Practice Midterm 1

Math 61, Section 2, Fall 2009

• The Midterm will take place 12:00-12:50 pm on Monday, April 20 in Boelter 3400. Note this location is not our regular class meeting room.

• You will need to bring a pencil and your student ID card to the midterm. No other materials or devices will be allowed.

• All of the following sections from Discrete Source are fair game for the midterm: Mathematical Induction, Sets, Functions, Sequences and Strings, Relations, Equivalence Relations, Matrices of Relations, Counting Methods: Basic Principles, and Permutations and Combinations.

• Disclaimer: questions on the practice midterm may not be similar to those on the actual midterm!

1. Consider 5 distinct math books and 4 distinct computer science books, to be placed on a shelf. You may leave answers in un-simplified form.

   (a) How many different ways can we place the books on the shelf?
   (b) How many different ways can the books be placed on the shelf if we start with a math book, then place a computer science book to its right, and continue alternating types of books to the right until we run out?
   (c) Suppose we place the books, then an earthquake occurs and 3 books fall onto the floor. How many different possible combinations of books can end up on the floor?

2. Prove the following:
   (a) Assume $Q$ is a transitive relation. Whenever $(x, y) \in Q \circ Q$, then $(x, y) \in Q$.
   (b) Assume $T$ is a relation and that $T \circ T$ is a subset of $T$. Then $T$ is transitive.

3. Let $X = \{a, b, c, d\}$ and let $R$ be a relation on $X$ given by $R = \{(c, a), (a, d), (c, d), (b, b)\}$.
   (a) Is $R$ a partial ordering on $X$? Justify your answer.
   (b) Give the matrix representation of $R$.
   (c) Use the matrix of $R$ to calculate the matrix of the relation $R \circ R$.

4. Prove, using induction, that the following is true for all $n = 1, 2, 3, \ldots$:

   \[
   \frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \cdots + \frac{1}{(n + 1)^2 - 1} = \frac{3}{4} - \frac{1}{2(n + 1)} - \frac{1}{2(n + 2)}
   \]

5. Let $S$ be the set of all binary strings of length 6. (for example, 110111 is in $S$.)

   (a) What is the value of $|S|$?
   (b) What is the value of $|\mathcal{P}(S)|$? You may leave your answer in un-simplified form.
   (c) Let $E$ be the relation on $S$ defined by: $sEt$ if, reading left to right, $s$ contains its first “1” in the same position as $t$. Is $E$ an equivalence relation on $S$? Justify your answer.