Consider minimizing \( f(x) \) for \( x \in S \) where \( S \) is the set of integers. Prove that every point in \( S \) is a local minimizer of \( f \).

Choose any \( y \in S \), and take \( \epsilon = \frac{1}{2} \). Now note: the set
\[
D = \{ x : x \in S \text{ and } \| x - y \| < \frac{\epsilon}{2} = \frac{1}{4} \}
\]
By the definition, \( y \) is a local minimizer if
\[
f(y) \leq f(x) \quad \forall x \in D
\]
and this is trivially true, since \( D \) consists of only the point \( y \). Thus \( y \) is a local minimizer.