Some More Linear Congruences

For each of the congruences below, list all possible values of $x$ modulo $m$, and check which values give solutions to the congruence.

1. $2x \equiv 1 \pmod{3}$
2. $3x \equiv 7 \pmod{2}$
3. $2x \equiv 2 \pmod{4}$
4. $4x \equiv 3 \pmod{6}$
5. $4x \equiv 2 \pmod{6}$
6. $4x \equiv 4 \pmod{6}$
Some Facts to Prove

Prove each of the statements below:

1. If \( 3x \equiv 1 \pmod{4} \), then \( x \equiv 3 \pmod{4} \).
2. If \( x \equiv 2 \pmod{3} \), then \( x \) is not divisible by 6.
3. If \( x \equiv 4 \pmod{12} \), then \( x \) is not a prime number.
4. If \( x \) ends in a 0, then \( x \equiv 0, 5, 10, 15, \) or \( 20 \pmod{25} \).
5. If \( x \equiv 2 \pmod{6} \) and \( x \equiv 3 \pmod{5} \), then \( x \equiv 8 \pmod{30} \).
6. The smallest positive integer \( x \) which is congruent to 4 \( \pmod{100} \), and congruent to 2 \( \pmod{7} \), is 604.
7. In the previous problem, there is no largest such positive integer \( x \).