the relationship between your results in parts (a.) and (b.)?