Suppose we want to find the shortest path from \( a \) to \( z \).

1. \( L(a) = 0 \), \( L(b) = L(z) = \infty \)

2. a) \( \text{Circle } L(a) = 0 \)
   
   b) \( L(b) = \min \{ \infty, 0 + 3 \} = 3 \)

   \( L(z) = \min \{ \infty, 0 + 2 \} = 2 \)

2. a) \( \text{Circle } L(z) = 2 \).

Algorithm is done! But the shortest path is \((a, b, z)\), with length 1.
2) \( a_n = 3a_{n-1} - 2a_{n-2} - 2n \)

1) \( \text{solve } \begin{align*} \left( t^2 - 3t + 2 \right) &= 0 \\ (t - 1)(t - 2) &= 0 \\ t &= 1, 2 \\ \Rightarrow u_n &= b + d2^n \end{align*} \)

2) \( g(n) = An + B \)

3) \( \text{replace } g(n) \text{ due to conflict w/ } u_n \text{ terms:} \)

\[ g(n) = An^2 + Bn \]

4) \( \text{plug in } g(n): \)

\[ \begin{align*} An^2 + Bn &= 3 \left[ A(n-1)^2 + B(n-1) \right] - 2 \left[ A(n-2)^2 + B(n-2) \right] \\ -2n \\ An^2 + Bn &= 3A(n^2 - 2n + 1) + 3B(n-1) - 2A(n^2 - 4n + 4) - 2B(n-2) \\ -2n \\ 0 &= \left[ -A + 3A - 2A \right] n^2 + \left[ -B - 6A + 3B + 8A - 2B - 2 \right] n \\ &\quad + \left[ 3A - 3B - 8A + 4B \right] \\ \text{each term in brackets must } &= 0; \text{ so:} \\
2A - 2 &= 0 \quad &-5A + B &= 0 \\
A &= 1 \quad &B &= 5A = 5 \\ \Rightarrow g(n) &= n^2 + 5n \]
2) (cont'd)

a) \( W_n = U_n + g(n) = b + d \cdot 2^n + n^2 + 5n \)

b) \( W_0 = b + d = 1 \Rightarrow b = 1 - d \)
\( W_1 = b + 2d + 1 + 5 = 2 \)
\( d = 2 - 1 - 1 - 5 = -5 \)
\( b = 1 + 5 = 6 \)

\( \Rightarrow W_n = -5 + 6(2^n) + n^2 + 5n \)

C) THIS R.R. IS NOT LINEAR W/ CONST. COEFFS.,
SO USE ITERATION:

\[ a_n = 3n \cdot a_{n-1} \]
\[ = 3n \cdot 3(n-1) \cdot a_{n-2} \]
\[ = 3^n \cdot n \cdot (n-1) \cdot (n-2) \cdot a_{n-3} \]
\[ = \ldots \]
\[ = 3^n \cdot n! \cdot a_0 \]

\( \Rightarrow a_0 = 2 \Rightarrow a_n = 2 \cdot 3^n \cdot n! \)
\[
\binom{35}{29} (3x)^6 (-y)^{29} \\
= \frac{35!}{29!6!} \cdot 3^6 (-1)^{29} x^6 y^{29} \\
= -3^6 \cdot \frac{35!}{29!6!} x^6 y^{29}
\]

So coefficient is \[\frac{-3^6 (35!)}{29!6!}\]
4) Give 5 bars to Alf, then distribute the remaining 31:

Label each bar A, B, C, or D according to who gets it.

So \( t = 4 \), \( k = 31 \)

\[
\binom{k+t-1}{k} = \binom{34}{31} = \frac{34!}{31!3!} = \frac{34 \cdot 33 \cdot 32}{6}
\]

b) First, compute the # ways to distribute when Alf gets at least 16 bars.
We give 16 to Alf then distribute the remaining 20:

\( t = 4 \), \( k = 20 \)

\[
\binom{k+t-1}{k} = \binom{23}{20} = \frac{23!}{20!3!} = \frac{23 \cdot 22 \cdot 21}{6}
\]

So Alf getting 25 bars but < 16 bars has

\[ \# \text{ distributions} = (\# \text{ dist. w/ Alf 25}) - (\text{ Alf 25}) \]

\[ = \frac{34 \cdot 33 \cdot 32 - 23 \cdot 22 \cdot 21}{6} \]

\[ = \frac{23 \cdot 22 \cdot 21}{6} \]

\[ = \frac{45}{5 \cdot 20 \cdot 16} \]

\[ = \frac{45!}{8! \cdot 20! \cdot 16!} \]
a) Try to divide \( V \) into two sets \( V_1, V_2 \) satisfying bipartite def.

Suppose \( d \in V_1 \). Then since \( d \) is adjacent to \( a, c, \) and \( e \), we must have \( a, c, e \in V_2 \).

But \( c \) is adjacent to \( e \), which cannot happen if \( c, e \in V_2 \). Thus \( G \) cannot be bipartite.

b) Yes. We have drawn it so edges do not cross so it must be planar.

Alternatively: \( G \) is homeomorphic to \( G' \):

\( G' \) has only 4 vertices, so it must be planar by Kuratowski's thm., since it cannot contain any subgraphs homeomorphic to \( K_{3,3} \) or \( K_5 \).
c) We can find a Hamiltonian cycle if we can delete edges until every vertex has degree 2.

Deleting the 2 marked edges, we get the desired subgraph.

=> YES

d) We need every vertex to have even degree.

b, c, d, e all have odd degree, so we have to add at least 2 edges to make them all even. We have found 2 edges here, so we are done.
There are three possible functions $f$ that satisfy this:

$$f_1 = \mathcal{E}(a, a), (b, d), (d, b), (c, c), (e, e)$$

$$f_2 = \mathcal{E}(a, a), (b, b), (c, e), (e, c), (d, d)$$

$$f_3 = \mathcal{E}(a, a), (b, d), (d, b), (c, e), (e, c)$$
Let \( G_1 = (V_1, E_1) \), \( G_2 = (V_2, E_2) \) and let \( f : V_1 \to V_2 \), \( g : E_1 \to E_2 \) define the isomorphism.

We want to prove: given \( v, w \in V_2 \), there is a path from \( v \) to \( w \) in \( G_2 \).

Since \( f \) is a bijection, it is onto, so there exist \( x, y \in V_1 \) such that \( f(x) = v \) and \( f(y) = w \). By the assumption, there exists a path from \( x \) to \( y \) in \( G_1 \), since \( G_1 \) is connected:

\[
(x, e_1, x_1, e_2, x_2, \ldots, x_{n-1}, e_n, y)
\]

Then

\[
(u, g(e_1), f(x_1), g(e_2), f(x_2), \ldots, f(x_{n-1}), g(e_n), v)
\]

defines a path from \( u \) to \( v \) in \( G_2 \), since by definition of isomorphism, each \( e_i \) is incident on \( x_{i-1} \) and \( x_i \).

\[
\Rightarrow\text{ each } g(e_i) \text{ is incident on } f(x_{i-1}) \text{ and } f(x_i).
\]

Thus we have found the desired path.