Your Name: ____________________________

UCLA id: ______________________________

Math 61 Section: ______

Date: __________

The rules:
You MUST simplify completely and BOX all answers with an INK PEN.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no web access. You MUST write your name and UCLA id.
Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught writing after time get automatic 10% score deduction.

Points:

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Total: (out of 100)
Problem 1. (20 points)
Compute the number of (shortest) grid walks from (0, 0) to (9, 9) which:
\(a\) do not go through any of the other diagonal points \((1, 1), (2, 2), \ldots, (8, 8)\)
\(b\) stay on or above \(y = x - 1\) diagonal
\(c\) stay on or above \(y = x\) diagonal AND do not go through \((6, 6)\)
\(d\) stay on or above \(y = x\) diagonal AND on or below \(y = x + 1\) diagonal.

Note: You can use the formula for Catalan numbers or write the answer in terms of \(C_n\)’s.
\[
C_n = \frac{1}{n+1} \binom{2n}{n}.
\]
Problem 2. (20 points)

Compute the number of subgraphs of $G$ isomorphic to $H$, where

a) $G = K_{7,9}, H = C_4$

b) $G = K_{7,9}, H = P_4$

c) $G = C_9, H = P_4$

d) $G = K_9, H = K_{2,3}$
Problem 3. (15 points)
Let $a_1 = 2$, $a_2 = 7$, $a_{n+1} = a_n + 2a_{n-1}$. Solve this LHRR and find a closed formula for $a_n$. 
**Problem 4.** (15 points)

Decide whether the following pairs of graphs on 8 vertices are isomorphic or non-isomorphic.

\(a)\)

\[\text{Graph 1}\]

\[\text{Graph 2}\]

\(b)\)

\[\text{Graph 3}\]

\[\text{Graph 4}\]

**Important:** In case of isomorphism, you must write a bijection in the figure above (in ink). No need for further arguments. In case non-isomorphism, you must say so and present an argument why two graphs are not isomorphic.
Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T F (1) Isomorphic graphs have the same number of edges.

T F (2) Isomorphic graphs have the same number of connected components.

T F (3) Isomorphic graphs have the same number of 4-cycles.

T F (4) $F_n \leq C_n$ for all integer $n$.

T F (5) Sequence $(3, 3, 3, 3, 3)$ is a valid score of a simple graph.

T F (6) Sequence $(4, 4, 4, 4, 2)$ is a valid score of a simple graph.

T F (7) Sequence $(4, 4, 4, 2, 2)$ is a valid score of a simple graph.

T F (8) Sequence $(4, 4, 2, 2, 2)$ is a valid score of a simple graph.

T F (9) Sequence $(2, 2, 2, 0, 0)$ is a valid score of a simple graph.

T F (10) Graph $C_8$ is a subgraph of $K_{7,7}$.

T F (11) Graph $C_8$ is a subgraph of $K_{9,3}$.

T F (12) Graph $P_8$ is a subgraph of $K_{9,3}$.

T F (13) Graph $K_4$ is a subgraph of $K_{7,7}$.

T F (14) Graph $K_9$ has 72 edges.

T F (15) Catalan numbers modulo 2 are periodic with period 6.